

# Outline for Today

1. Introduction to MRI

---

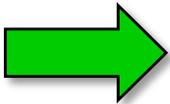
  - 'Quantum' NMR and MRI in 0D
  - Magnetization,  $m(x,t)$ , in a Voxel
  - Proton T1 Spin Relaxation in a Voxel
  - Proton Density MRI in 1D
  - MRI Case Study, and *Caveat*
2. 

---

  - Sketch of the MRI Device
  - 'Classical' NMR in a Voxel
  - Free Induction Decay in 1D
  - T2 Spin-Relaxation
3. 

---

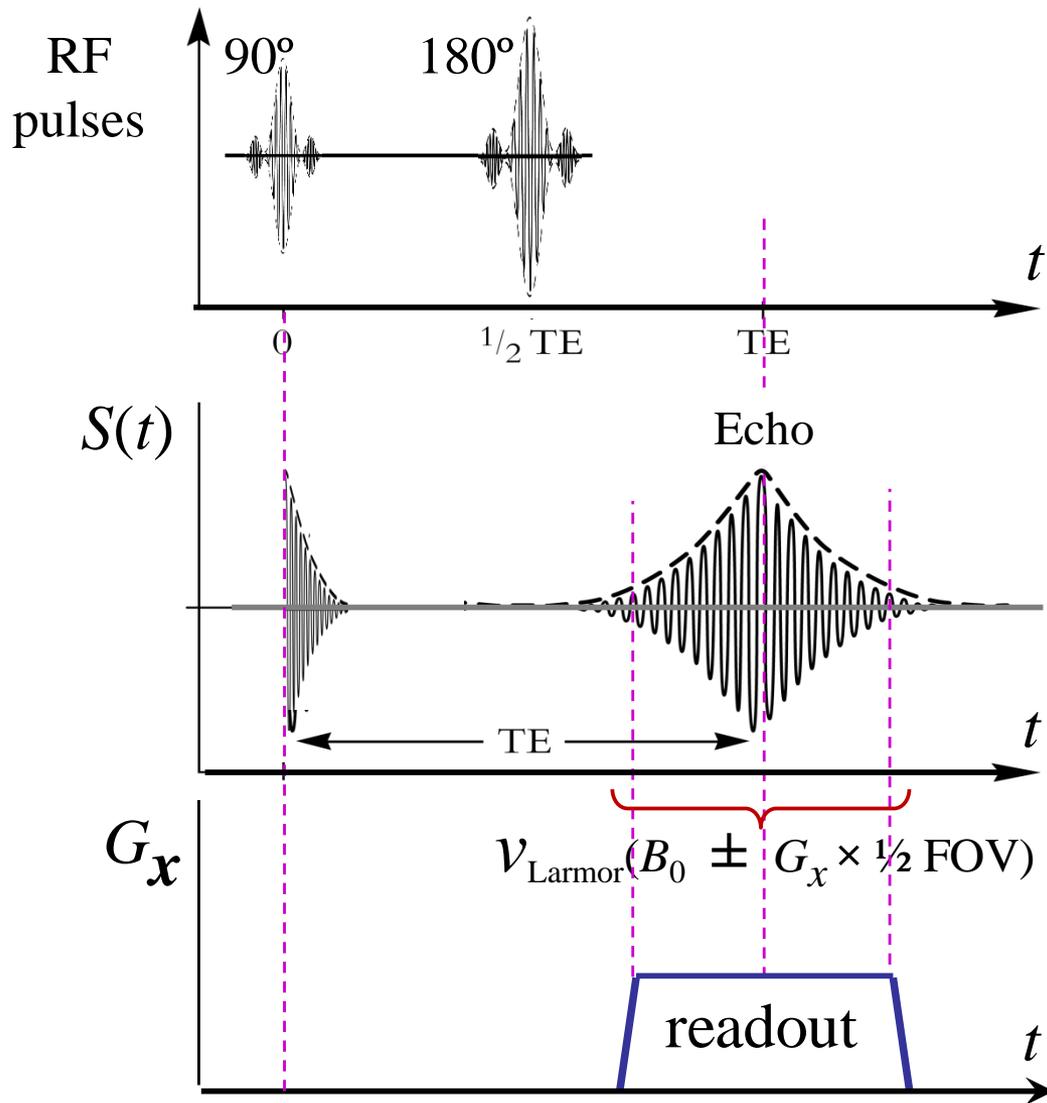
  - Spin-Echo Acquisition
  - Tissue Contrast-Weighting in SE
  - Spin-Echo / Spin-Warp in 2D



