Advanced Medical Image Registration Techniques

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AAPM 2016, 58th Annual Meeting & Exhibition, Session in Memory of Jean Pouliot: Next-Generation Deformable Image Registration

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Outline

- Introduction to Image Registration
- Symmetric Image Registration
- Jacobian of a Transformation
- Large Deformation Diffeomorphic Image Registration
- Currents
- Applications

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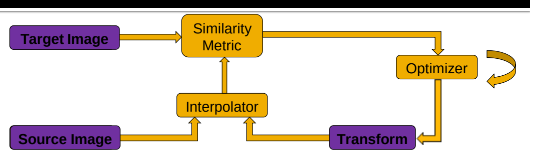
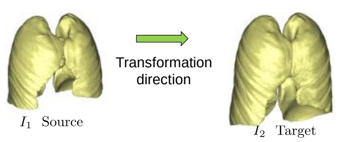
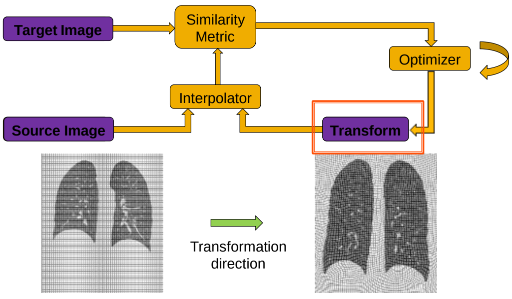
Registration Framework





3

Registration Framework



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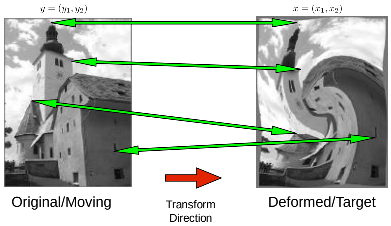
Transformation

- A transformation establishes a unique correspondence between each (y_1,y_2) coordinate in the original image with a point (x_1,x_2) in the deformed/transformed image.



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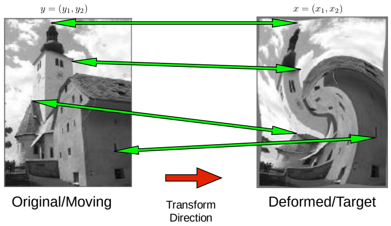
Transformation

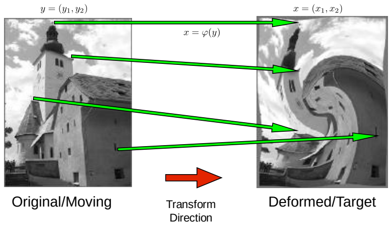
- There are two types of transformations that produce the same deformed image:
  - A push-forward transformation pushes an intensity from (y_1,y_2) to (x_1,x_2).
  - A pullback transformation pulls an intensity from (y_1,y_2) to (x_1,x_2).
- Push-forward and pullback transformations are inverses of each other.
- It is convenient to use a push-forward transformation when describing how an image transforms from its original to its deformed shape.
- However, a pullback transformation is used to compute the deformed image.



Push-forward Transformation

A push-forward transformation is a vector valued function φ that maps a point (y_1, y_2) in the original image to a point (x_1, x_2) in the deformed image.

$$x_1 = \varphi_1(y_1, y_2)$$
$$x_2 = \varphi_2(y_1, y_2)$$



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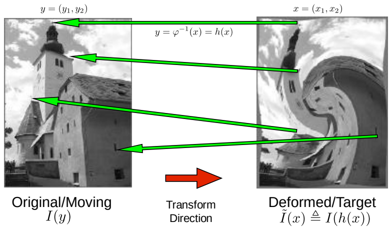
Pullback Transformation

- A pullback transformation is a vector valued function $\varphi^{-1} = h$ that maps a point x to y.
- It is used to pull back the intensity value at y to x to create the deformed image.



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Suppose we want to actually apply a transform to an image. How should we do it?

Directly using the push-forward transform may not work as you would expect...

