PET Physics and Instrumentation

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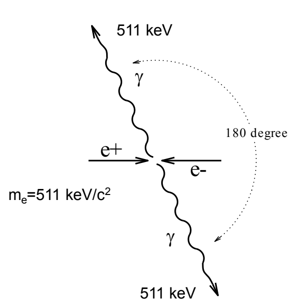
Positron Decay

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z-1}Y_{N+1} + e^{+} + \upsilon $$

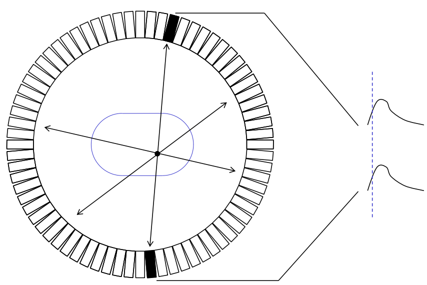
e.g., $^{18}F \rightarrow {}^{18}O + e^{+} + \nu$

Nuclide	half-life
C-11	20.3 min
N-13	10 min
O-15	124 sec
F-18	110 min
Rb-82	*75 sec*
Ga-68	*68 min*

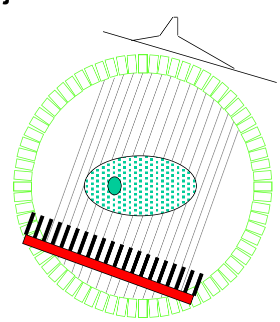
Positron Annihilation



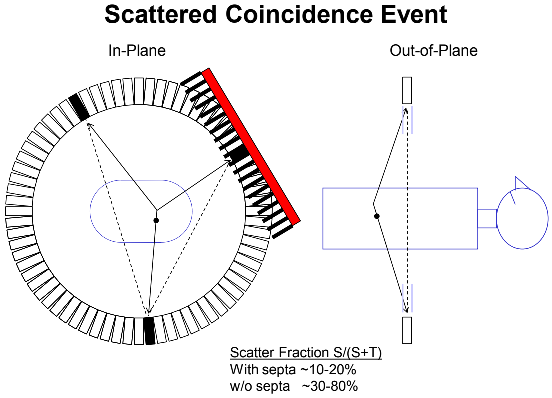
Coincidence Event



Projections



Scattered Coincidence Event



Random Coincidence Event

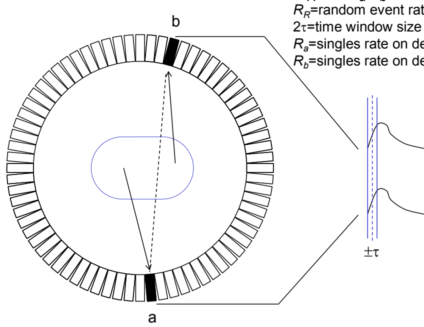
$R_R = 2\tau R_a R_b$

R_R=random event rate

2τ=time window size

R_a=singles rate on detector *a*

R_b=singles rate on detector *b*



Correcting Background; Noise Equivalent Counts

$$P_{prompts} = T_{trues} + S_{scatter} + R_{randoms}$$

$$T' = P - S' - R' \quad \textit{(Estimation of true events by subtracting S and R estimates)}$$

$$\langle T' \rangle = \langle P \rangle + \langle S' \rangle + \langle R' \rangle = P + \left|\begin{matrix}0\\R\end{matrix}\right| \geq P \geq T$$

(Variance propagation and Poisson properties. 0 vs. R depends on randoms correction method.)

$$SNR \equiv \frac{T'}{\sqrt{\langle T' \rangle}} \approx \frac{T}{\sqrt{P + \left|\begin{matrix}0\\R\end{matrix}\right|}}; NEC = \frac{T^2}{P + \left|\begin{matrix}0\\R\end{matrix}\right|} = \frac{T}{(1 + S/T + (2?)R/T)}$$

S and R refer to scattered and random events on LORs that subtend the imaged object.

More background → more statistical image noise.

