Outline for Today

Introduction to MRI

1.

‘Quantum’ NMR and MRI in 0D
Magnetization, $\boldsymbol{m}(x,t)$, in a Voxel
Proton T1 Spin Relaxation in a Voxel
Proton Density MRI in 1D
MRI Case Study, and *Caveat*

2.

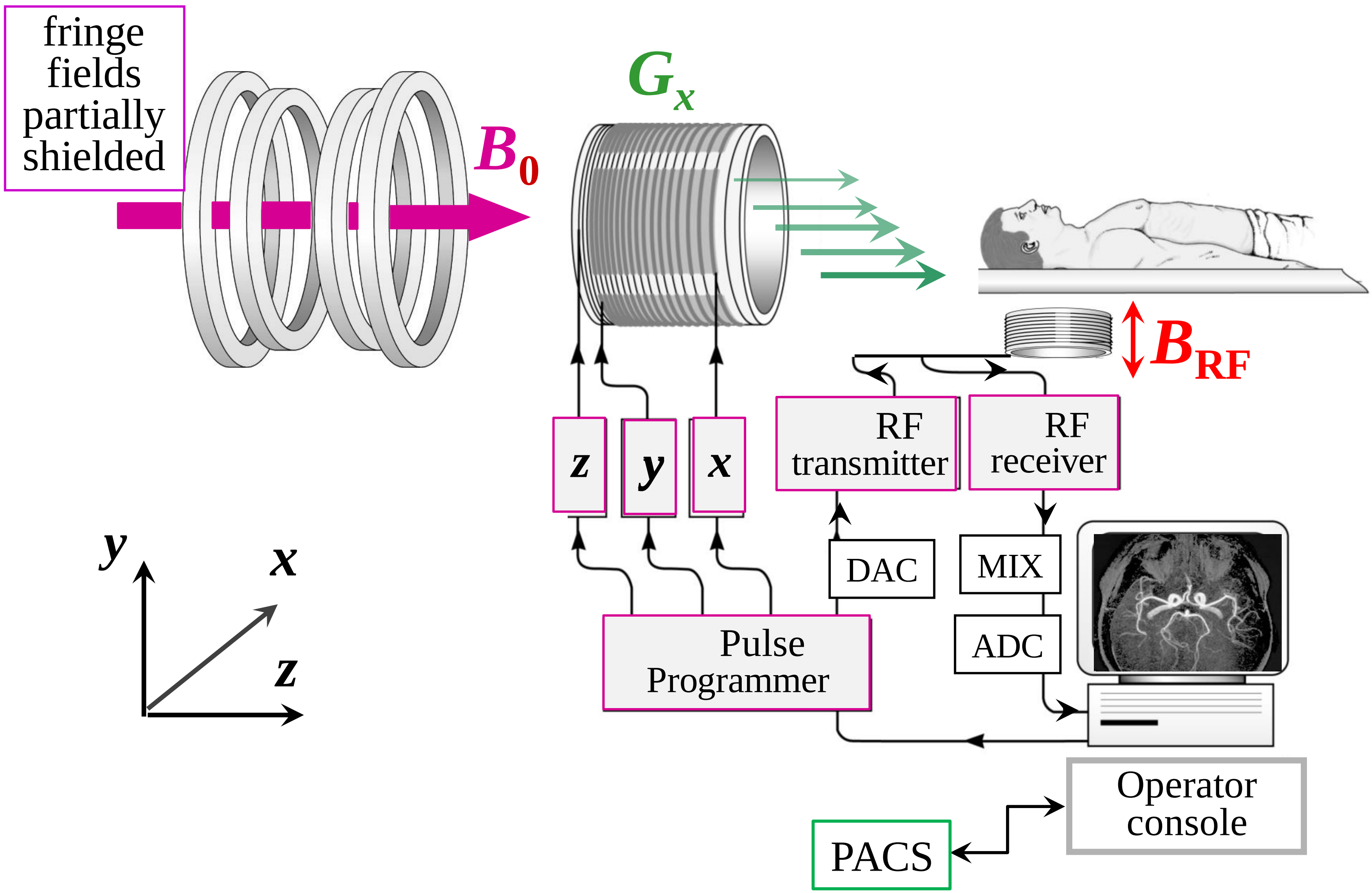
Sketch of the MRI Device
‘Classical’ NMR in a Voxel
Free Induction Decay in 1D

3.

T2 Spin-Relaxation
Spin-Echo Reconstruction in 1D
Tissue Contrast-Weighting in SE
Spin-Echo / Spin-Warp in 2D

Sketch of the MRI Device

Major Components of a ***Superconducting*** MRI System

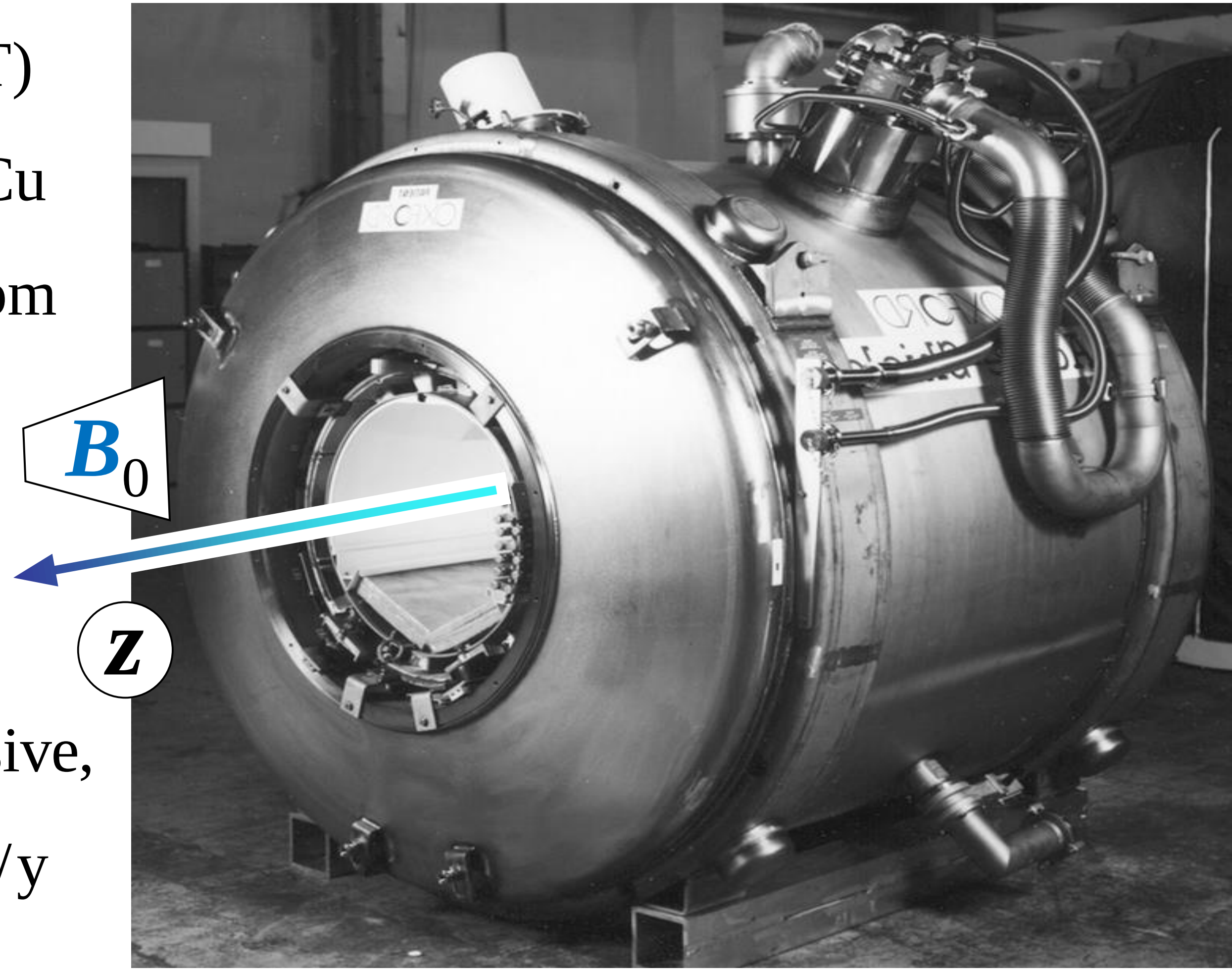


Cylindrical Superconducting Magnets

B_0 : 1.5 T, 3.0 T, (7 T)