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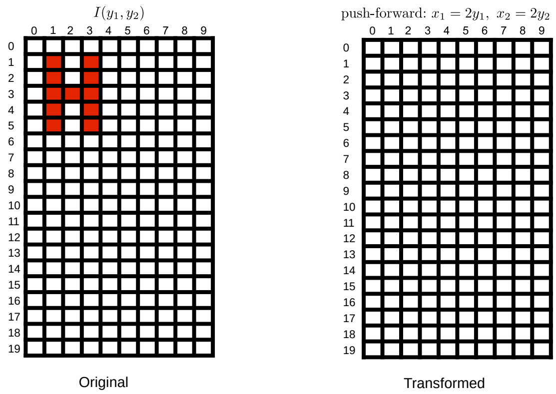
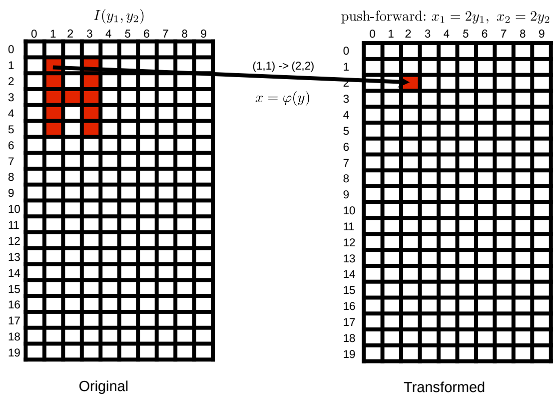
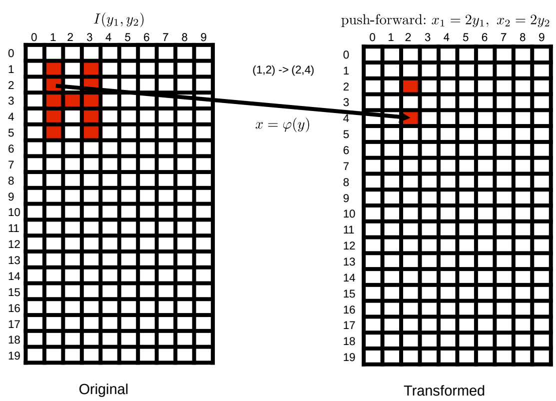
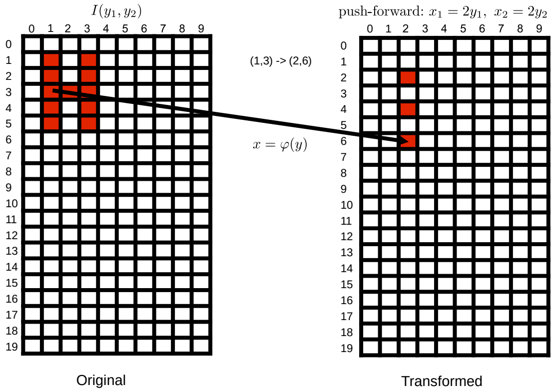
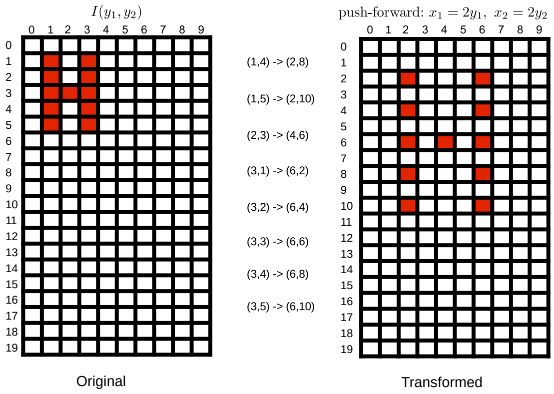
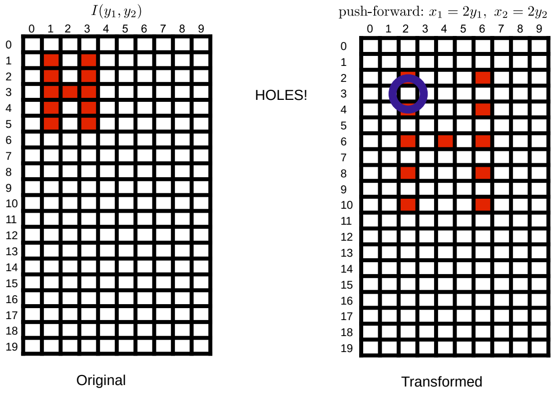

$I(y_1, y_2)$
push-forward: $x_1 = 2y_1,\ x_2 = 2y_2$
Original
Transformed


$I(y_1, y_2)$
push-forward: $x_1 = 2y_1,\ x_2 = 2y_2$
(1,1) -> (2,2)
$x = \varphi(y)$
Original
Transformed


$I(y_1, y_2)$
push-forward: $x_1 = 2y_1,\ x_2 = 2y_2$
(1,2) -> (2,4)
$x = \varphi(y)$
Original
Transformed


$I(y_1, y_2)$
push-forward: $x_1 = 2y_1,\ x_2 = 2y_2$
(1,3) -> (2,6)
$x = \varphi(y)$
Original
Transformed


$I(y_1, y_2)$
push-forward: $x_1 = 2y_1,\ x_2 = 2y_2$
(1,4) -> (2,8)
(1,5) -> (2,10)
(2,3) -> (4,6)
(3,1) -> (6,2)
(3,2) -> (6,4)
(3,3) -> (6,6)
(3,4) -> (6,8)
(3,5) -> (6,10)
Original
Transformed


$I(y_1, y_2)$
push-forward: $x_1 = 2y_1,\ x_2 = 2y_2$
HOLES!
Original
Transformed

Suppose we want to actually apply a transform to an image. How should we do it?

Now lets see what happens using the pullback transformation.