# Spin-Echo

# SE Sequence: 90°, 180° RF Pulses, Readout Gradient

e.g., 256 voxels in a row



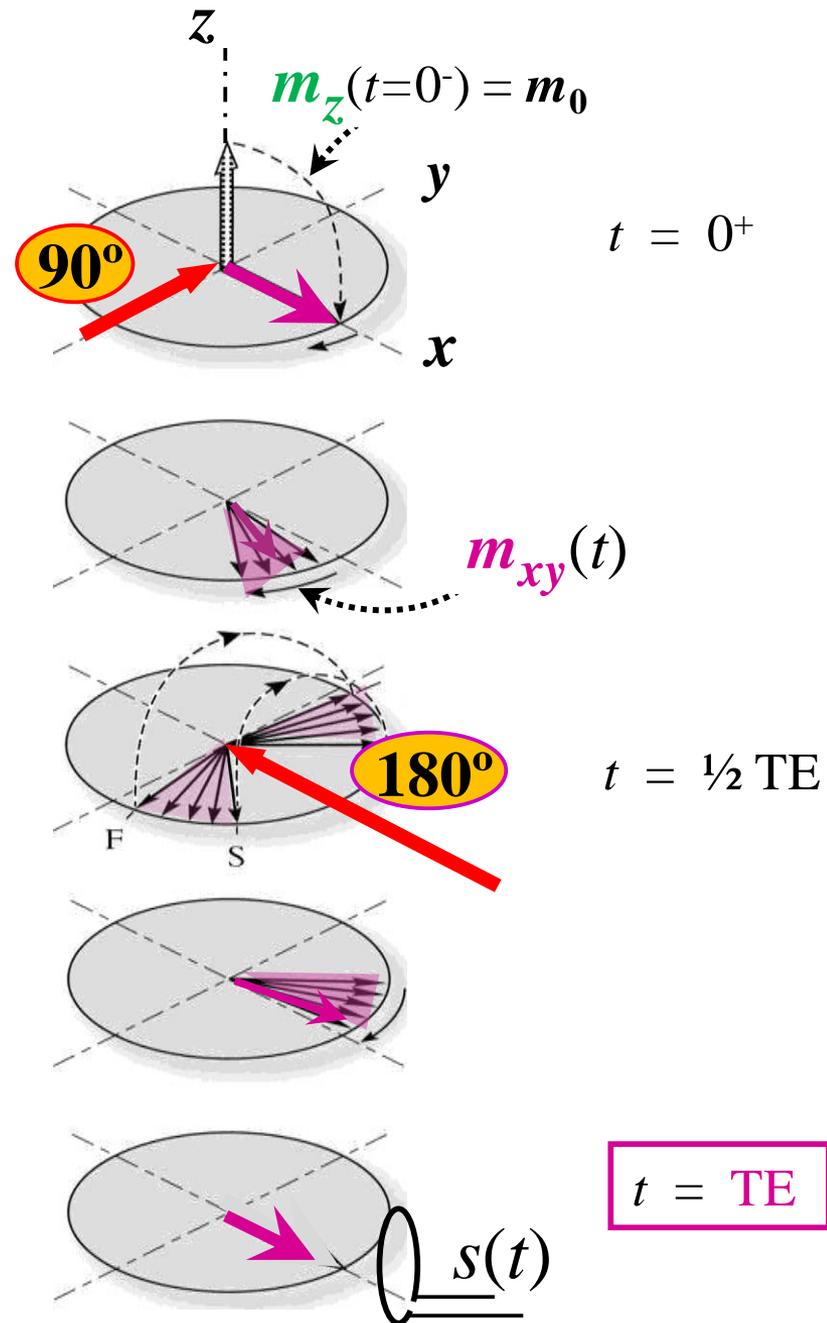