50 km Nb-Ti wire in Cu

Homogeneity: <10 ppm



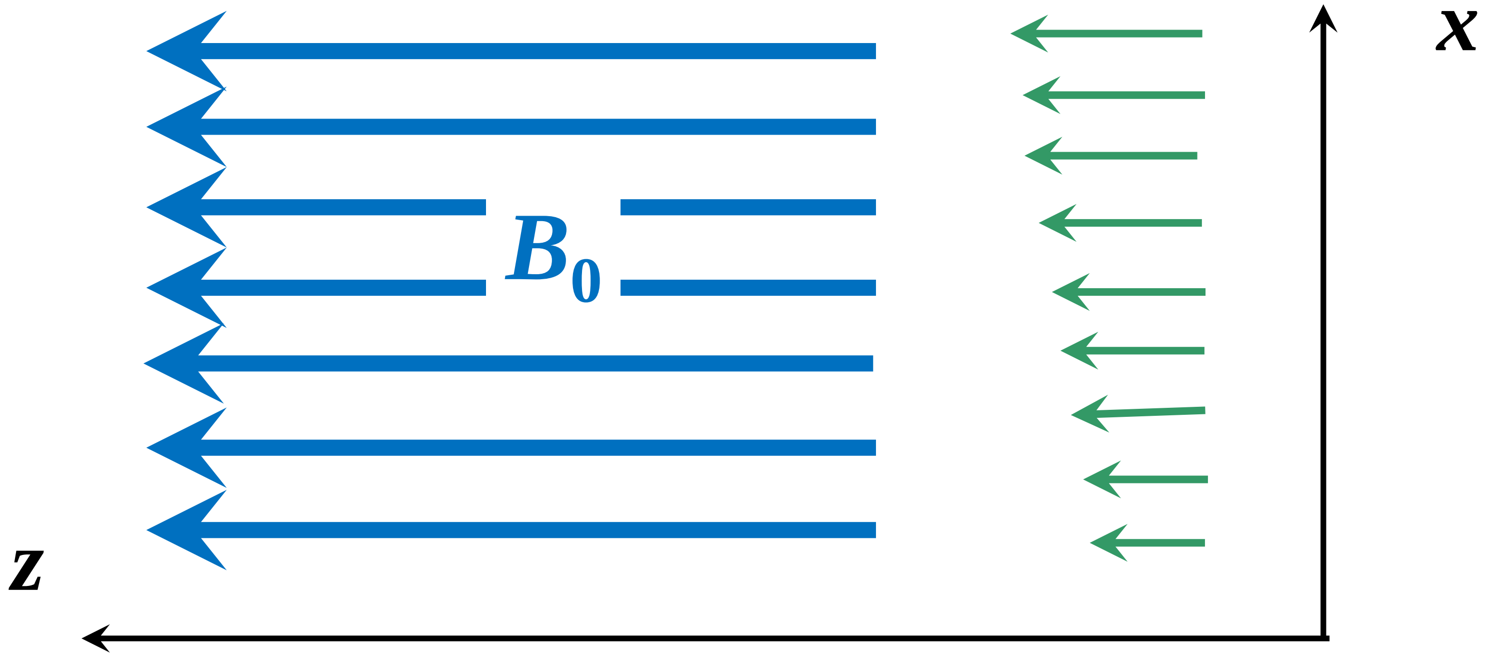
Shielding: active, passive,

Cryogen: 0.1 liter He/y

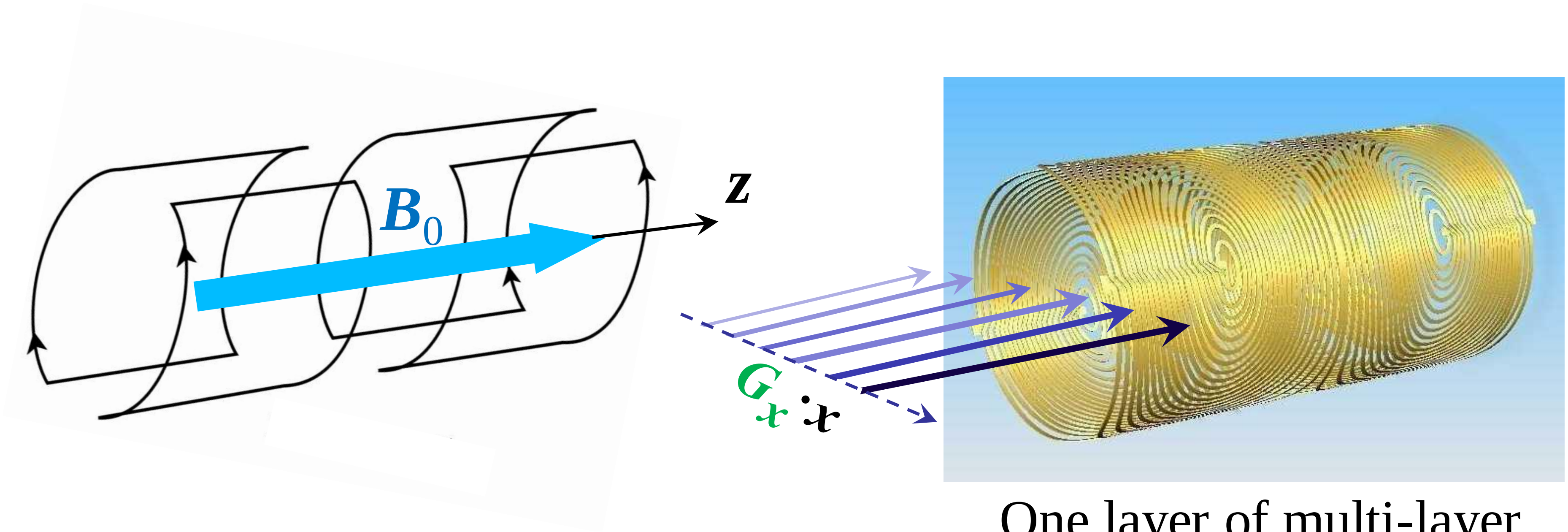
Weight: 4 tons

External Magnetic Fields

$$G_x \equiv \Delta B_z(x)/\Delta x$$



x-Gradient Magnet Winding for Superconducting Magnet

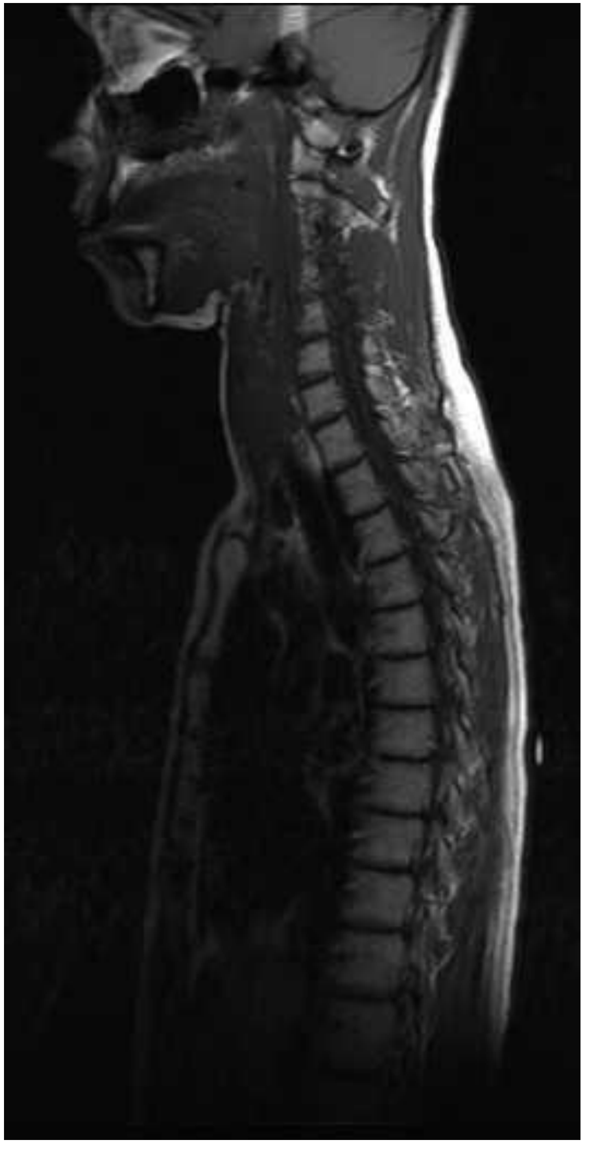


One layer of multi-layer
x-gradient coil

x-Gradient $dB_{\mathbf{z}}/dx$	20 − 60 mT/ m
Rise time	0.3 ms (to reach 10 mT/ m)
Slew rate	50 − 200 mT/m/ms

Artifact: Gradient Non-Linearity

correctable



RF Coils

B_{RF}: 20 μT
Pulse on-time: 3 msec
RF power: ~15 kW
SAR: ~2W/ kg

‘Parallel’ RF Receiving Coils for Much Faster Imaging

transmit parallel coils in the works

‘Classical’ NMR in a Voxel

The Two Approaches to NMR/MRI

quantum state function

$|\psi\rangle$

now this

Simple QM

$|\uparrow\rangle, |\downarrow\rangle$

transitions between spin-up, spin-down states

f_{Larmor}, m_0, T1

oversimplified

Classical Bloch Eqs.

for expectation values

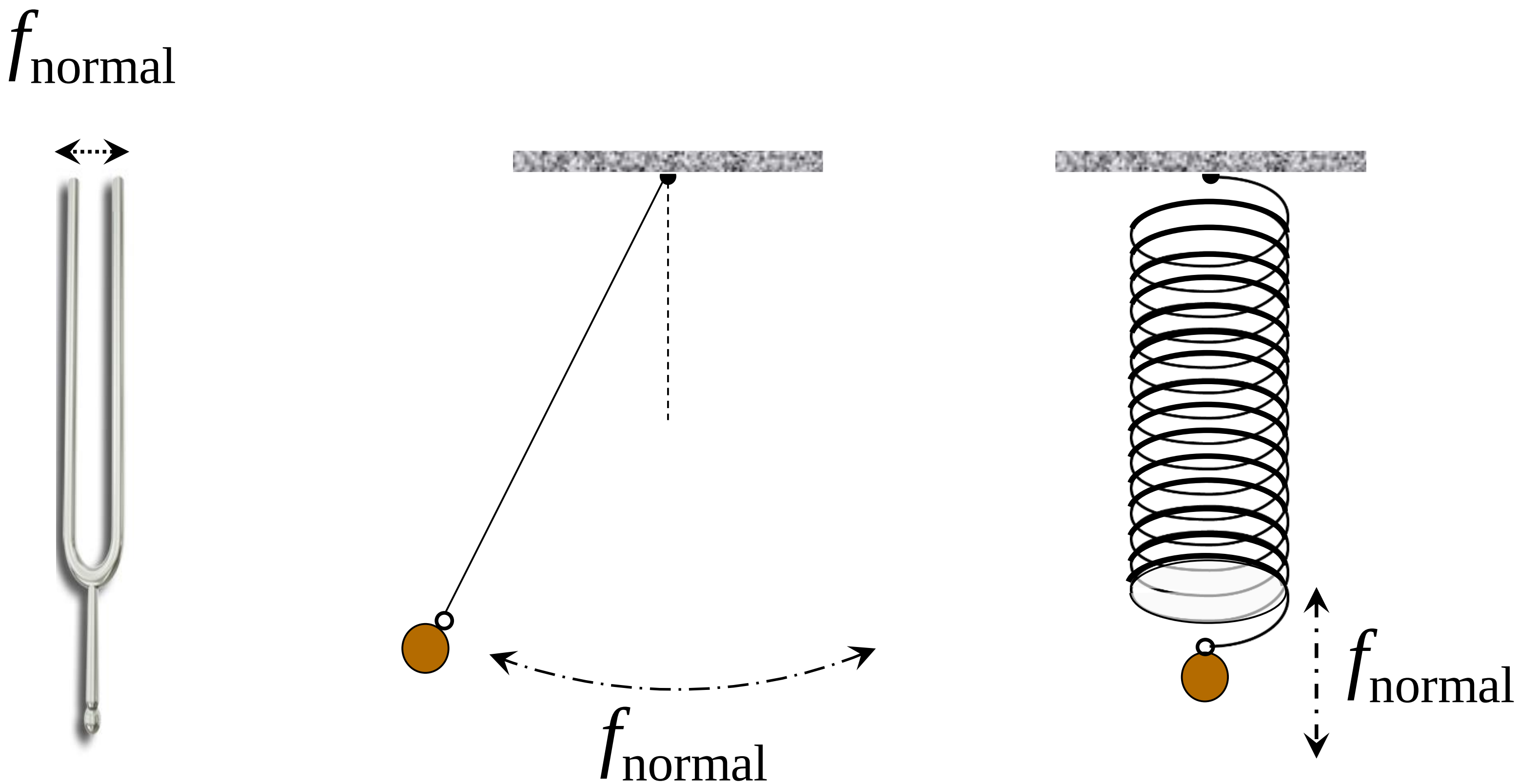
precession, nutation of voxel magnetization, $\boldsymbol{m}(t)$

f_{Larmor}, T2, $\boldsymbol{k}$-space

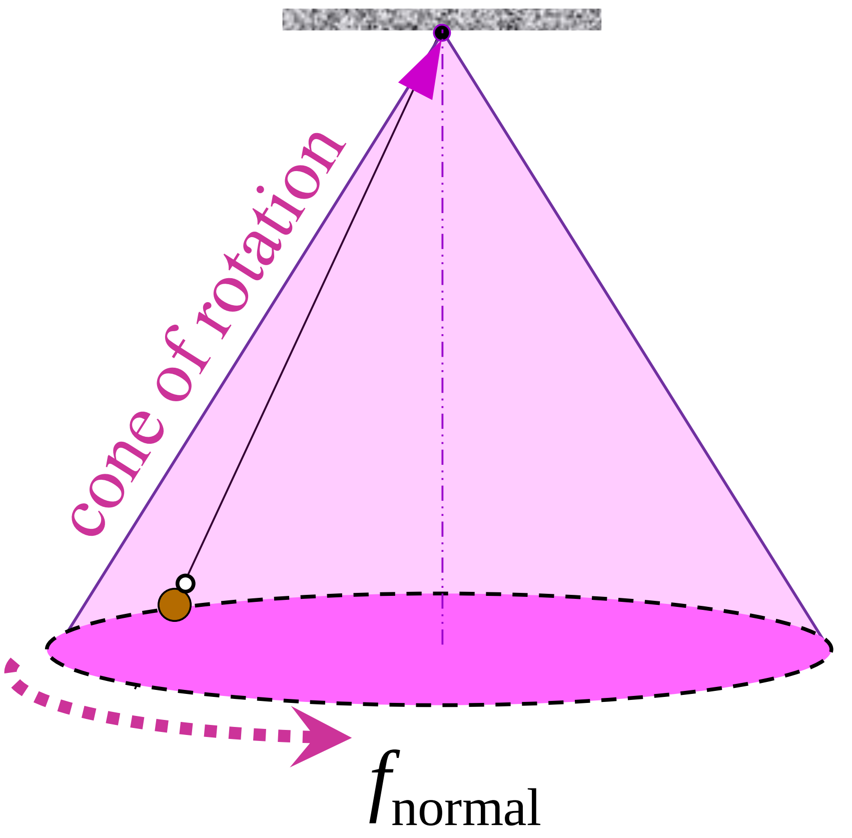
exact; from full QM

Normal Mode, at ν_{normal}

relaxation time T



A Normal Mode of a 2-D Pendulum



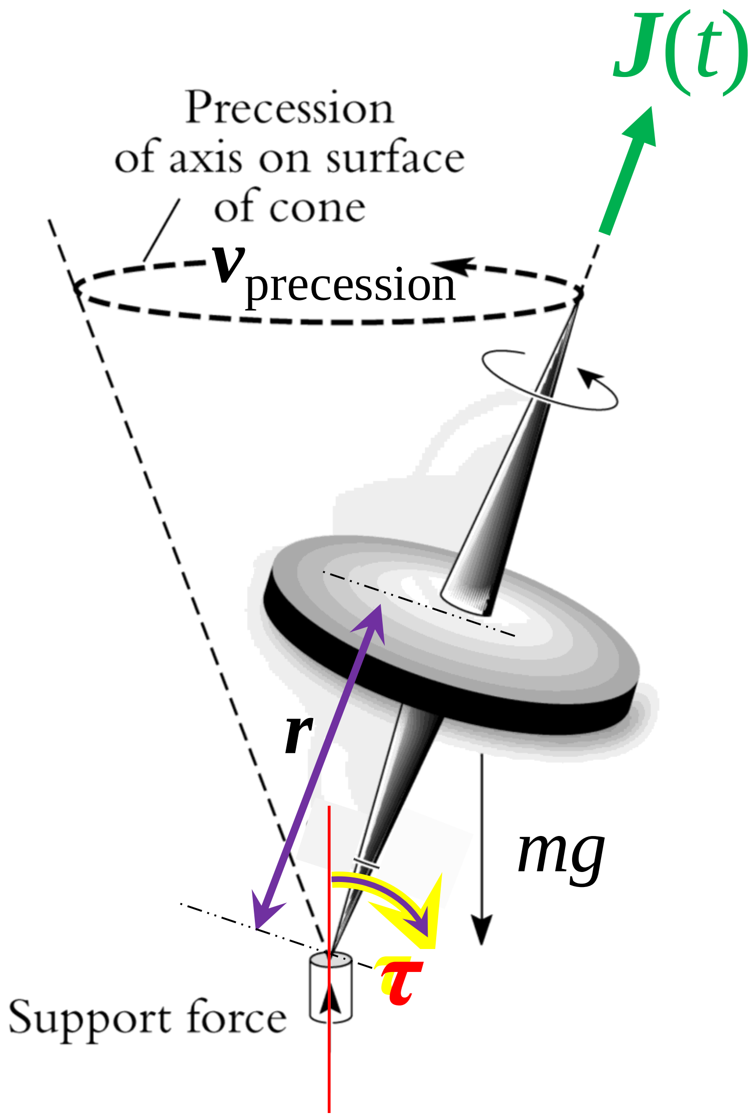
Normal Mode ***Precession*** about External Gravitational Field

($d\mathbf{p}/dt = \boldsymbol{F}$)