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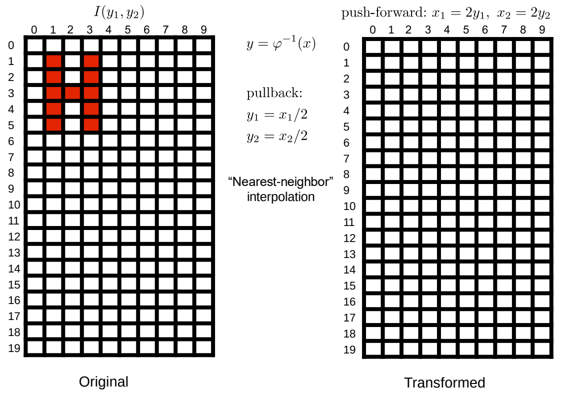

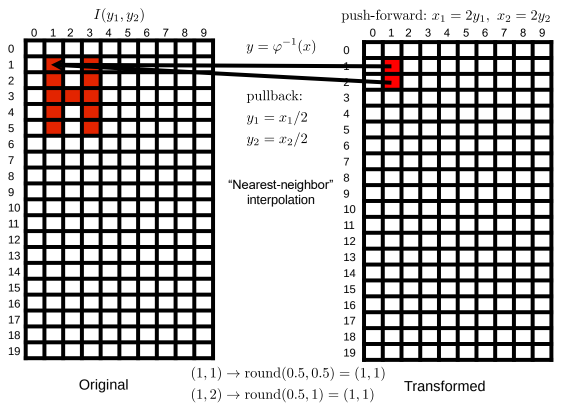

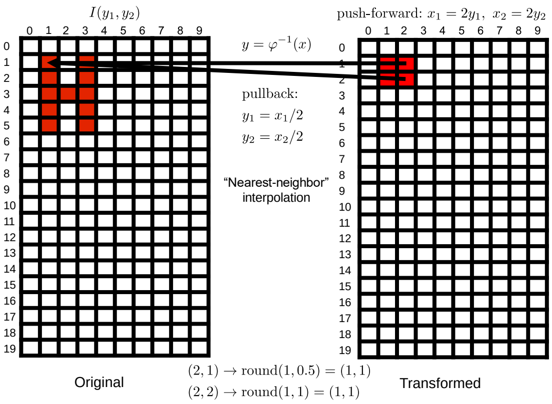

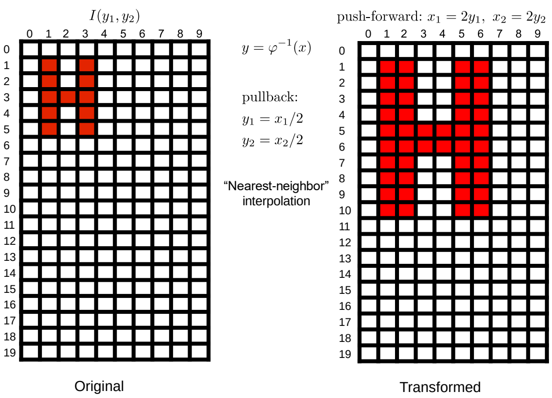

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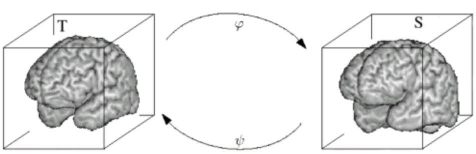
Symmetric Image Registration

- Many image registration algorithms are not symmetric. That is, registering T to S (called the forward direction) gives a different correspondence than registering S to T (called the reverse direction).
- The **inverse consistency error** measures the error in the correspondence between the forward transformation φ and the reverse transformation ψ.



Christensen and Johnson, Consistent Image Registration, IEEE TMI 20(7), July 2001, pp. 568-582.

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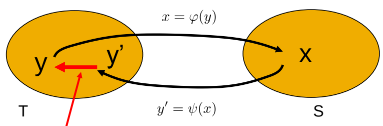
Inverse Consistency Error*

The **inverse consistency error** is defined to be the difference between the composition of the forward and reverse transformations $\varphi \circ \psi$, and the **identity transformation**.

NOTE: φ and ψ are both push-forward transformations. The inverse consistency error can equivalently be stated using pullback transformations.



Inverse Consistency Error $= ||y - y'||$ where $y' = \psi(\varphi(y))$

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Push-forward Transformation Jacobian Determinant*

The determinant of the push-forward transformation Jacobian describes the pointwise **expansion or contraction** produced by the transformation at y.

$$J(y) = \left[\frac{\partial \varphi(y)}{\partial y}\right] = \begin{bmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} & \frac{\partial x_1}{\partial y_3} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} & \frac{\partial x_2}{\partial y_3} \\ \frac{\partial x_3}{\partial y_1} & \frac{\partial x_3}{\partial y_2} & \frac{\partial x_3}{\partial y_3} \end{bmatrix} = \text{Push-forward Jacobian}$$

Example interpretations of the push-forward Jacobian determinant:

- det(J) = 0 -> transformation is singular (no inverse)
- 0 < det(J) < 1 -> area becomes smaller
- det(J) = 1 -> area is preserved
- det(J) > 1 -> area becomes larger

NOTE: The Jacobian determinant is occasionally referred to as "the Jacobian."