NEC Examples

$$NEC = \frac{T}{(1 + S/T + R/T)}$$

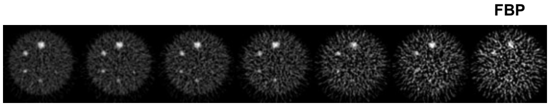
Prompts	Trues	Scatter	SF	NEC
100	100	0	0	100
200	100	100	0.5	50
400	200	200	0.5	100

PET Design Goal

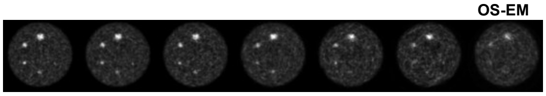
- Highest possible sensitivity
- Lowest possible scatter and random fractions.
- Maximize NECR

Image Noise and Lesion Detection

FBP



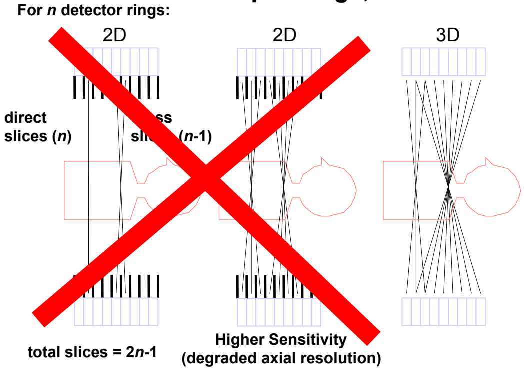
OS-EM



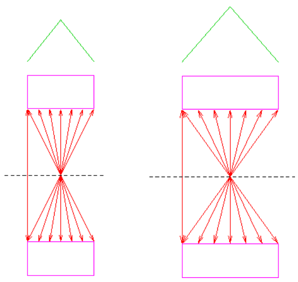
600s 300s 150s 75s 38s 19s 10s

Multiple Rings, 2D – 3D

For *n* detector rings:

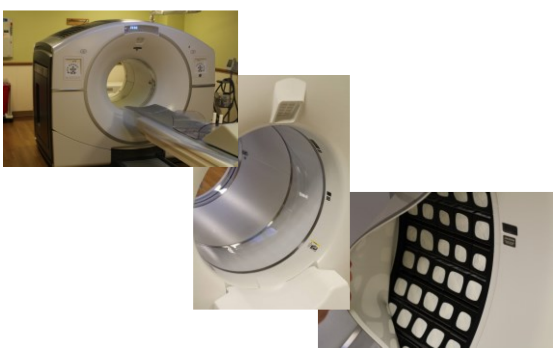


3D Sensitivity vs. axial FOV

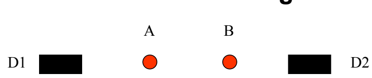


Sens $\propto$ (FOV)^2

GE Discovery IQ



The influence of background



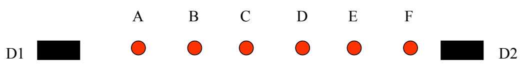
<u>What is SNR of measurement of counts from A?</u>

Detectors measure counts C_A+C_B.

S.D. of measurement is $\sqrt{C_A + C_B}$

SNR = $C_A/\sqrt{C_A + C_B}$

The influence of even more background



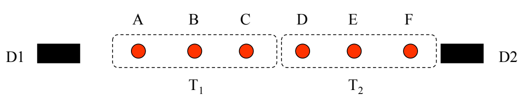
<u>SNR of measurement of A?</u>

SNR = $C_A / \sqrt{C_A + C_B + C_C + C_D + \ldots}$

If all N sources are equal,

SNR = $C_A / \sqrt{N C_A}$

Magical new detectors



Can distinguish between counts originating in segment T_1 and counts originating in segment T_2.

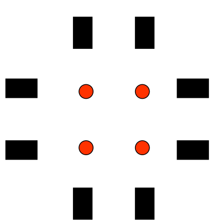
If all N sources are equal,

SNR = $C_A / \sqrt{n C_A}$

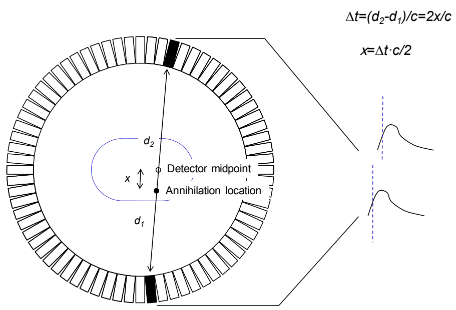
where n is the number of sources within zone T_1.

SNR improvement of sqrt(N/n); similar to counting longer by N/n.