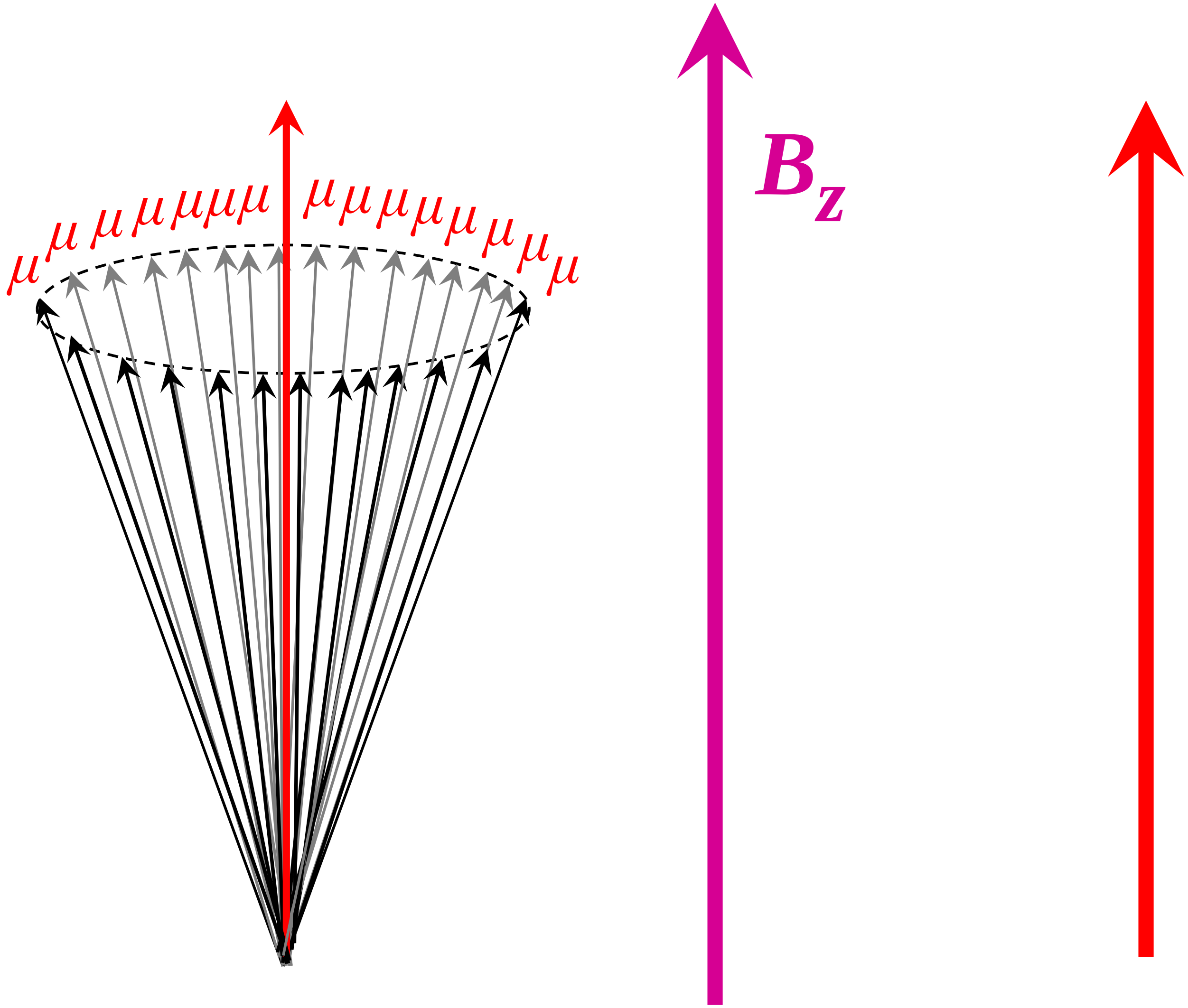
$d\boldsymbol{J} / dt = \boldsymbol{\tau}$ (torque)

$\boldsymbol{J}(t)$: Angular momentum

Precession at v_{normal}



Packet of Protons in Voxel Acts Like Classical $\boldsymbol{m}(t)$ Vector

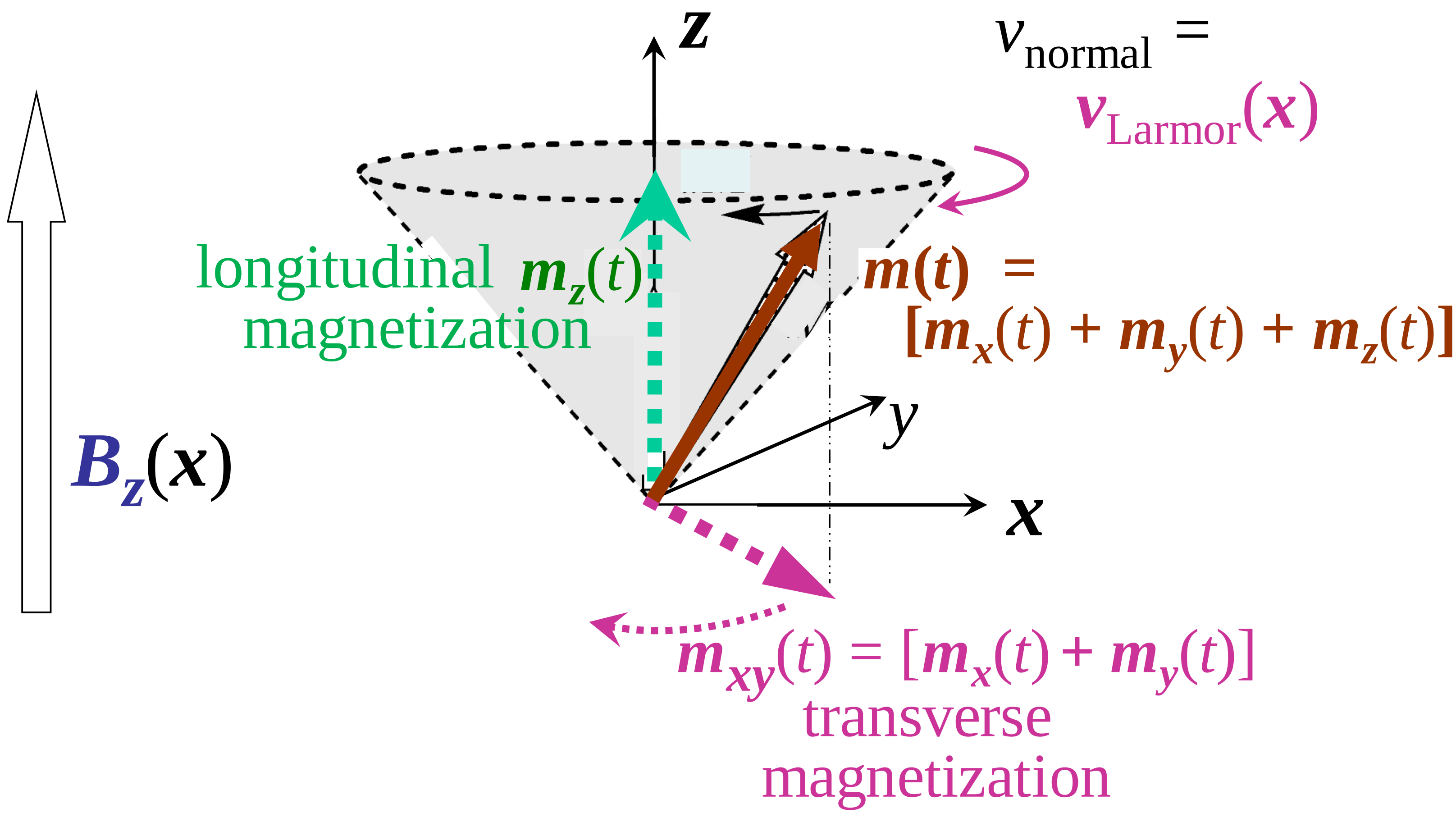


Think
classical:
$\langle \boldsymbol{m}(t) \rangle$

!!!!!

Normal Mode Precession of Voxel's $\boldsymbol{m}(\boldsymbol{x},t)$ in $\boldsymbol{B}_0$

can be derived rigorously from quantum mechanics



Bloch Equations of Motion for $m(t,x)$ in $\mathbf{B_z}(\mathbf{x})$

$$dJ/dt = \boldsymbol{\tau}$$ but $\boldsymbol{\mu} = \gamma \mathbf{J}$, so…

$$d(\boldsymbol{\mu}/\gamma)/dt = \boldsymbol{\tau}$$

Lorentz torque on spins with magnetic moment $\boldsymbol{\mu}$ in $\mathbf{B_z}$:

$$\boldsymbol{\tau} = \boldsymbol{\mu}\times\mathbf{B_z}$$ (***vector*** cross product)

Equation of motion becomes:

$$d\boldsymbol{\mu}(t)/dt = \gamma\,\boldsymbol{\mu}(t)\times\mathbf{B_z} \quad .$$

Sum/average over all protons in bundle:

$$d\mathbf{m}(t)/dt = \gamma\,\mathbf{m}(t)\times\mathbf{B_z}$$

Expectation Value, $\langle\mathbf{m}(t)\rangle$, behaves ***classically***

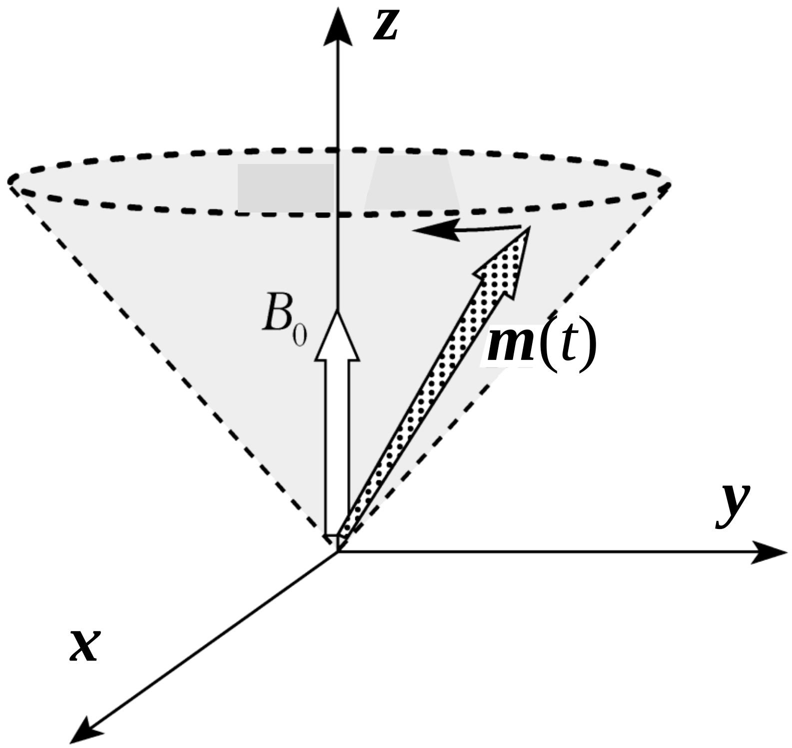
With T1 relaxation along ***z***-axis:

$$d\mathbf{m}(t)/dt = \gamma\,\mathbf{m}(t)\times\mathbf{B_z} + [\mathbf{m}(t) - \mathbf{m_0}]\hat{\mathbf{z}}\,/\,\mathrm{T1}$$

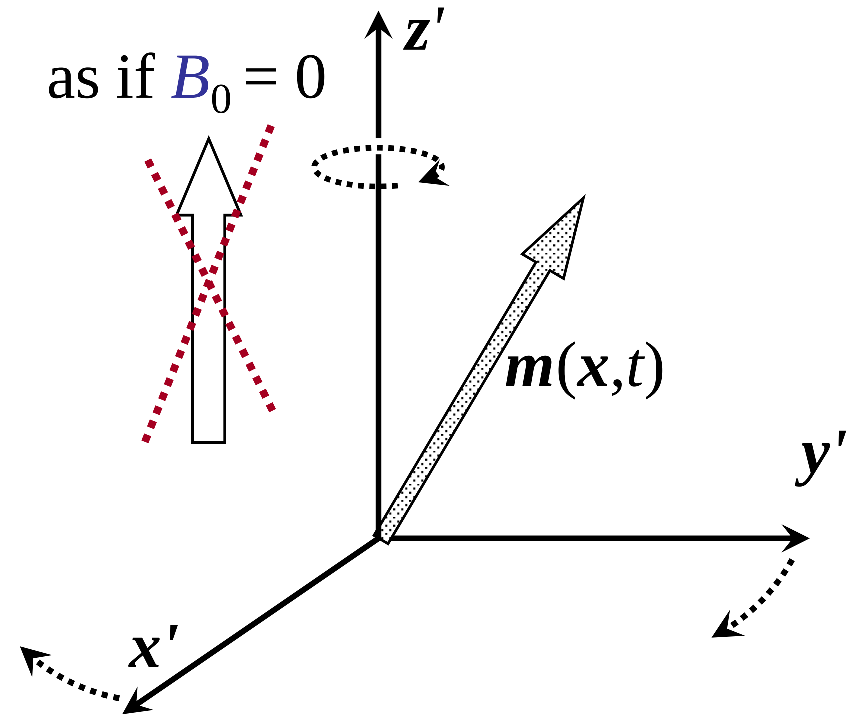
Precession of $\boldsymbol{m}(t)$ about $\boldsymbol{B_z}(\boldsymbol{x})$ at $\nu_{\text{Larmor}} = (\gamma/2\pi)\, B_z(x)$

as seen from:

Fixed Frame

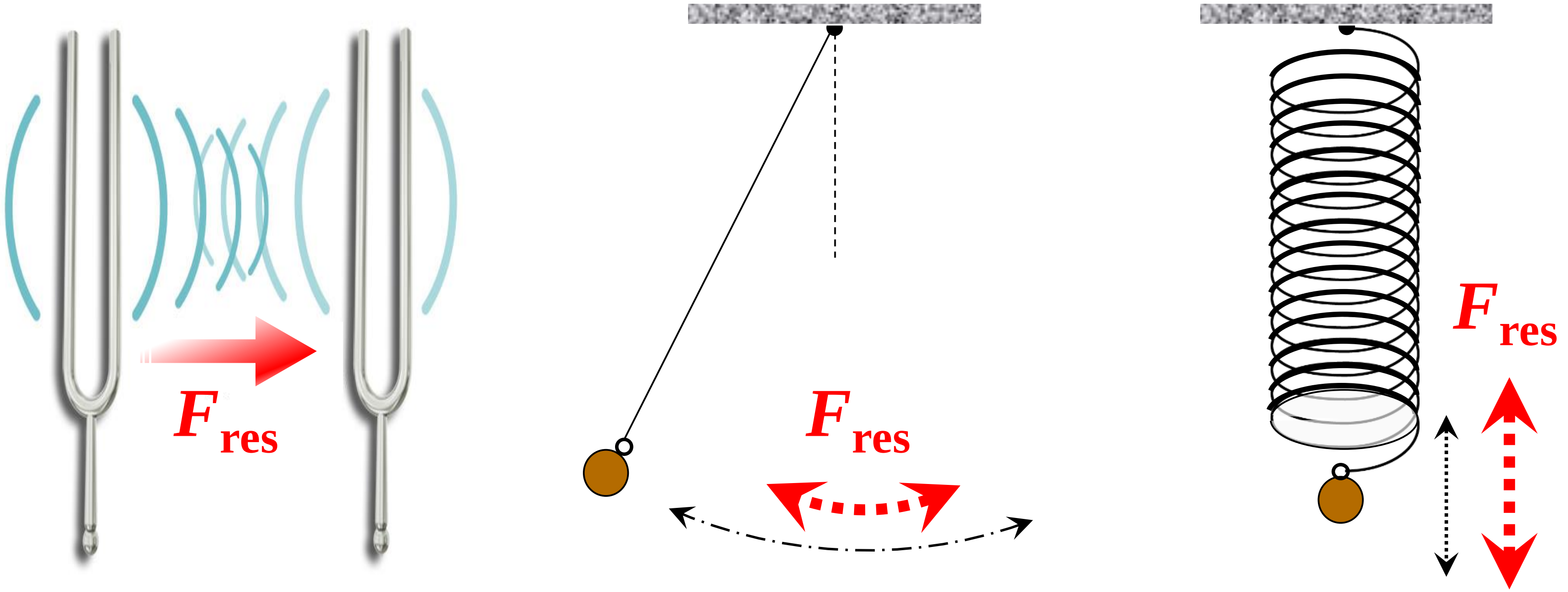


Rotating at $\nu_{\text{Larmor}}(\boldsymbol{x})$



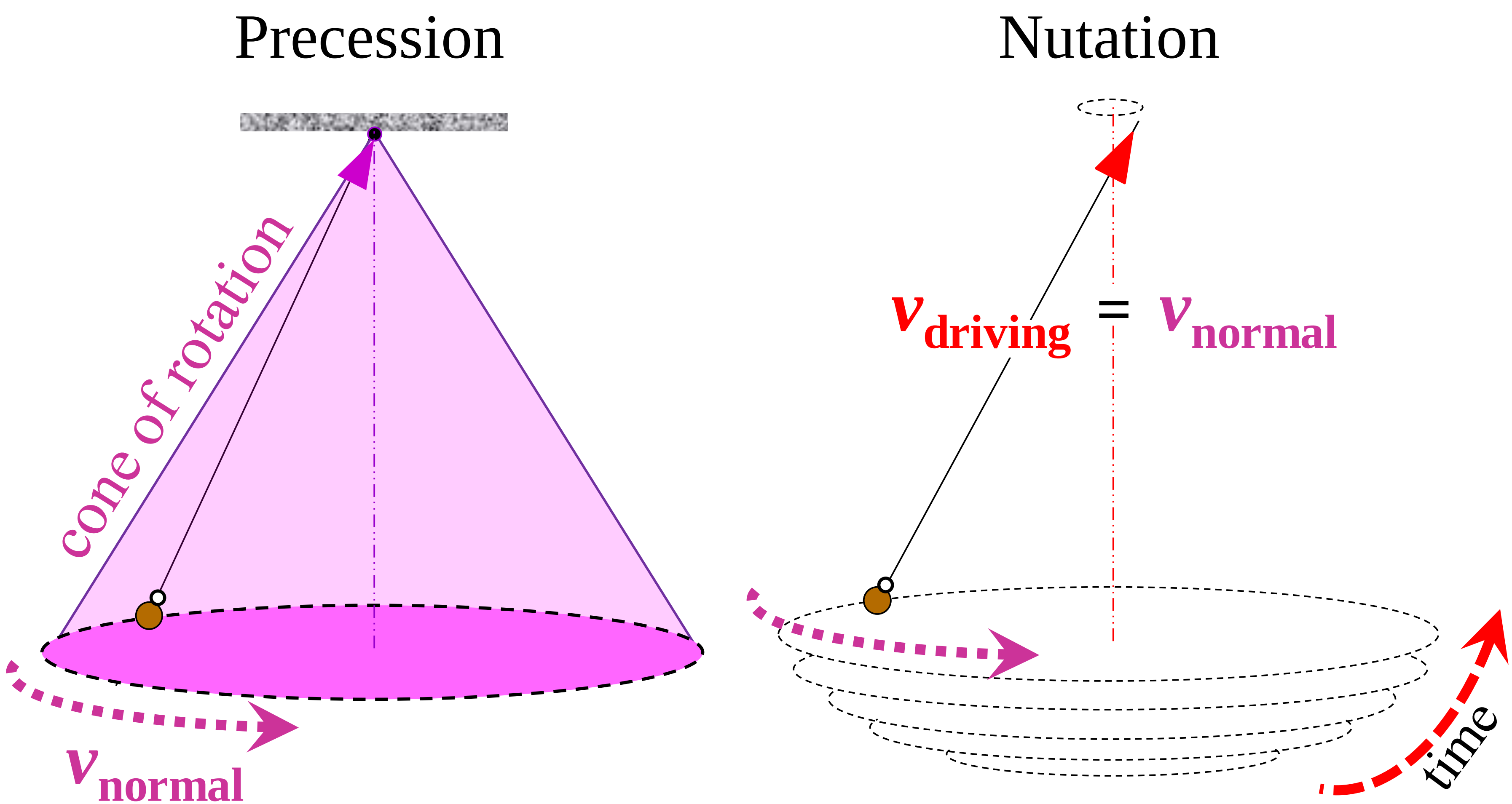
The ponies don’t advance when you’re *on* the carousel.

Resonance Energy Transfer when $\nu_{driving} = \nu_{normal}$



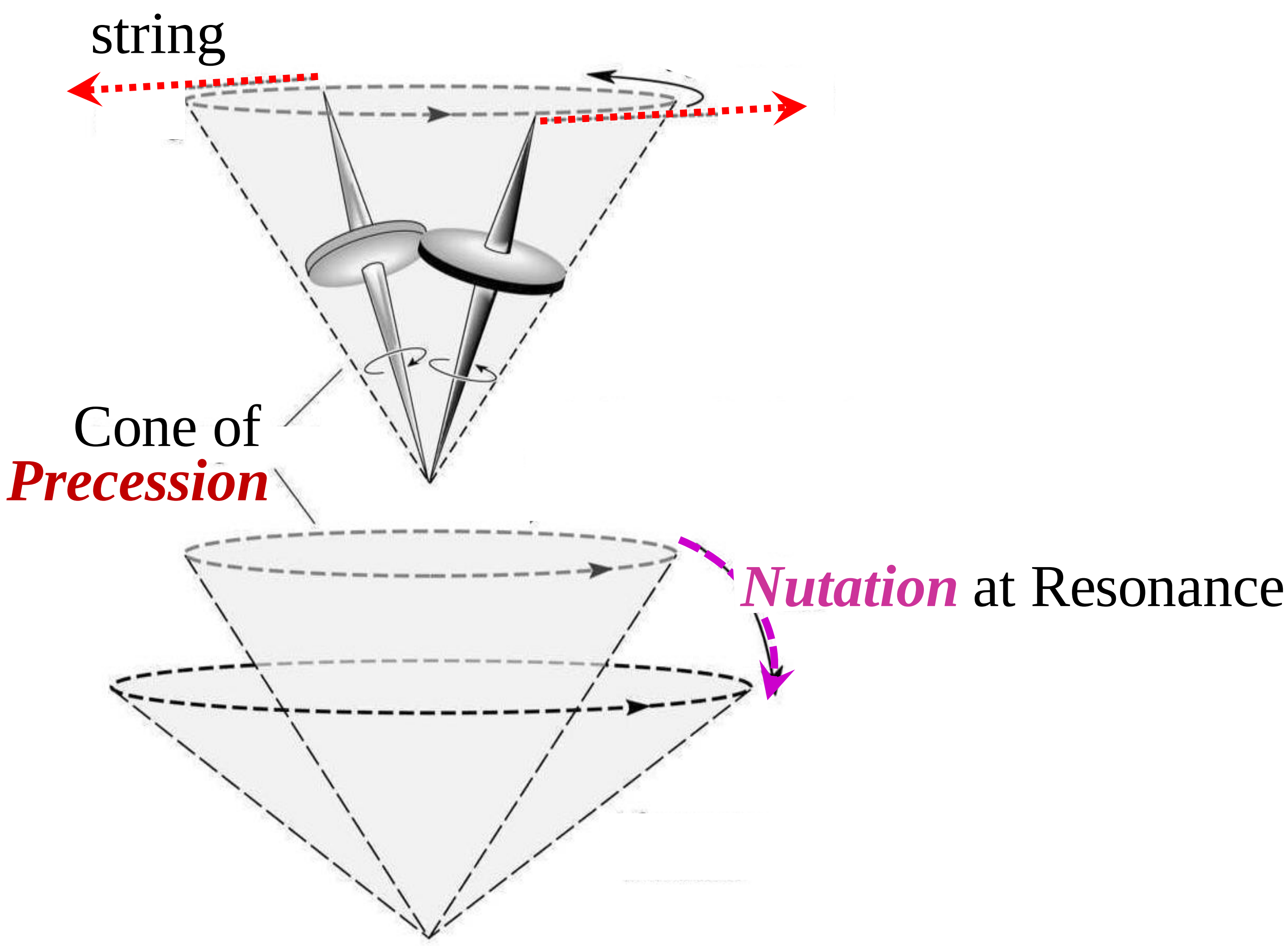
Resonance and *Nutation* of a 2-D Pendulum

net $\nu_{resonance}$ power input



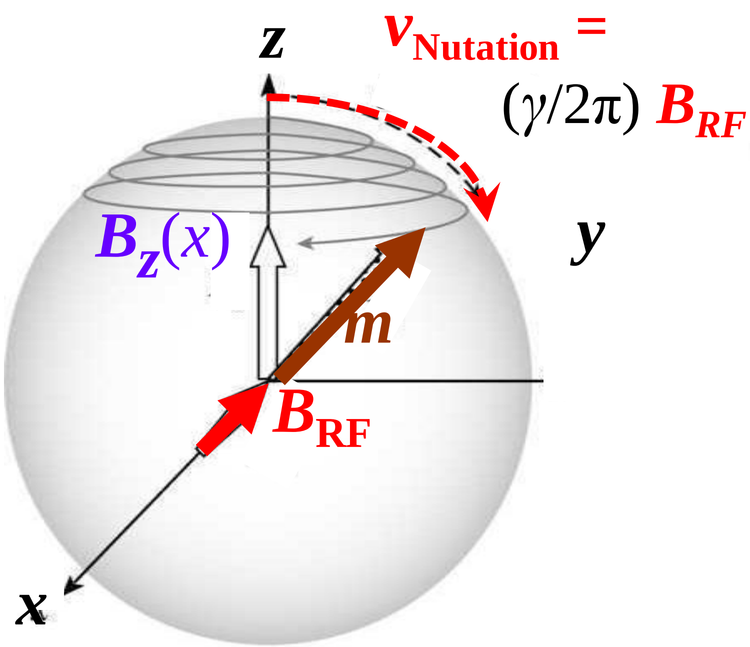
Resonance and Nutation of a Gyroscope

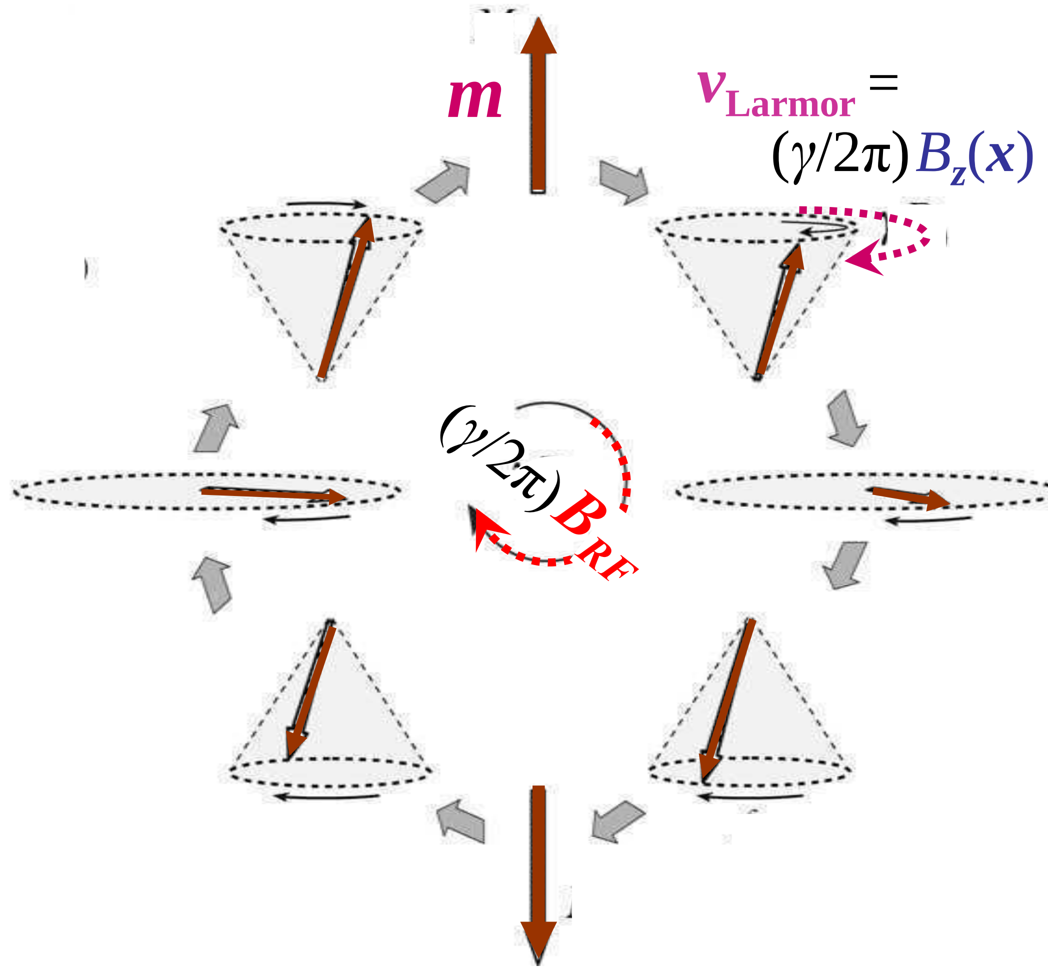
net $\boldsymbol{v}_{\text{resonance}}$ power input



Nutation of the Voxel's Magnetization, $\boldsymbol{m}(\boldsymbol{x},t)$

$$B_{RF} / B_0 \sim 10^{-5} - 10^{-4}$$





precesses at

$\nu_{\text{Lar}} = (\gamma/2\pi) B_z(x)$ (always!)