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Pullback Transformation Jacobian Determinant*

The determinant of the pullback transformation Jacobian describes the pointwise **expansion or contraction** produced by the transformation at x.

$$J(x) = \left[\frac{\partial \varphi^{-1}(x)}{\partial x}\right] = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \frac{\partial y_1}{\partial x_3} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \frac{\partial y_2}{\partial x_3} \\ \frac{\partial y_3}{\partial x_1} & \frac{\partial y_3}{\partial x_2} & \frac{\partial y_3}{\partial x_3} \end{bmatrix} = \text{Pullback Jacobian}$$

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NOTE: The matrix inverse of the push-forward Jacobian matrix is the Jacobian matrix of the pullback transformation if the transformation is a diffeomorphism.

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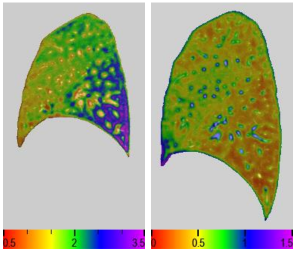
Jacobian Maps



Inhalation Exhalation

- Jac > 1: Expansion (purple)
- Jac < 1: Contraction (red)
- Jac = 1: No volume change

- An obvious dorsal to ventral gradient is noticed
- Vessels have small volume change.

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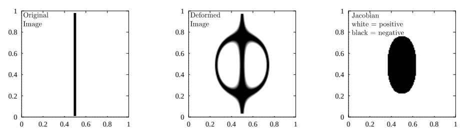
Diffeomorphism*

Definition: A **diffeomorphism** is a differentiable transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$, a bijection (a one-to-one and onto map) and its inverse is differentiable.

Transforming an image with a diffeomorphism ensures that

- Objects do not rip apart.
- Neighborhood structure is maintained.
- Objects/structures change size but do not disappear or get turned inside out.

Example:
$h(x) = (x_1 + \frac{1}{4}\sin 2\pi x_1 \sin \pi x_2, x_2)$ for $\vec{x} \in \Omega = [0,1]^2$ is not a diffeomorphism.



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Parameterizing Diffeomorphisms*

In the large deformation setting:

- A diffeomorphic transformation is most often parameterized by a **velocity field.**

Beg, Miller, Trouve, Younes. Computing Large Deformation Metric Mappings via Geodesic Flows of Diffeomorphisms. IJCV, 61(2); 2005.
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Parameterizing Diffeomorphisms*

In the large deformation setting:

- A diffeomorphic transformation is most often parameterized by a **velocity field.**
- A flow ϕ_t is computed from a given a time-dependent velocity vector field v_t using the evolution equation

$$\frac{d}{dt}\phi_t(x) = v_t(\phi_t(x)) \qquad 0 \leq t \leq 1$$

- The solution to this equation is a flow ϕ_t with initial point $\phi_0 = \mathit{Identity\ map}$ and ending point $\phi_1 =$ Desired transformation φ.

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- The solution to this equation is a flow ϕ_t with initial point $\phi_0 = \mathit{Identity\ map}$ and ending point $\phi_1 =$ Desired transformation φ.
- Thus, the goal is to find the velocity field v_t that generates the diffeomorphism φ that registers the moving image to the target image.

Beg, Miller, Trouve, Younes. Computing Large Deformation Metric Mappings via Geodesic Flows of Diffeomorphisms. IJCV, 61(2); 2005.

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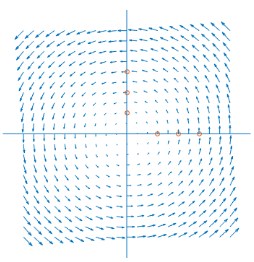
Simple Flow Example



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LDDMM Image Registration

LDDMM = Large Deformation Diffeomorphic Metric Mapping

The goal is to find v_t that minimizes

$$C(v_t) = \int_\Omega (I_0 \circ \phi_1^{-1} - I_1)^2 dx + \alpha \int_0^1 ||v_t||_V^2 dt$$

Such that

$$\frac{d}{dt}\phi_t(x) = v_t(\phi_t(x))$$

Beg, Miller, Trouve, Younes. Computing Large Deformation Metric Mappings via Geodesic Flows of Diffeomorphisms. IJCV, 61(2); 2005.

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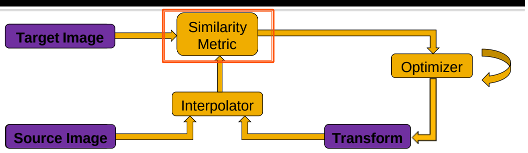
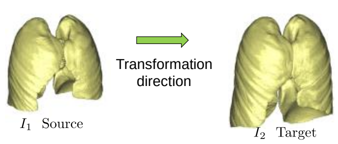
Registration Framework





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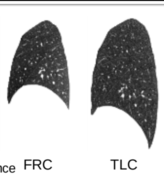
Similarity Metrics

- Intensity-based similarity metrics
  - SSD --- Sum of Squared intensity Difference
  - MI --- Mutual Information
  - **SSTVD** --- Sum of Squared Tissue Volume Difference/ Mass preserving image registration
  - **SSVMD** --- Sum of Squared Vesselness Measure Difference
- Curve and Surface-based similarity metric:
  - **Currents**
- 4D similarity metric
  - **Group SSD**



Consider CT intensity change due to different air content during respiration!