# Spin-Echo

Generates an *Echo*

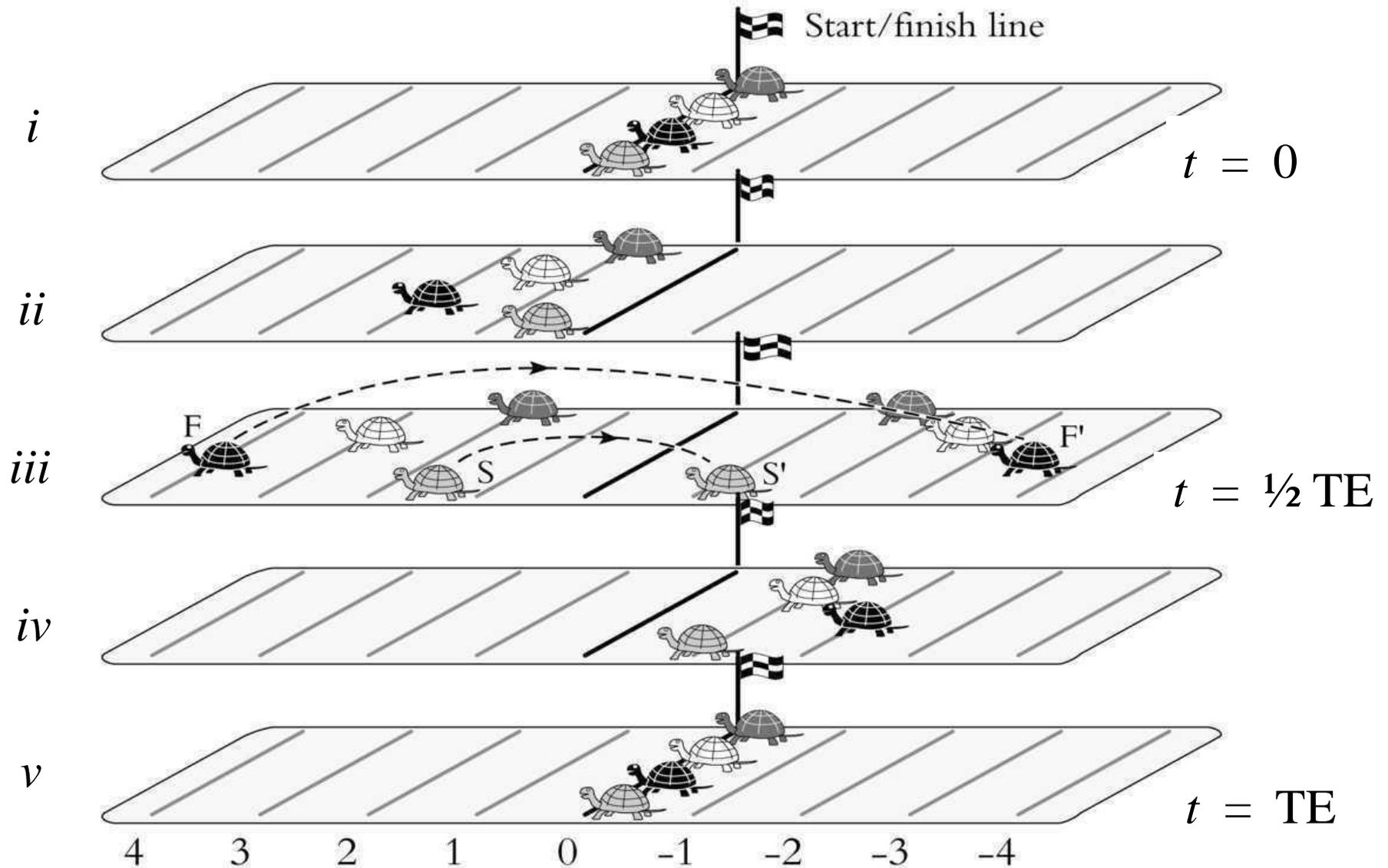
SE introduces a  $180^\circ$  inversion pulse to refocus the spins and create an echo.