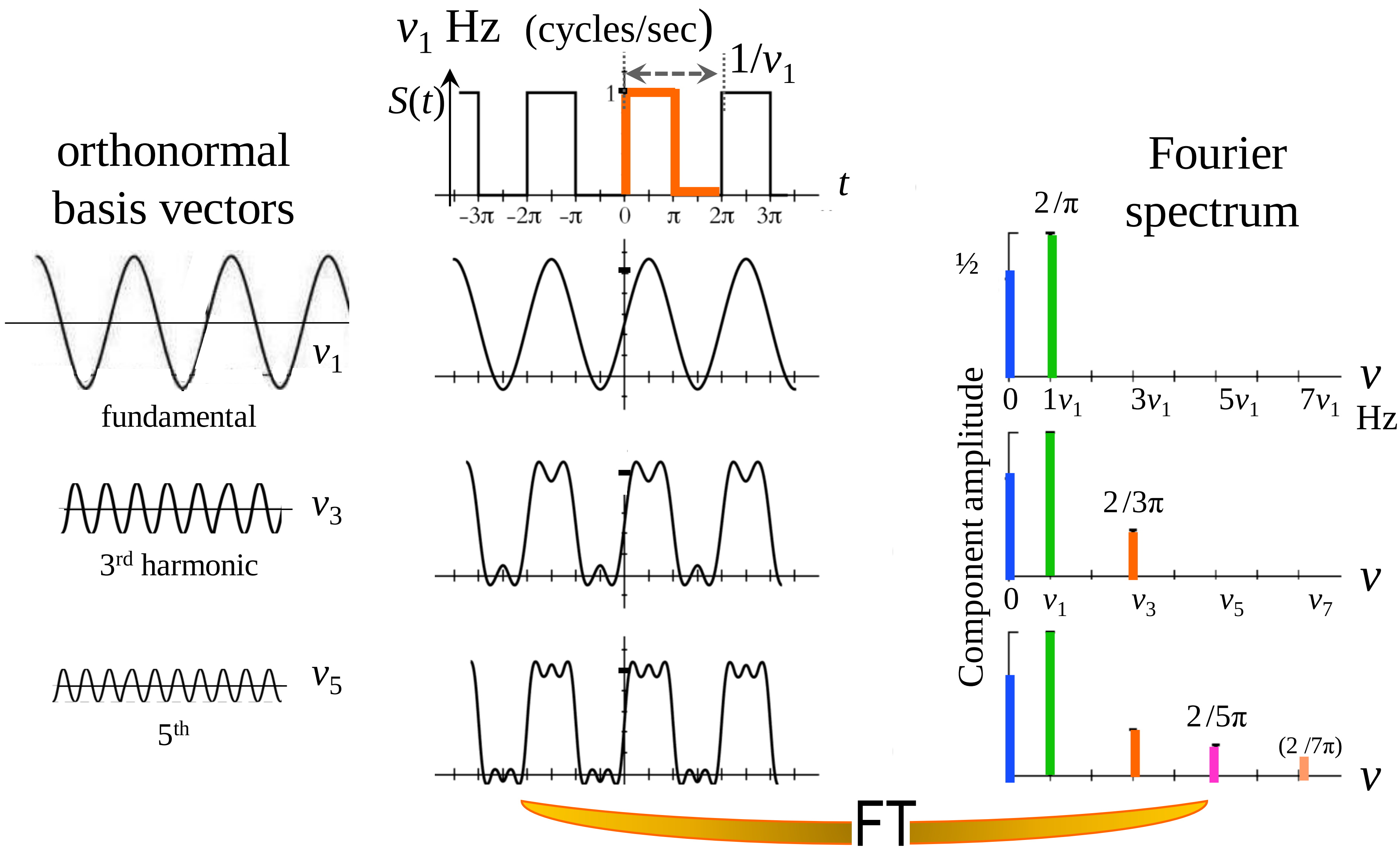
nutates at

$\nu_{\text{nut}} = (\gamma/2\pi) B_{RF}$ (only when B_{RF} is on!)

Free Induction Decay in 1D

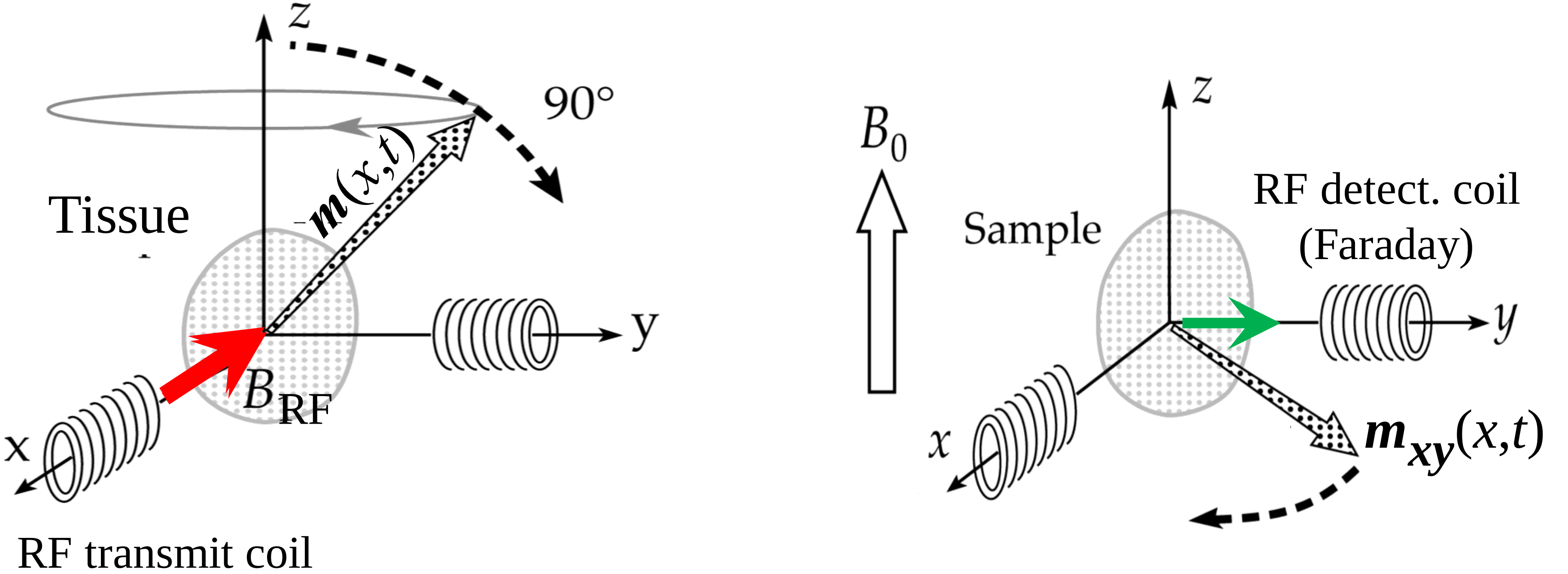
Reminder: Fourier Decomposition of Periodic Signal

$$S(t) \;\sim\; \tfrac{1}{2} + (2/\pi)\{\sin(2\pi\, \nu_1 t) + \tfrac{1}{3}\sin(6\pi\, \nu_1 t) + 1/5\,\sin(10\pi\, \nu_1 t) + \ldots\}$$



FID: $\boldsymbol{m}(\boldsymbol{x},t)$ for a *Single* Voxel at $\boldsymbol{x}$,

following a 90º pulse, precessing in the ***x-y*** plane

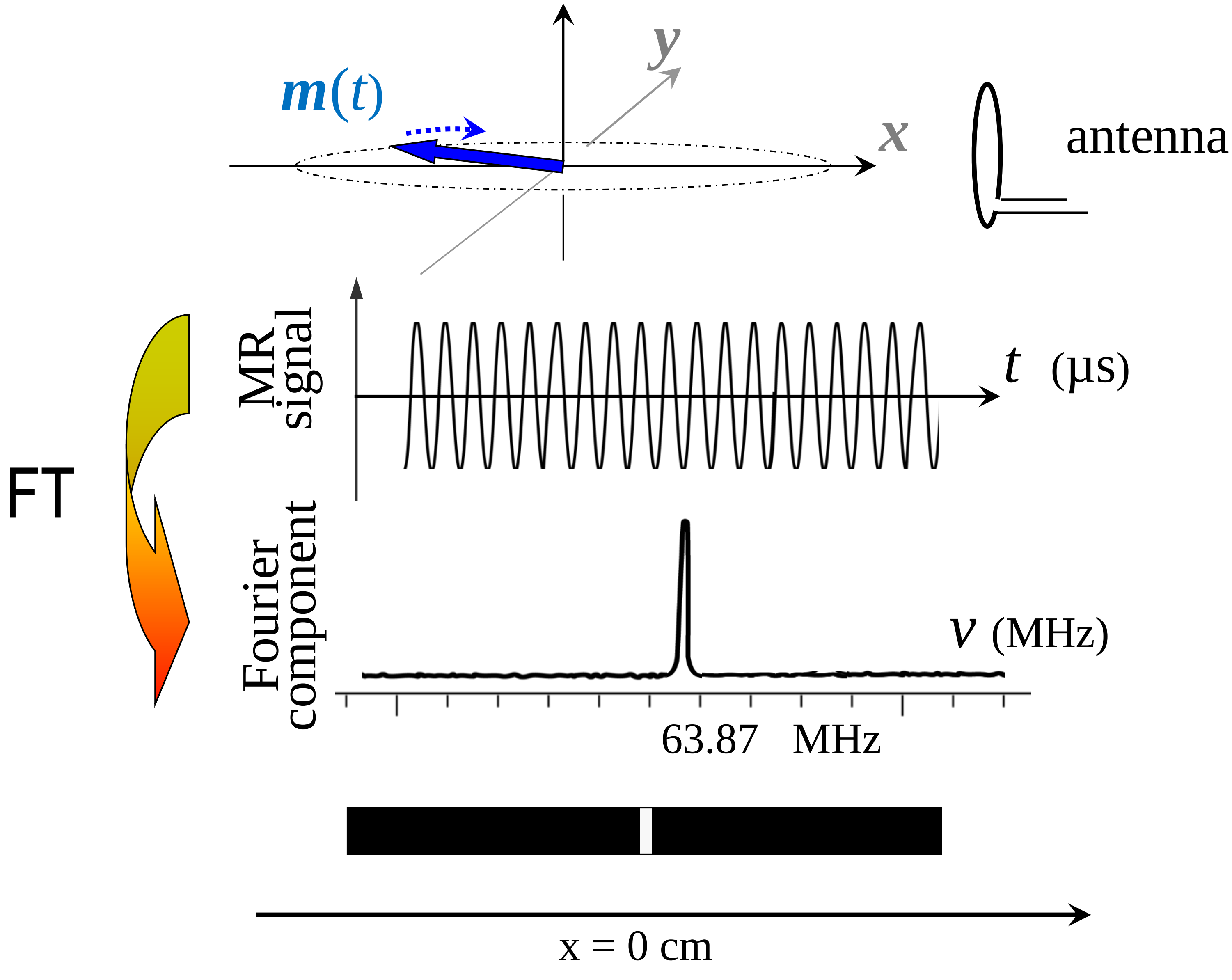


In MRI, the *only* signal the detector *ever* sees comes from the *set* $\{m(x,t)\}$, *all* precessing in the *x-y* plane

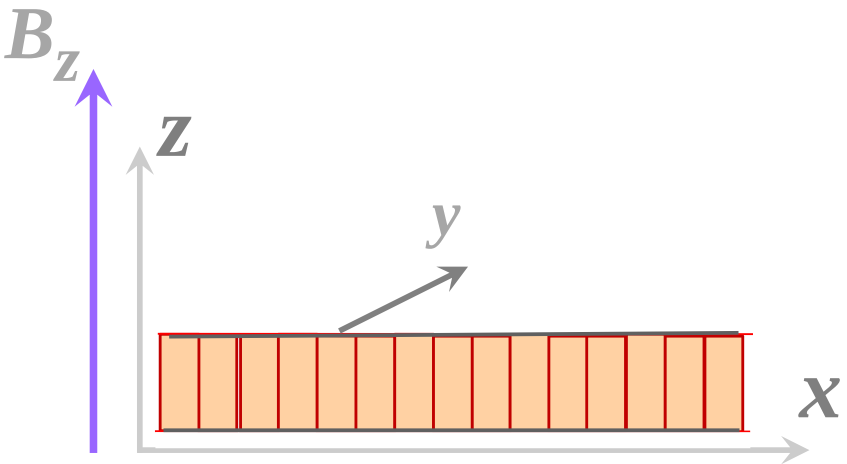
!!!