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Curve and Surface Registration Using Currents

SECOND INTERNATIONAL WORKSHOP ON PULMONARY IMAGE PROCESSING -15-

Curve- and Surface-based Registration of Lung CT images via Currents

Vladlena Gorbunova[1], Stanley Durrleman[2,3], Pechin Lo[1], Xavier Pennec[2], and Marleen de Bruijne[1,4]

[1] Department of Computer Science, University of Copenhagen, Denmark
[2] Asclepios, INRIA Sophia Antipolis, France
[3] Centre de Mathematique et Leurs Application, ENS Cachan, France
[4] Biomedical Imaging Group Rotterdam, Erasmus MC, Rotterdam, the Netherlands

And

S. Durrleman. Statistical models of currents for measuring the variability of anatomical curves, surfaces and their evolution. PhD thesis, 2009.

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Currents

- In 2005, J. Glaunès and M. Vaillant introduced the concept of currents in the field of Computational Anatomy.
- The currents similarity measure does not assume point correspondence between meshes or polygonal curves.
- They proposed to use the framework of reproducing kernel Hilbert spaces (RKHS) to give a tractable formula of this metric as well as its derivatives.

Glaunès. Transport par difféomorphismes de points, de mesures et de courants pour la comparaison de formes et l'anatomie numérique . PhD thesis, Université Paris 13, 2005.
Vaillant and Glaunès. Surface Matching via Currents . In Proceedings of IPMI 2005, LNCS 3565. Springer.

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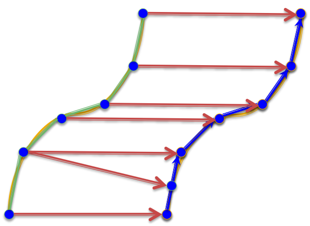
How to Represent a Curve?



Points and line segments

Points, line segments and tangents

How do we define correspondence between two lines?

Pointwise?

What if there are a different number of points?

One solution is to use currents to represent a curve.
Currents do not assume correspondence between curves

But, what is a current?

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Vessel Tree Currents

The current of a curve L is defined as the path integral of a vector field ω along the curve L and is given by

$$L(\omega) = \int_L \omega(x)^t \tau(x) d\lambda(x)$$

where τ is the tangent to L at x and $d\lambda$ the Lebesgue measure on L.

- Think of the object L as the current.
- L is acting on ω to produce a real number.

- Abuse of notation: L represents the curve.

Vessel Tree Currents

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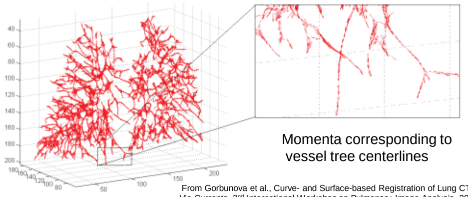


Momenta corresponding to vessel tree centerlines

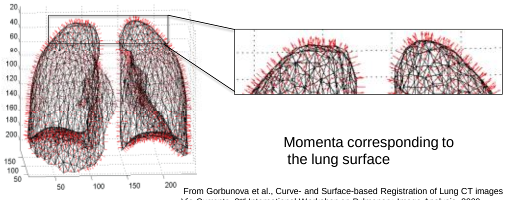
From Gorbunova et al., Curve- and Surface-based Registration of Lung CT images Via Currents. 2nd International Workshop on Pulmonary Image Analysis, 2009

Lung Surface Currents

The current of a surface S is defined by the flux of a vector field ω through S and is given by

$$S(\omega) = \int_S \omega(x)^t n(x) d\lambda(x)$$

where n is the normal to S at x and $d\lambda$ the Lebesgue measure on S.



Momenta corresponding to the lung surface

From Gorbunova et al., Curve- and Surface-based Registration of Lung CT images Via Currents. 2nd International Workshop on Pulmonary Image Analysis, 2009

Reproducing Kernel Hilbert Space

- Let W denote the space of test functions which is the set of square integrable functions convolved with a smoothing kernel.
- Formally, W is defined as a Reproducing Kernel Hilbert Space (RKHS).
- Assume a Gaussian kernel: $K^W(x,y) = \exp(-|x-y|^2/\lambda_W^2)Id$.

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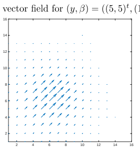
- The RKHS of vector fields W have two important properties:
  - W is the closed span of the vector fields of the form $\omega(x) = K^W(x,y)\beta$ for any fixed points y and vector β. The pair (y,β) is called momentum.