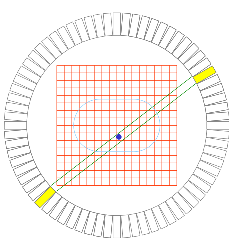
Tomography



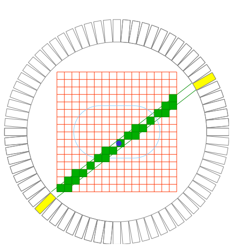
Time-of-Flight PET



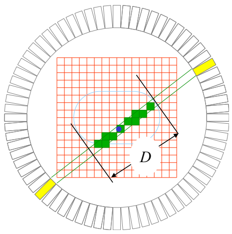
Where this radioactivity could give a count:



All of the locations that could give counts in this LOR:

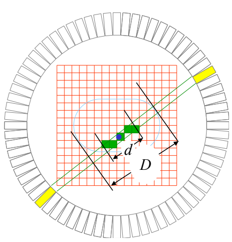


All of the locations that could give counts from this body on this LOR:



D = diameter of body along line of response

All of the locations that could give counts on this LOR if we have TOF information:



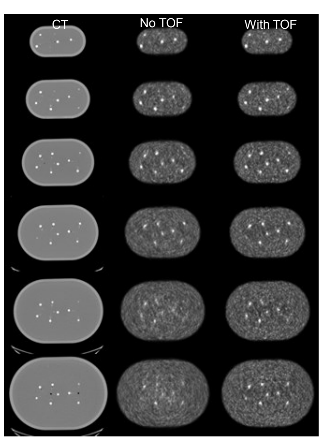
D = diameter of body along line of response

d = time-of-flight resolution

The TOF measurement leads to reduction in background, thereby improving count statistics by ~ D/d

Fillable, Tapering Phantom with 1 cm lesions

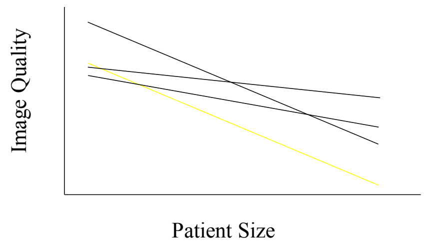
Josh Wilson, PhD



80 s

240 s

Image Quality vs. Size



Intrinsic Spatial Resolution Limits

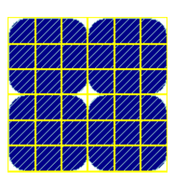
- **positron range**
  Depends on nuclide
- **opening angle**
  (Not exactly 180 degrees)
- **detector size**
- **block effect**
- **depth of interaction**

$$R_{sys} = \sqrt{R_{det}^2 + R_{acol}^2 + R_{range}^2 + b^2}$$

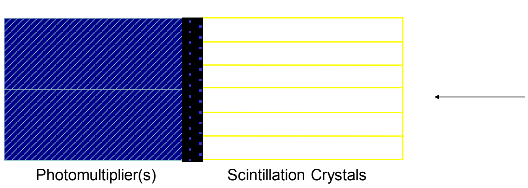
R_{det} = resolution of detectors ($\leq d$)
R_{acol} = resolution from photon acollinearity (=0.0022D)
R_{range} = resolution from positron range
b = block effect

Currently ~4-5 mm FWHM -- point source response - ramp filter only
This is NOT the smallest detectable lesion size!!!

Block Detector (GE,Siemens)