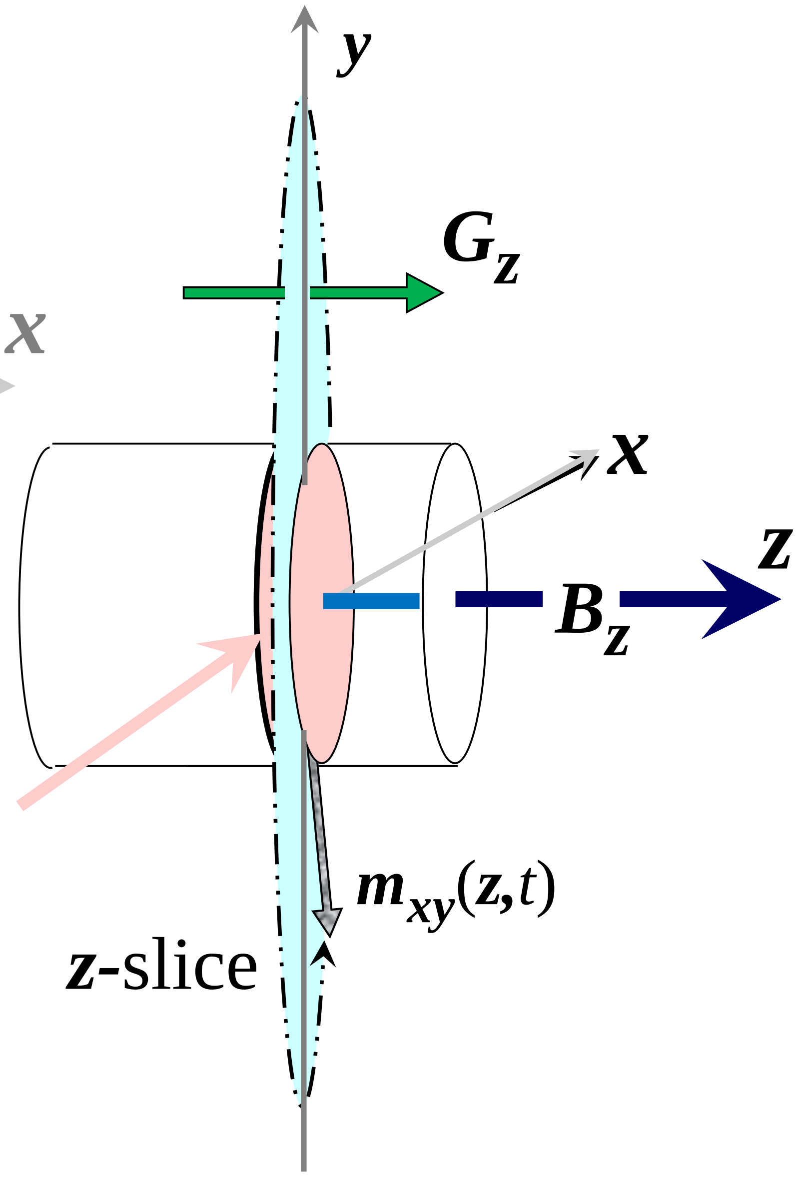
FID: Precession, Reception, Fourier Analysis (single voxel)

n.b. detect induced $V(t)$, not power absorption (as before)

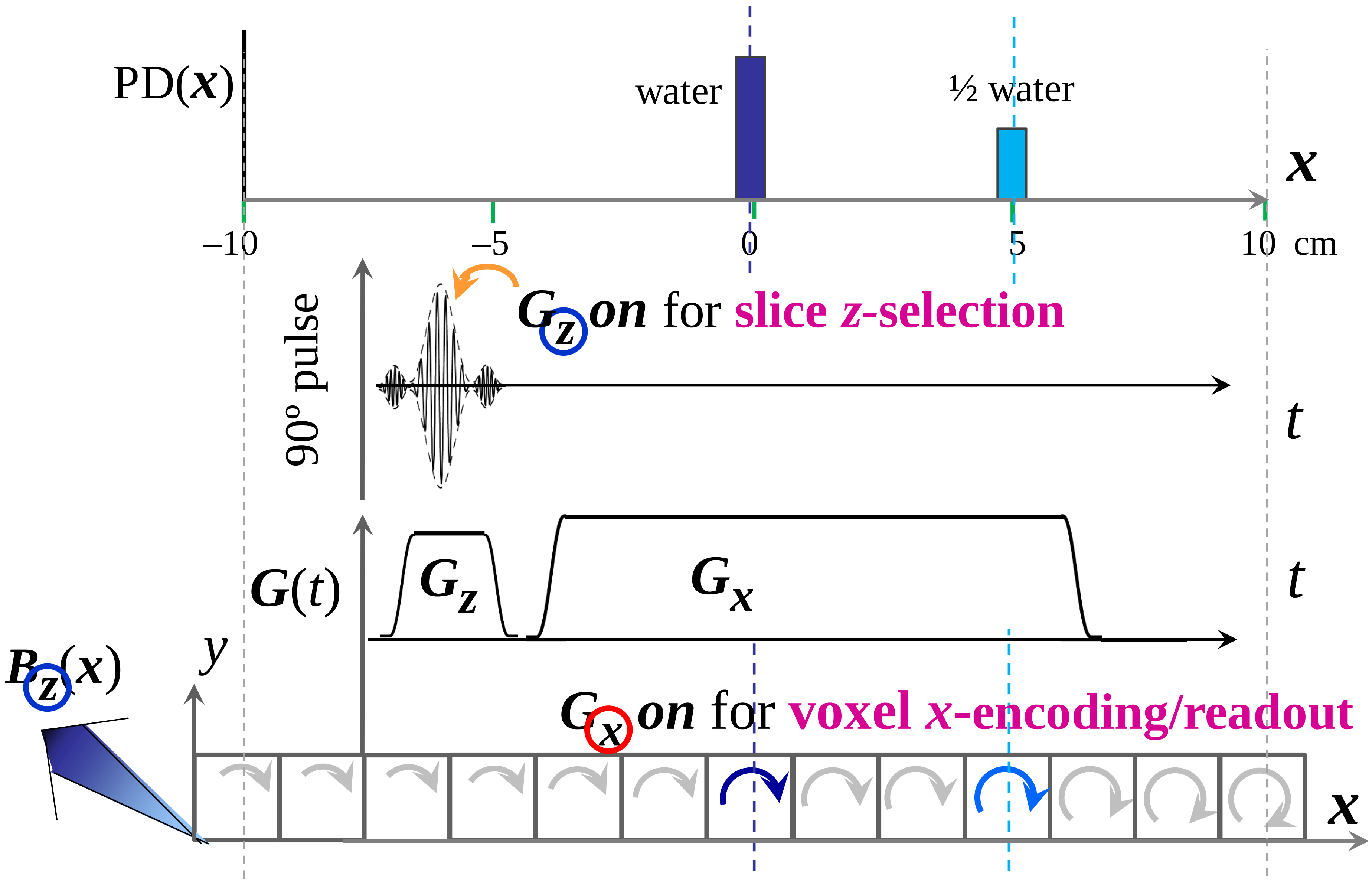


FID: Selecting the *z*-Slice that Contains the *x*-Row



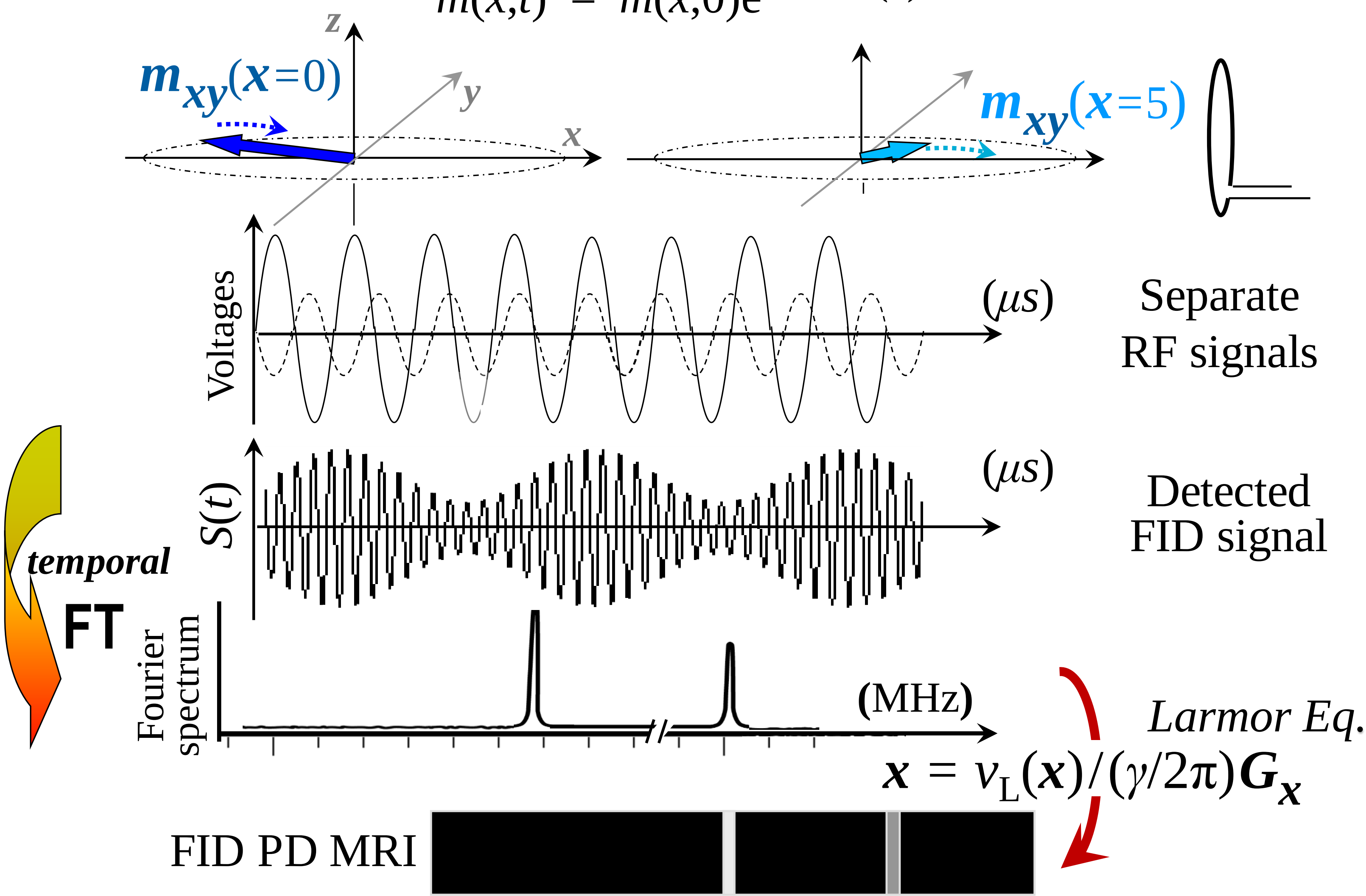


FID: 90°& G_z for *z-Slice Selection*, then G_x for *x-Encoding/Readout of Voxels*



FID: Following 90° Pulse, Precession at $\nu_{Larmor}(\boldsymbol{x})$

$$m(x,t) = m(x,0)e^{-2\pi i\,\nu(x)\,t}$$

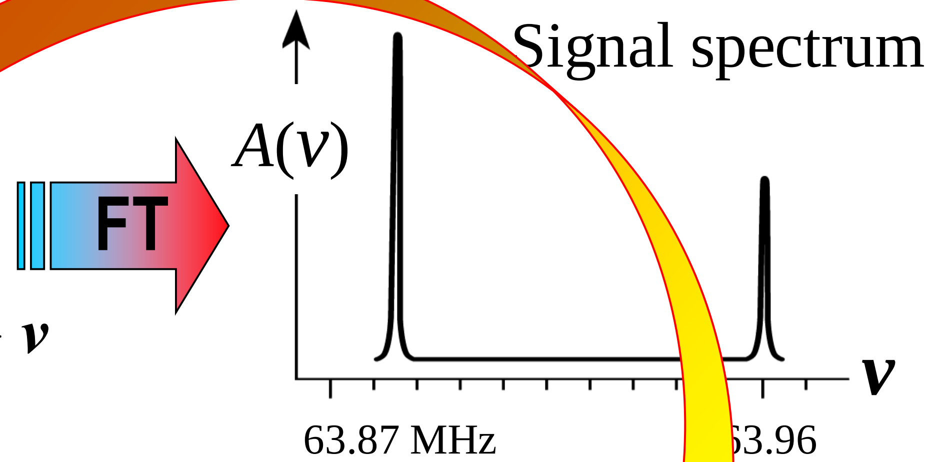


To Summarize What We Have Done So Far with FID

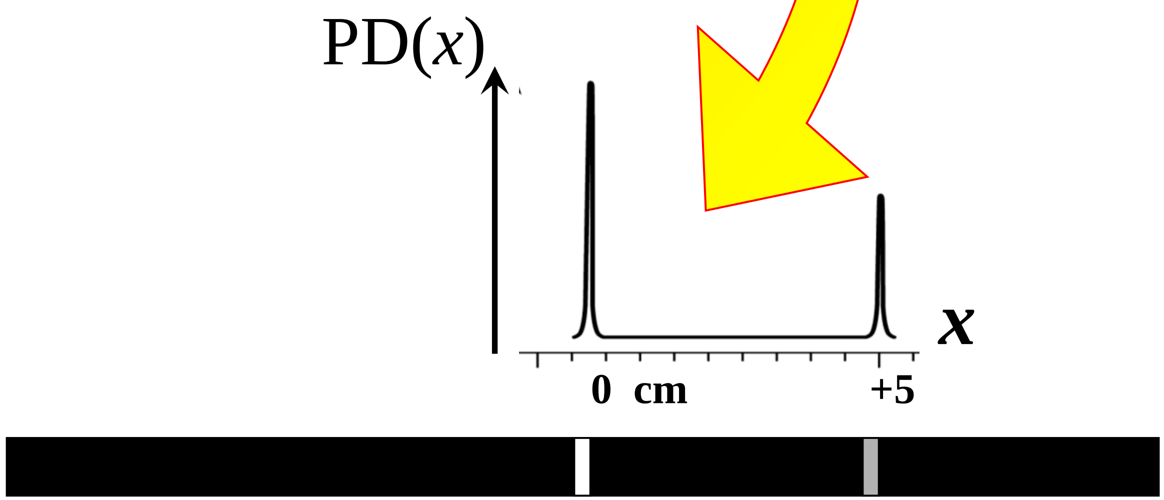
follow temporal FT with isomorphism of $\boldsymbol{A}(\boldsymbol{v})$ to real-space, $\boldsymbol{R}(\boldsymbol{x})$