Example:
momentum $= (y,\beta) = ((5,5)^t,(1,1)^t)$
kernel: $\lambda_W = 2.82$

$$\omega(x) = \begin{bmatrix}1\\1\end{bmatrix} \exp\left(-\frac{(x_1-5)^2+(x_2-5)^2}{\lambda_W^2}\right)$$

basis vector field for $(y,\beta) = ((5,5)^t,(1,1)^t)$



Reproducing Kernel Hilbert Space

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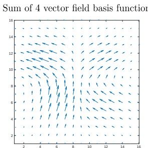
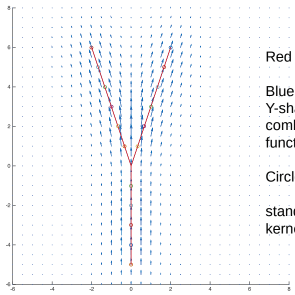
Example 2:
momentum 1 $= (y_1,\beta_1) = ((4,10)^t,(-3,1)^t)$
momentum 2 $= (y_2,\beta_2) = ((10,10)^t,(2,1)^t)$
momentum 3 $= (y_3,\beta_3) = ((5,5)^t,(1,3)^t)$
momentum 4 $= (y_4,\beta_4) = ((11,5)^t,(-2,1)^t)$
kernel: $\lambda_W = 2.82$

$$\omega(x) = \sum_{i=1}^{4} \beta_i \exp\left(-\frac{|x-y_i|^2}{\lambda_W^2}\right)$$

Sum of 4 vector field basis functions



Reproducing Kernel Hilbert Space



Red = Y-shaped object

Blue = vector field representation of Y-shaped object via a linear combination of Gaussian kernel basis functions.

Circle = location of a momentum

standard deviation of Gaussian kernel = 1.2

Reproducing Kernel Hilbert Space

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  - W is the closed span of the vector fields of the form $\omega(x) = K^W(x,y)\beta$ for any fixed points y and vector β. The pair (y,β) is called momentum.
  - W is provided with an inner product defined by

$$< K^W(.,x)\alpha, K^W(.,y)\beta >_W = \alpha^t K^W(x,y)\beta \qquad (1)$$

Example:

$$\left\langle \begin{bmatrix}1\\2\end{bmatrix} \exp\left(-\frac{(x_1-5)^2+(x_2-6)^2}{4}\right), \begin{bmatrix}3\\0\end{bmatrix} \exp\left(-\frac{(x_1-7)^2+(x_2-8)^2}{4}\right)\right\rangle_W$$

$$= \begin{bmatrix}1 & 2\end{bmatrix} \begin{bmatrix}3\\0\end{bmatrix} \exp\left(-\frac{(5-7)^2+(6-8)^2}{4}\right) = 0.4$$

Reproducing Kernel Hilbert Space

- Let W denote the space of test functions which is the set of square integrable functions convolved with a smoothing kernel.
- Formally, W is defined as a Reproducing Kernel Hilbert Space (RKHS).
- Assume a Gaussian kernel: $K^W(x,y) = \exp(-|x-y|^2/\lambda_W^2)Id$.
- The RKHS of vector fields W have two important properties:
  - W is the closed span of the vector fields of the form $\omega(x) = K^W(x,y)\beta$ for any fixed points y and vector β. The pair (y,β) is called momentum.
  - W is provided with an inner product defined by

$$< K^W(.,x)\alpha, K^W(.,y)\beta >_W = \alpha^t K^W(x,y)\beta \qquad (1)$$

  - If ω denotes the vector field $K^W(.,y)\beta$ in Eq. 1, this equation can be written as

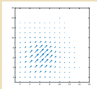
$$< K^W(.,x)\alpha, \omega >_W = \alpha^t \omega(x) \qquad (2)$$

which is called the "reproducing property".

Reproducing Kernel Hilbert Space

W = RKHS

Basis: $\omega = K^W(\cdot, y)\alpha$
Smooth vector fields

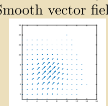


$< \omega, \omega' >_W$

Dual Space = Space of Currents

W = RKHS

Basis: $\omega = K^W(\cdot, y)\alpha$
Smooth vector fields



$< \omega, \omega' >_W$

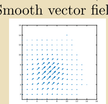
$\mathcal{L}_W$ →

W* =
Dual Space of W
= Space of Currents

Currents Basis

W = RKHS

Basis: $\omega = K^W(\cdot, y)\alpha$
Smooth vector fields



$< \omega, \omega' >_W$

$\mathcal{L}_W$ →

W* =
Dual Space of W
= Space of Currents

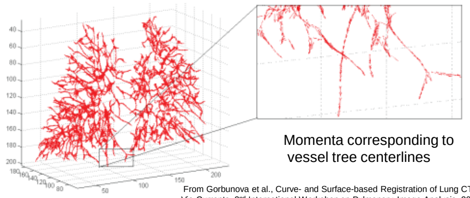
Basis: $\delta_x^\alpha = \mathcal{L}_W(K^W(., x)\alpha)$
Linear functionals

Discrete Vessel Tree Currents

The discrete approximation of the current L is given by

$$L(\omega) = \int_L \omega(x)^t \tau(x) d\lambda(x) \sim \sum_k \delta_{x_k}^{\tau_k}(\omega) = \sum_k \omega(x_k)^t \tau_k$$

where τ_k is the tangent to L at x_k.