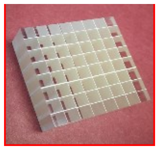


$$R_{sys} = \sqrt{R_{det}^2 + R_{acol}^2 + R_{range}^2 + b^2}$$

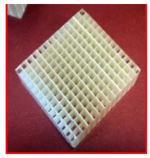


4 PMT's, or 2 dual-cathode PMT's, or 1 quad-cathode PMT.

Example Block Detectors

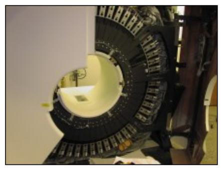


Standard Detector
6.4 mm x 6.4 mm
64 crystals/block

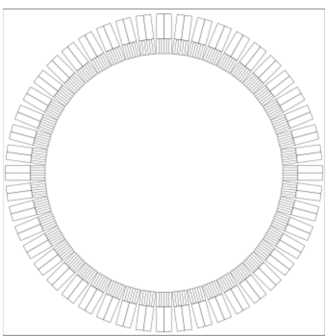


HI-REZ Detector
4.0 mm x 4.0 mm
169 crystals/block

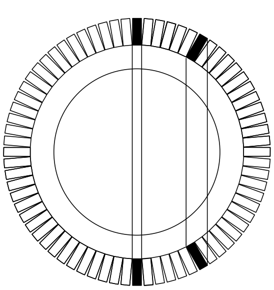
3 x 2 unit of block detectors



PET Ring with Block Detectors

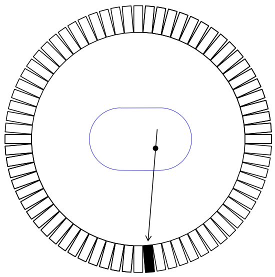


Depth of Interaction Uncertainty

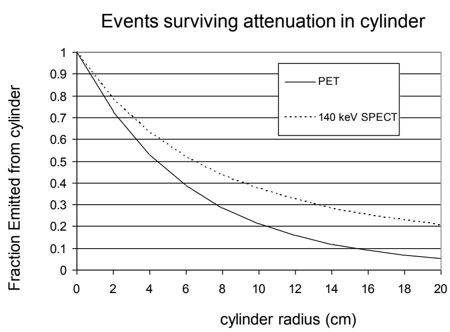


- High stopping power helps: Interactions more likely in front of detector
- Some high resolution systems sacrifice sensitivity by shortening detectors (to mitigate DOI effects.)
- Some systems (HRRT) use two layers of detectors to lessen effect. Others propose a measurement of the DOI.

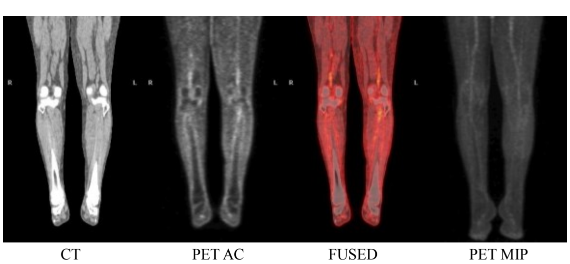
Attenuated Event



Attenuation losses - PET and SPECT

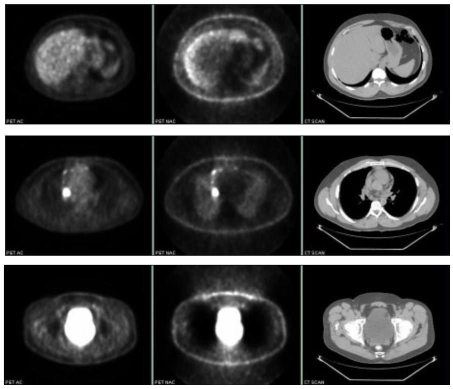


"Legs"



Attenuation Effects

AC NAC x-ray CT



PET Design Goals

- Maximize NEC by maximizing sensitivity (Trues/s/activity) while minimizing background (S/T and R/T)

$$NEC = \frac{T^2}{P} = \frac{T}{(1 + S/T + (2?)R/T)}$$

- Good intrinsic resolution without compromising sensitivity
- Provide accurate corrections for attenuation, scatter, randoms, deadtime, normalization (uniformity), …
- Good TOF without compromising sensitivity (too much)
- Stability, service, low cost

Some Available Scintillators

Scint.	ρ (g/cm3)	Eff. Z	Hygro-scopic?	Decay (ns)	light out (relative)	ΔE/E
NaI(Tl)	3.67	51	Yes	230	100	10%*
LSO	7.4	65	No	40	75	15%*
GSO	6.71	59	No	60	30	14%*
BGO	7.13	75	No	300	15	25-17%*
BaF2	4.88	53	No	0.8	12	NG
CsF	4.64	53	Very	4	5	NG

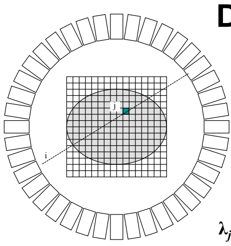
sodium iodide (thallium doped)
lutetium oxyorthosilicate
gadolinium oxyorthosilicate
bismuth germanate

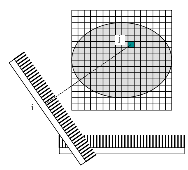
* depends on detector configuration, quality of material. No standard way to measure.

Melcher, J Nucl Med, 41:1051-1055

Image Reconstruction

Detection Model





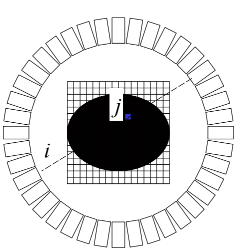
$$y_i = b_i + \sum_{j=1}^{npix} p_{ij}\lambda_j$$

$\boldsymbol{\lambda_j}$ = radioactivity at voxel $\boldsymbol{j}$
$\boldsymbol{p_{ij}}$ = probability of emission from $\boldsymbol{j}$ being measure in $\boldsymbol{i}$
$\boldsymbol{b_i}$ = background contributing to $\boldsymbol{i}$
$\boldsymbol{y_i}$ = expected counts on projection $\boldsymbol{i}$
$\boldsymbol{m_i}$ = measured counts on proj. $\boldsymbol{i}$

Image reconstruction: $\lambda = \boldsymbol{p}^{-1}(m_i - b_i)$ (What is $\boldsymbol{p}^{-1}$?)