$$A(v) = \int S(t)\, e^{-2\pi i\, v(t)\, t}\, dt$$

$\boldsymbol{t} \leftrightarrow \boldsymbol{v}$

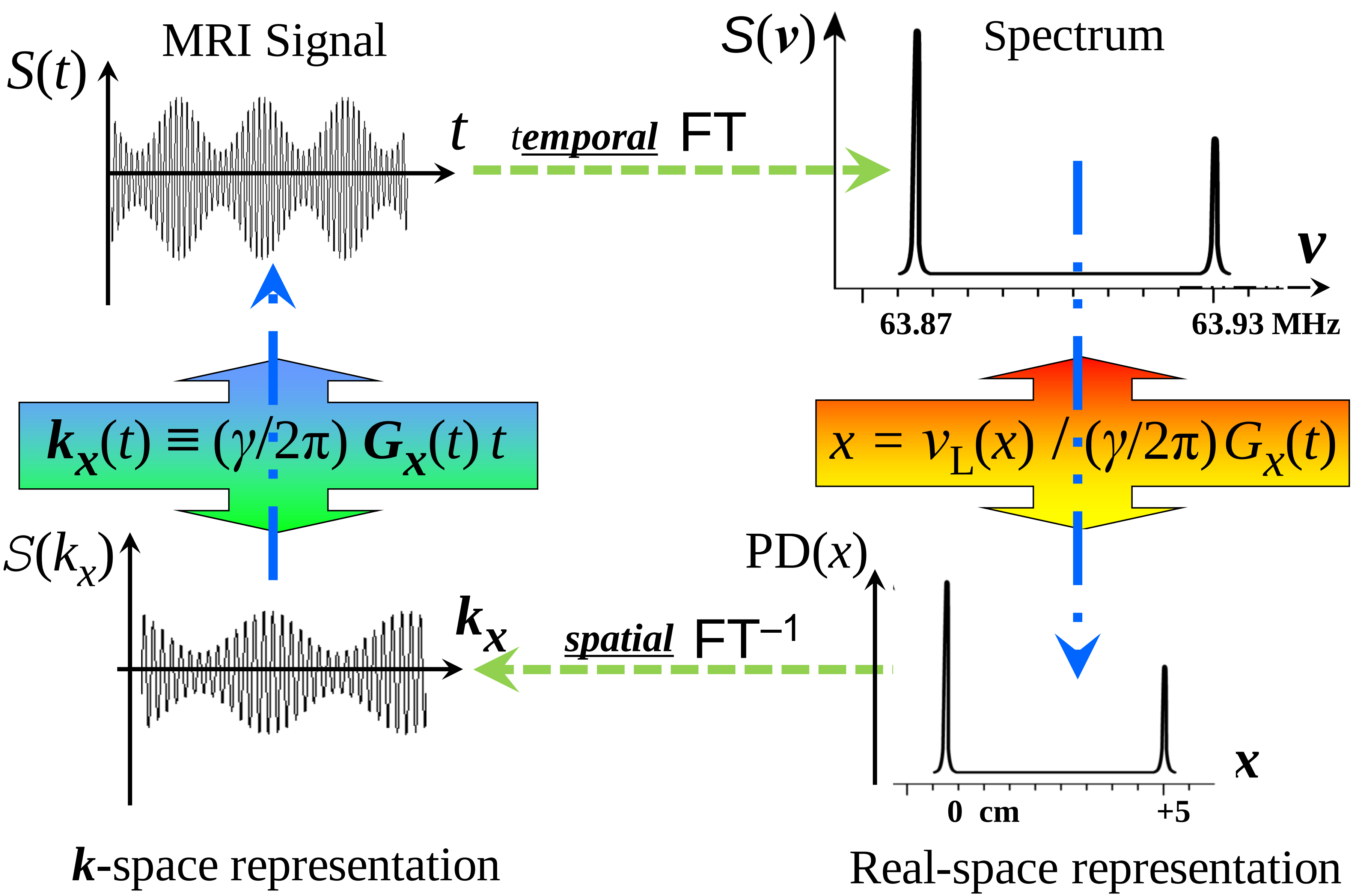


$$x = v_L(x) \;/\; (\gamma/2\pi)G_x$$



Real-space representations

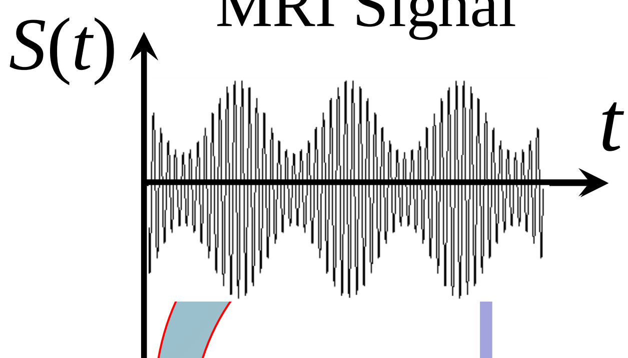
New: Keep on Going, Closing the Loop



Try Going the Other Way…. *k*-Space Approach!

necessary for 2D, 3D

MRI Signal

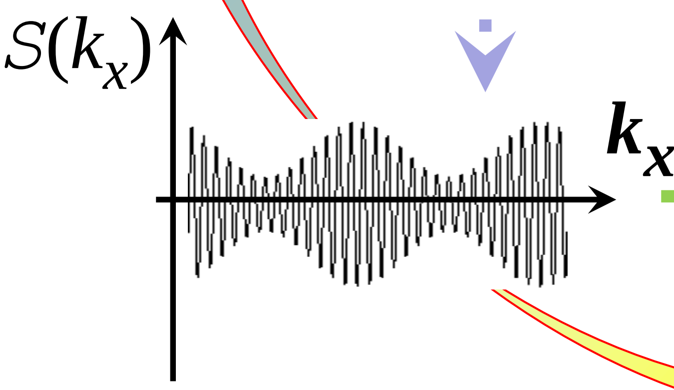


$k_x \equiv$ spatial frequency, $1/\lambda$.

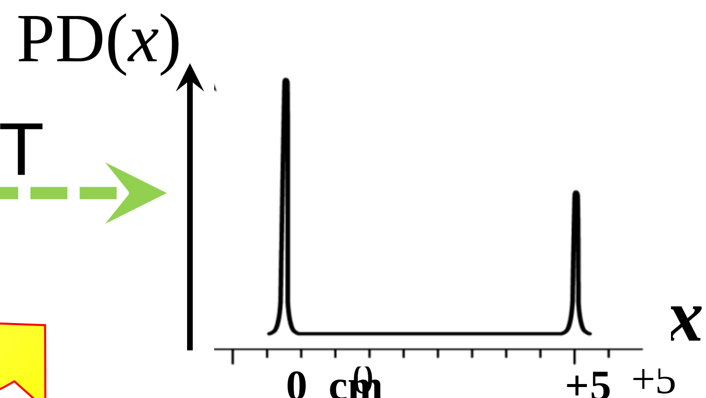
but also…

$$k_x(t) \equiv (\gamma/2\pi)\, G_x(t)\, t$$

$$PD(x) = \int S(x)\, e^{+2\pi i\, k_x x}\, dx$$



spatial FT



***k*-space representation**

Real-space representation

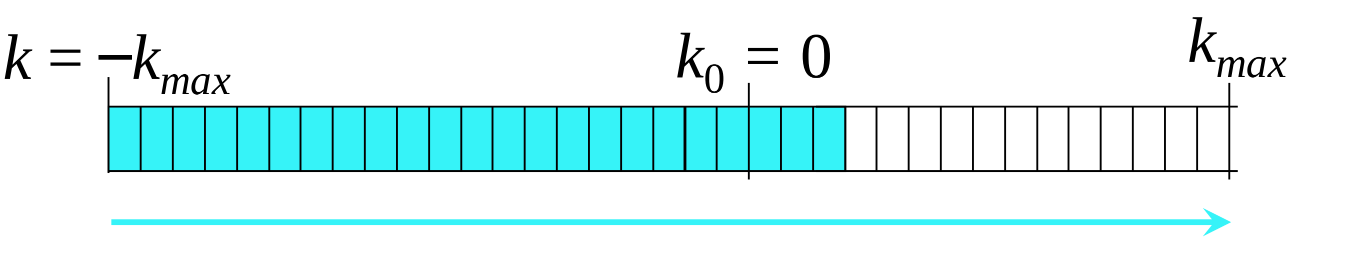
During Readout, k_x Increases Linearly with t

Signal is sampled sequentially 256 or 512 times spaced Δt apart.

$t_n = n\,\Delta t$ is the exact sampling time after G_x is turned on.

$\boldsymbol{k_x}(t)$ for all voxels increases linearly with t while the echo signal is being received and read: $\boldsymbol{k_n} = [\boldsymbol{G_x}\,\boldsymbol{\gamma} / 2\pi]\,\boldsymbol{t_n}$.

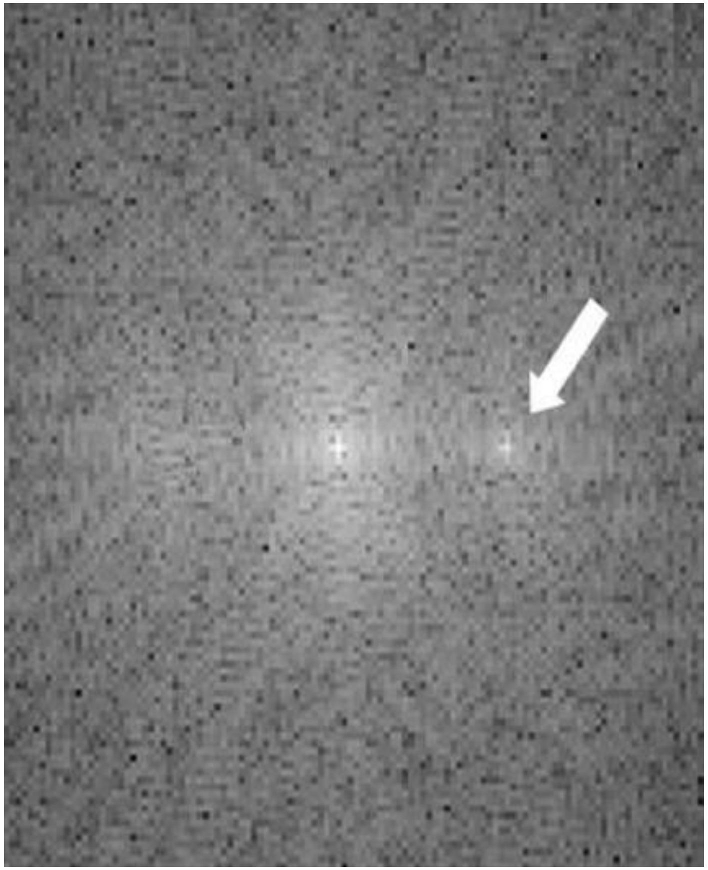
Larger magnitude k-values correspond to *greater spatial frequencies*!

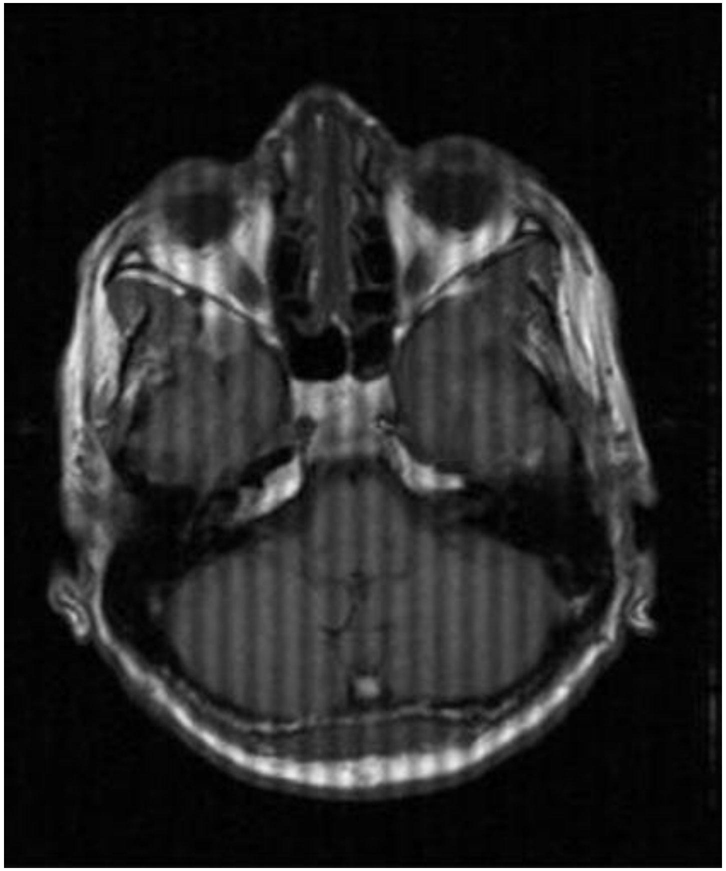


SE reads out from $k_n = -k_{max}$ to $k = 0$ to $k = k_{max}$

Herringbone Artifact

noise spike during data acquisition





T2 Spin Relaxation

T2 Relaxation

T2 Relaxation refers to the rate at which the transverse magnetization, $\boldsymbol{m}_{xy}(t)$, and the Echo signal it generates, Decay.

T2 relaxation results from ***T1-Events***, plus those from ***Non-Static, Random, Non-Reversible*** Proton-Proton Dipole Interactions.