Momenta corresponding to vessel tree centerlines

From Gorbunova et al., Curve- and Surface-based Registration of Lung CT images Via Currents. 2nd International Workshop on Pulmonary Image Analysis, 2009

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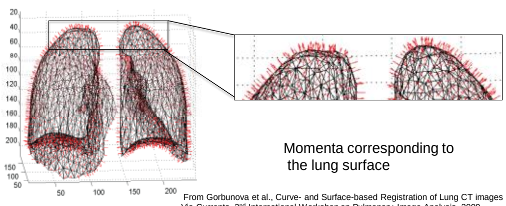
Discrete Lung Surface Currents

The discrete approximation of the current S is given by

$$S(\omega) = \int_S \omega(x)^t n(x) d\lambda(x) \sim \sum_k \delta_{x_k}^{n_k}(\omega) = \sum_k \omega(x_k)^t n_k$$

where n_k is the normal to S at x_k.



Momenta corresponding to the lung surface

From Gorbunova et al., Curve- and Surface-based Registration of Lung CT images Via Currents. 2nd International Workshop on Pulmonary Image Analysis, 2009

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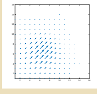
Currents Inner Product

W = RKHS

Basis: $\omega = K^W(\cdot, y)\alpha$
Smooth vector fields



$< \omega, \omega' >_W$

$\mathcal{L}_W$ →

W* =
Dual Space of W
= Space of Currents

Basis: $\delta_x^\alpha = \mathcal{L}_W(K^W(., x)\alpha)$
Linear functionals

$< \delta_x^\alpha, \delta_y^\beta >_{W^*}$

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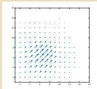
Currents Inner Product

W = RKHS

Basis: $\omega = K^W(\cdot, y)\alpha$
Smooth vector fields



$< \omega, \omega' >_W$

$\mathcal{L}_W$ →

← $\mathcal{L}_W^{-1}$

W* = Dual Space of W = Space of Currents

Basis: $\delta_x^\alpha = \mathcal{L}_W(K^W(.,x)\alpha)$
Linear functionals

$$< \delta_x^\alpha, \delta_y^\beta >_{W^*}$$
$$=< \mathcal{L}_W^{-1}(\delta_x^\alpha), \mathcal{L}_W^{-1}(\delta_y^\beta) >_W$$
$$=< K^W(.,x)\alpha, K^W(.,y)\beta >_W$$
$$= \alpha^t K^W(x,y)\beta$$

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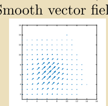
Induced Currents Norm

W = RKHS

Basis: $\omega = K^W(\cdot, y)\alpha$
Smooth vector fields



$< \omega, \omega' >_W$

$||\omega||_W^2 =< \omega, \omega >_W$

$\mathcal{L}_W$ →

← $\mathcal{L}_W^{-1}$

W* = Dual Space of W = Space of Currents

Basis: $\delta_x^\alpha = \mathcal{L}_W(K^W(.,x)\alpha)$
Linear functionals

$$< \delta_x^\alpha, \delta_y^\beta >_{W^*}$$
$$=< \mathcal{L}_W^{-1}(\delta_x^\alpha), \mathcal{L}_W^{-1}(\delta_y^\beta) >_W$$
$$=< K^W(.,x)\alpha, K^W(.,y)\beta >_W$$
$$= \alpha^t K^W(x,y)\beta$$

$$||T||_{W^*}^2 =< T, T >_{W^*}$$

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Current Similarity Cost Function

The joint cost function for registering both line and surface currents from source currents (L_1, S_1) to target currents (L_2, S_2) is defined by

$$C(v) = \left|\left|\varphi_* L_1 - L_2\right|\right|_{W_L^*}^2 + \alpha \left|\left|\varphi_* S_1 - S_2\right|\right|_{W_S^*}^2 + \beta \int_0^1 \left|\left|v_t\right|\right|_V^2 dt \qquad (1)$$

where the transformation $\varphi = \phi_1$ satisfies

$$\frac{d}{dt}\phi_t(x) = v_t(\phi_t(x)) \qquad (2)$$

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Current Similarity Cost Function

The joint cost function for registering both line and surface currents from source currents (L_1, S_1) to target currents (L_2, S_2) is defined by

$$C(v) = \left|\left|\varphi_* L_1 - L_2\right|\right|^2_{W_L^*} + \alpha\left|\left|\varphi_* S_1 - S_2\right|\right|^2_{W_S^*} + \beta \int_0^1 \left|\left|v_t\right|\right|^2_V dt \qquad (1)$$

where the transformation $\varphi = \phi_1$ satisfies

$$\frac{d}{dt}\phi_t(x) = v_t(\phi_t(x)) \qquad (2)$$

- Curve: push-forward action is given by: $\varphi_* \delta_x^\alpha = \delta_{\varphi(x)}^{d_x\varphi(\alpha)}$
- Surface: push-forward action is given by: $\varphi_* \delta_x^\alpha = \delta_{\varphi(x)}^{|d_x\varphi| d_x\varphi(x)^{-t}(\alpha)}$