Maximum Likelihood Expectation Maximization (ML-EM)

Shepp LA, Vardi Y., *IEEE Trans Med Imag* 1:113-121, 1982.
Lange K, Carson R., *J Comput Assist Tomo* 8:306-316, 1984.

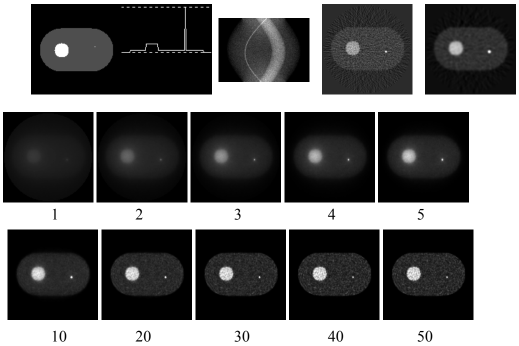


$$\lambda_j^{(n+1)} = \frac{1}{\sum_{i=1}^{nbin} p_{ij}} \sum_{i=1}^{nbin} \frac{p_{ij}\lambda_j^{(n)}}{b_i + \sum_{k=1}^{nvox} p_{ik}\lambda_k^{(n)}} m_i$$

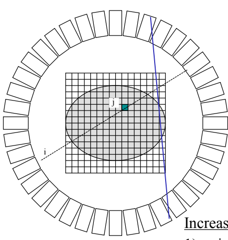
$$y_i = b_i + \sum_{j=1}^{npix} p_{ij}\lambda_j$$

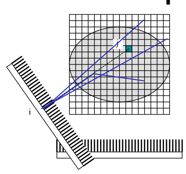
$\lambda_j^{(n)}$ is the estimated activity in voxel j at iteration n.

Extended Distribution Example



What is p?





$$y_i = b_i + \sum_{j=1}^{npix} p_{ij}\lambda_j$$

Increasing levels of complexity:

1) **p** is 1's and 0's - very sparse
2) **p** is fractional values, depending on how column intersects with voxel - still pretty sparse
3) **p** includes attenuation (lower values) – no less sparse
4) **p** includes resolution effects (collimator blurring, etc) - somewhat sparse – inherently 3D
5) **p** includes background - not sparse at all

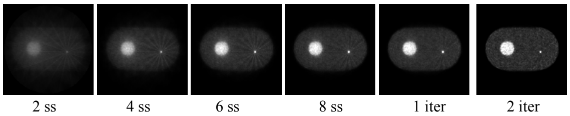
Ordered Subsets Expectation Maximization (OS-EM)

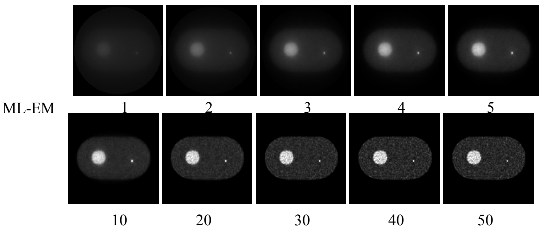
Hudson HM, Larkin RS . *IEEE Trans Med Imag* 13:601-609, 1994.

$$\lambda_j^{(n+1)} = \frac{1}{\sum_{i=1}^{nbin} p_{ij}} \sum_{i=1}^{nbin} \frac{p_{ij}\lambda_j^{(n)}}{b_i + \sum_{k=1}^{nvox} p_{ik}\lambda_k^{(n)}} m_i$$

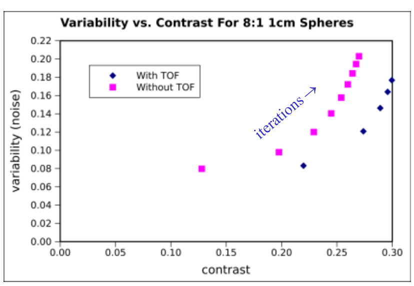
Instead of processing all projection for each image update, projections are divided into subsets, and the image is updated after processing each subset.

OS-EM 10 subsets





TOF vs NTOF : Hot Spheres



Final Comments

- Multiple factors (sometimes competing) lead to good PET design.
- The ideal system would have high sensitivity, low background fractions, excellent TOF performance, high spatial resolution, and low cost.
- It is important to have image reconstruction that makes the best use of the raw data the scanner provides.