Both Contribute to the Rate 1/T2***!***

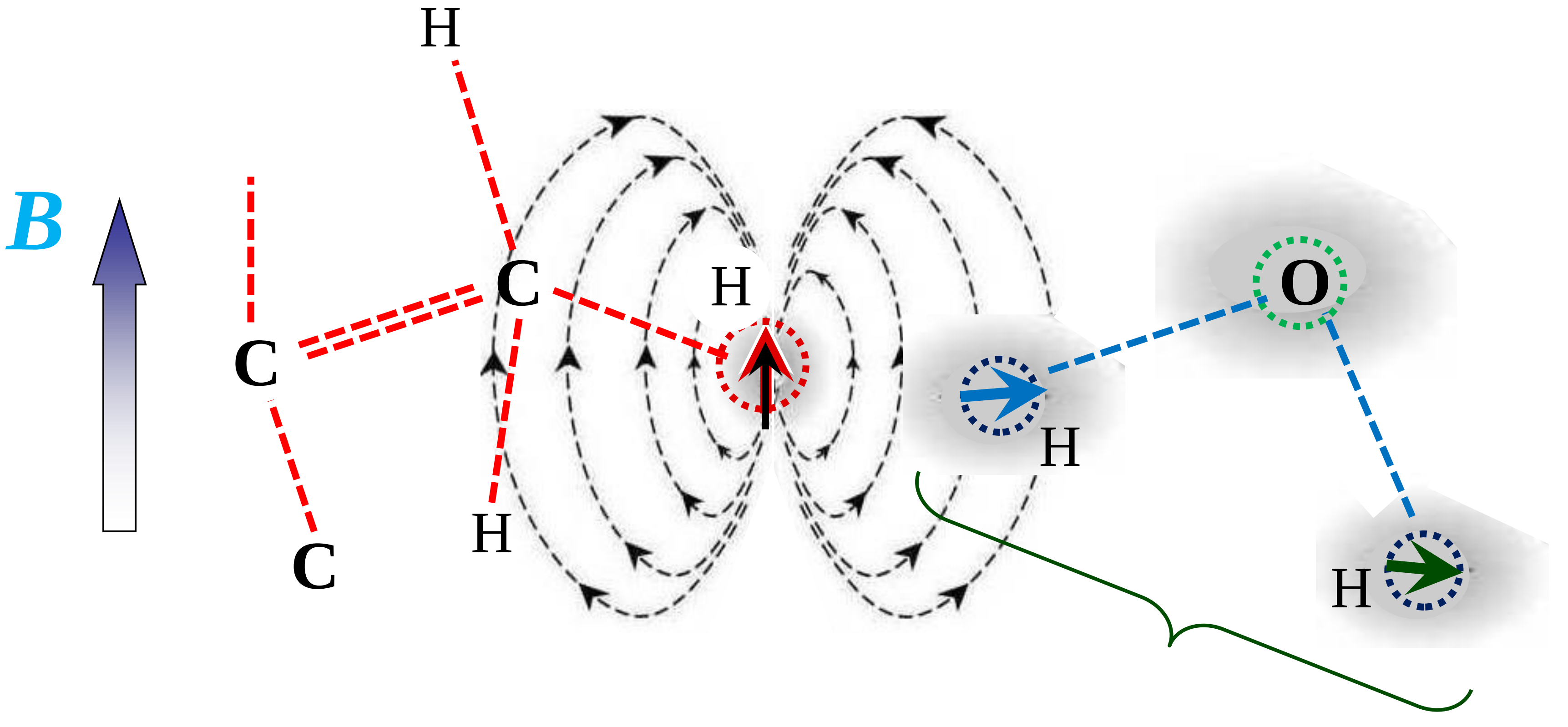
Dipole interactions:

1) Proton fields overlap, alter $\boldsymbol{\nu}_{\text{Larmor}}$;
2) Exchange of spin

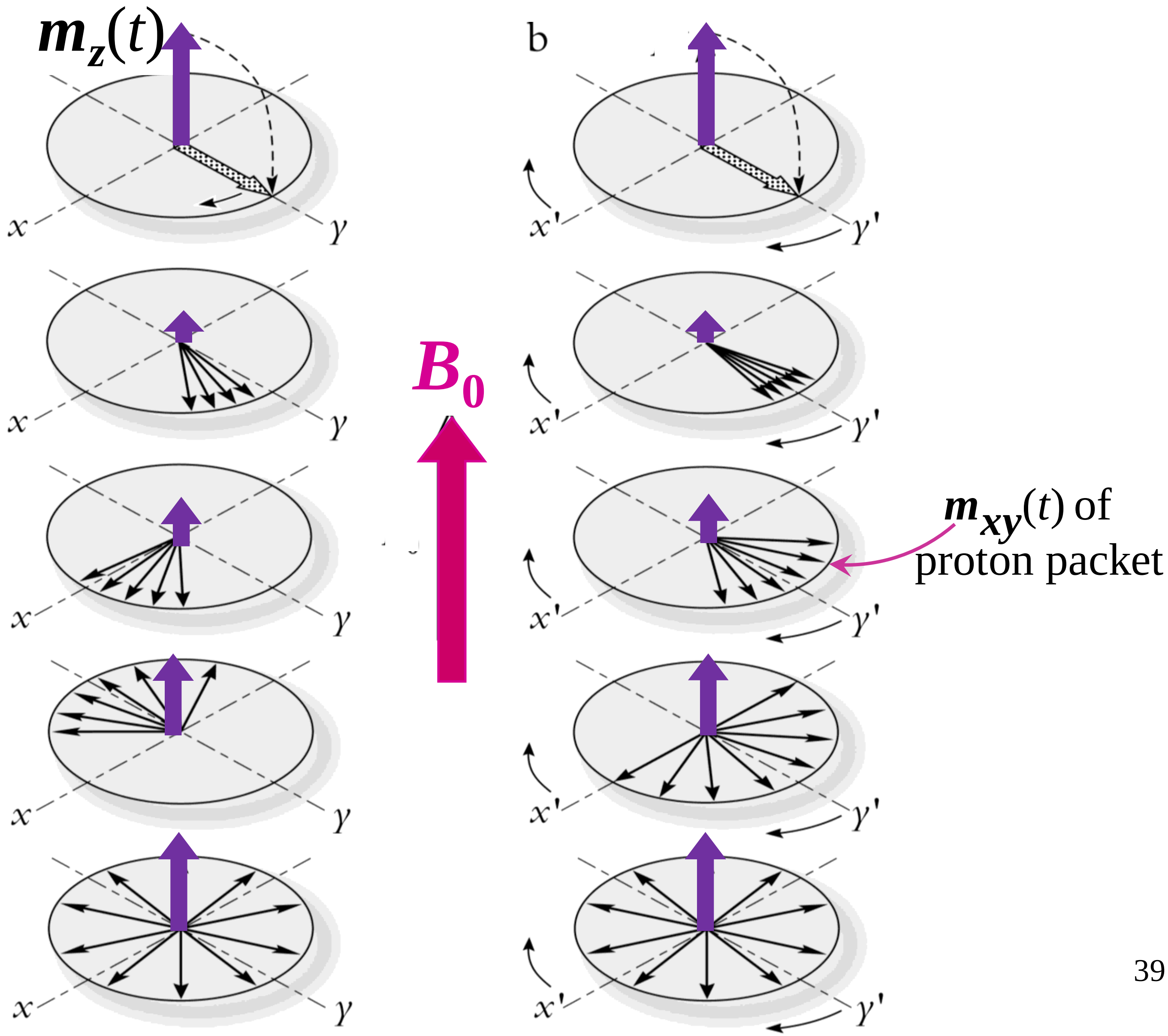
neither involves an energy transfer

Secular Component of T2 Relaxation Mechanism

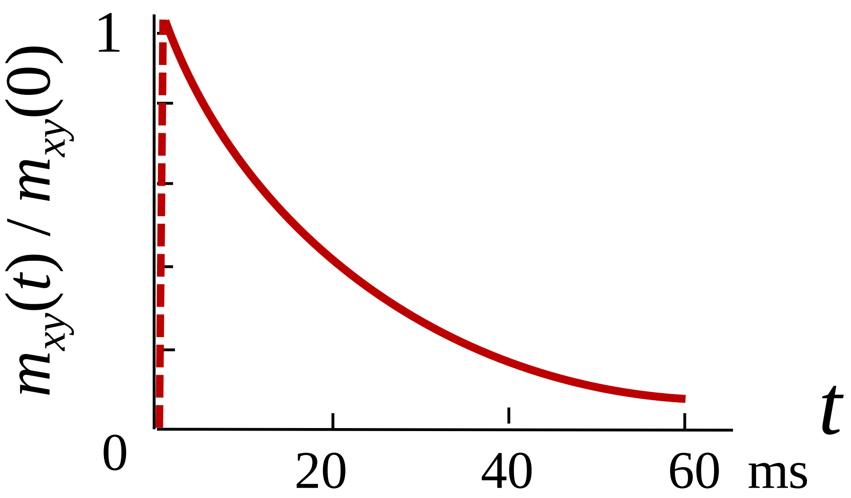
quasi-static spin-spin interactions
not spin flips.



Phase Loss: T2 De-coherence of Proton Packets in Voxel



Exponential T2-Caused De-Phasing of $\boldsymbol{m}_{xy}(x,t)$ in x-y Plane



$$d\boldsymbol{m}_{xy}(t) / dt = -(1/\text{T2})\, \boldsymbol{m}_{xy}(t)$$

$$m_{xy}(t) / m_{xy}(0) = e^{-t/\text{T2}}$$

$$d\boldsymbol{m}(x,t)/dt = \gamma\, \boldsymbol{m}(x,t) \times \boldsymbol{B_z}(x) - \frac{[m_z(x,t) - m_0(x)]\hat{\boldsymbol{z}}}{\text{T1}} - \frac{[m_x\hat{\mathbf{x}} + m_y\hat{\boldsymbol{y}}]}{\text{T2}}$$

T2: Loss of Phase of Voxel Packets in the *xy*-Plane

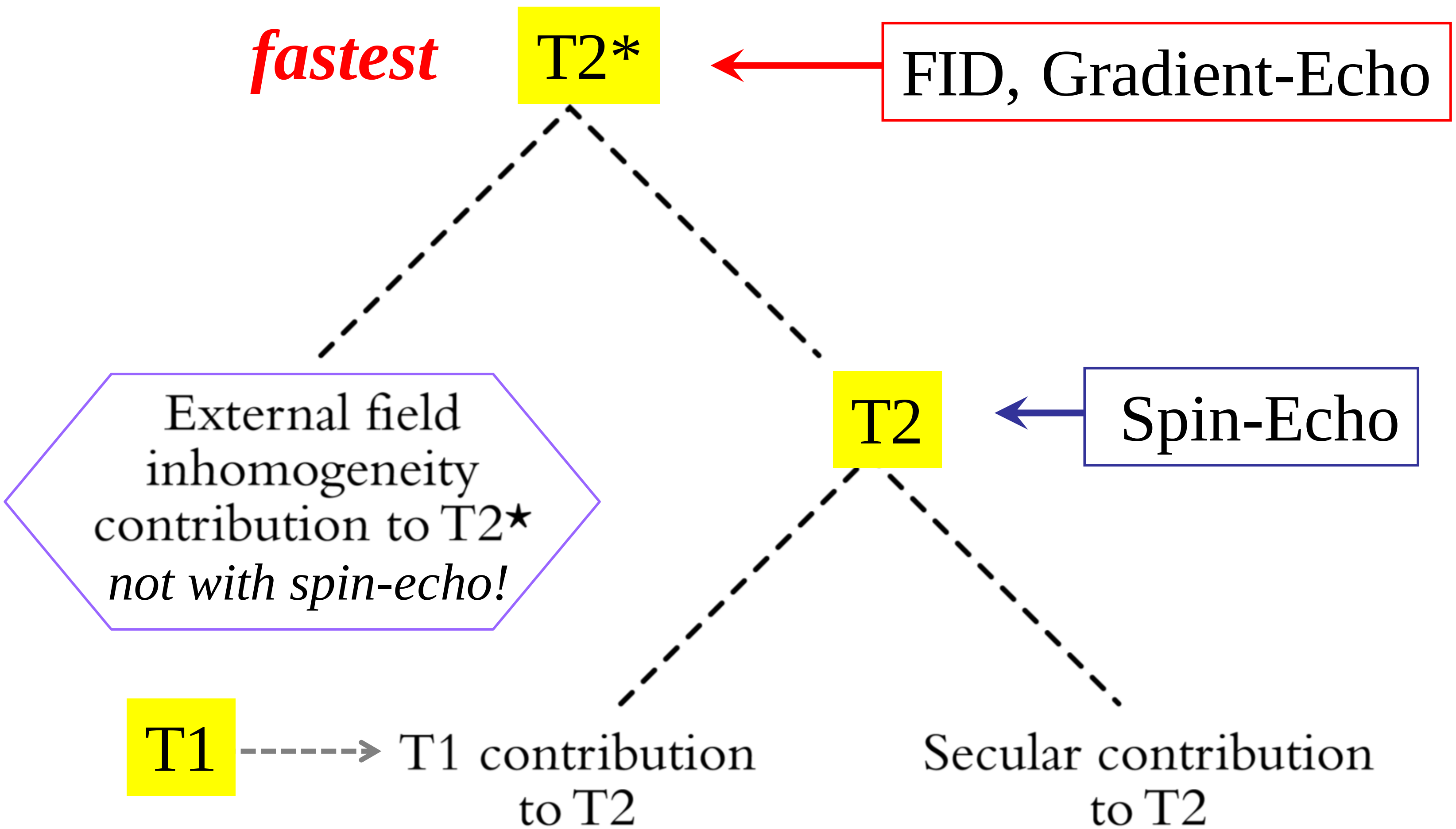
#3: $$\boldsymbol{m}_{xy}(t)\,/\boldsymbol{m}_{xy}(0) = e^{-t\,/\,T2}$$

Typical T1 and T2 Relaxation Times

relaxation rates: **1/T2 ~ 10 × (1/ T1)**

Tissue	PD p^+/mm^3, rel.	T1, *1 T* (ms)	T1, *1.5 T* (ms)	T1, *3 T* (ms)	T2 (ms)
pure H_20	1	4000		4000	4000
brain					
CSF	0.95	2500	2500	2500	200
white matter	0.6	700	800	850	90
gray matter	0.7	800	900	1300	100
edema			1100		110
glioma		930	1000		110
liver			500		40
hepatoma			1100		85
muscle	0.9	700	900	1800	45
adipose	0.95	240	260		60

One Last Member of the Spin-Relaxation Family Tree: T2*



Discrete spin dephasing, involves ΔE	Continuous spin dephasing, *no* ΔE!