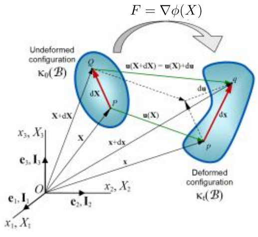
Mechanical Analysis

Deformation of a Continuum Body

- ϕ is the transformation from the undeformed configuration to the deformed configuration
- $x = \phi(X) = X + u(X)$ where u is the displacement field
- $F = \nabla\phi(X) = I + \nabla u(X)$ is the deformation gradient tensor
- F is the Jacobian of ϕ

$$F = \nabla\phi(X) = \begin{bmatrix} \frac{\partial\phi_1}{\partial x_1} & \frac{\partial\phi_1}{\partial x_2} & \frac{\partial\phi_1}{\partial x_3} \\ \frac{\partial\phi_2}{\partial x_1} & \frac{\partial\phi_2}{\partial x_2} & \frac{\partial\phi_2}{\partial x_3} \\ \frac{\partial\phi_3}{\partial x_1} & \frac{\partial\phi_3}{\partial x_2} & \frac{\partial\phi_3}{\partial x_3} \end{bmatrix}$$

$$x = \phi(X)$$
$$F = \nabla\phi(X)$$



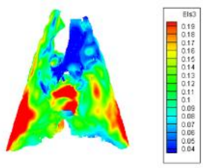
Max Principal Strain & Direction

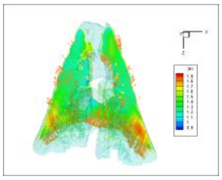
- Linear Strain Tensor

$$\epsilon_1 = \begin{bmatrix} \epsilon_x & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_y & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_z \end{bmatrix} = \frac{1}{2}\left[V + V^T\right] \qquad V = \begin{bmatrix} \partial u_1/\partial x_1 & \partial u_1/\partial x_2 & \partial u_1/\partial x_3 \\ \partial u_2/\partial x_1 & \partial u_2/\partial x_2 & \partial u_1/\partial x_3 \\ \partial u_3/\partial x_1 & \partial u_3/\partial x_2 & \partial u_1/\partial x_3 \end{bmatrix}$$

- Apply SVD on strain tensors to get

Principal Strains: $\lambda_1, \lambda_2, \lambda_3$ Principal Directions: d_1, d_2, d_3





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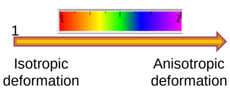
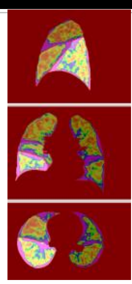
Anisotropic Deformation Index (ADI)

$$ADI = \frac{1 + \text{Max Principal Strain}}{1 + \text{Min Principal Strain}}$$





Tissues around fissures have more anisotropic deformation.

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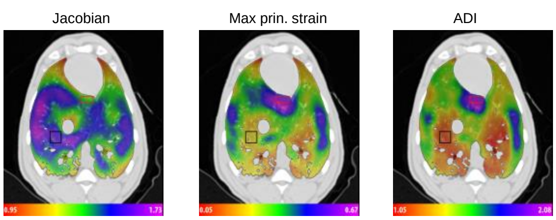
Summary of mechanical parameters

Jacobian --- Specific volume change

Strain Tensors --- Geometric information

(Def. magnitude, direction and pattern)

↓ prin. strain ↓ prin. direction ↓ ADI



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RT Applications

- Biomechanical analysis of lung motion using mechanical measures (PI Reinhardt, HL079406, Iowa).
- Tracking cumulative dose in pulmonary RT in presence of tumor regression and atelectasis (PI Hugo, CA166119, VCU and Iowa)
- 4DCT reconstruction using 4D B-spline registration (PI Hugo, CA166119, VCU and Iowa)
- Modeling and predicting dose response in pulmonary RT using pretreatment Jacobian determinant (PI Bayouth, CA166703, Wisconsin and Iowa)
- Starting Aug 2016 at Wisconsin, treating patients with Pulmonary RT plans that spare healthy lung tissue based on pretreatment Jacobian determinant (PI Bayouth, CA166703, Wisconsin and Iowa)

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Acknowledgements

- Oguz Durumeric
- Joseph Reinhardt
- Madhavan Raghavan
- Geoff Hugo
- Elisabeth Weiss
- John Bayouth
- Eric Hoffman
- Jeffrey Williamson
- Kunlin Cao
- Kai Ding
- Kaifang Du
- Ryan Amelon
- Chris Guy
- Youbing Yin
- Matt Fuld

We would like to thank K. Murphy and B. van Ginneken for use of the IX software.

This work was supported in part by the NIH grants HL079406, CA166119 and CA166703.

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Thank you!

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