Also results in higher SNR reduction of susceptibility contamination.

$$m_{xy}(t=TE) \sim m_0 \times e^{-TE/T_2}$$

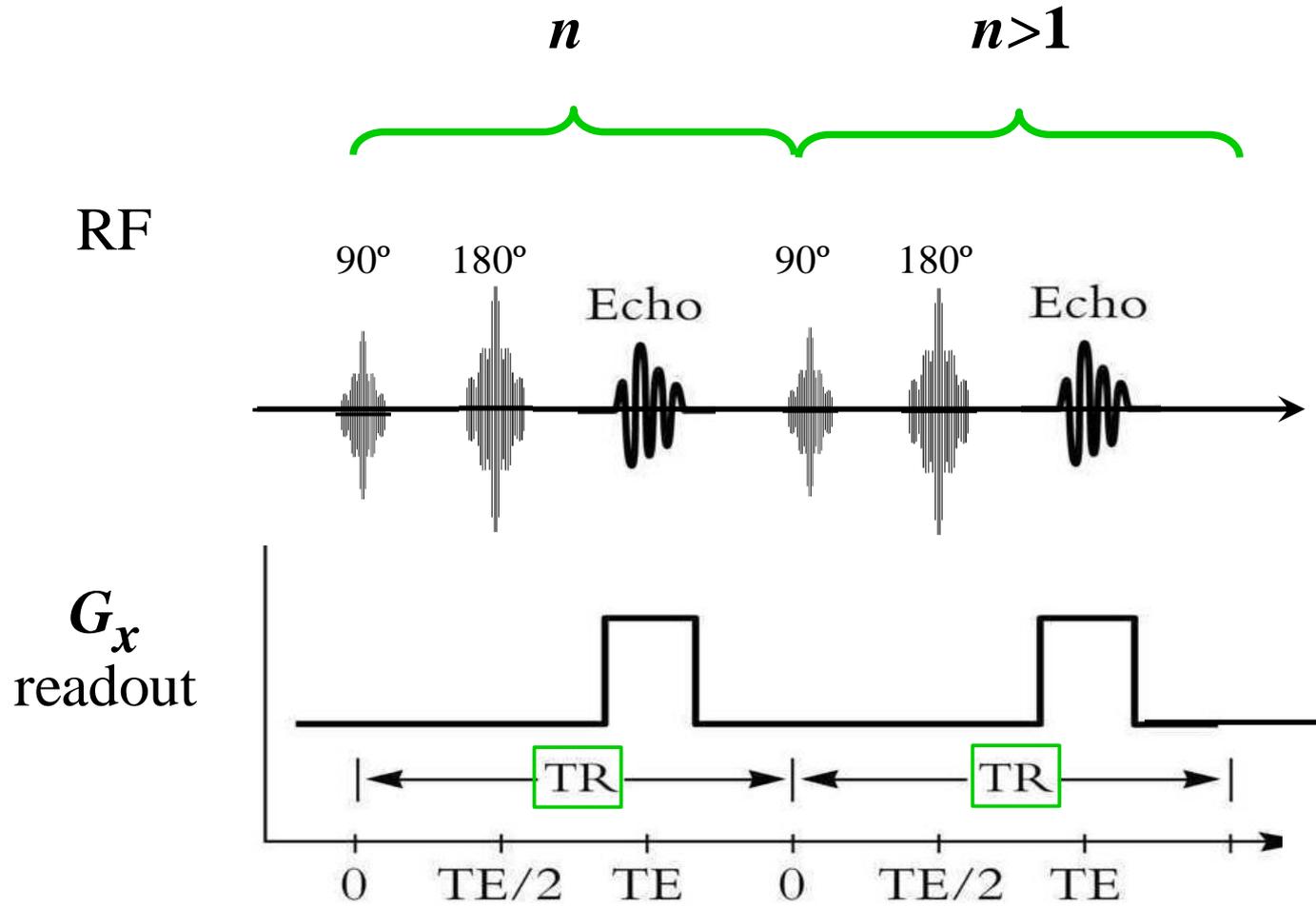


# Kentucky Turtle-Teleportation Derby (illustration of S-E)



# Multi-SE Pulse Sequence

(e.g., what happens to  $m_z$ ?)



# Spin-Echo

Generates an *Echo*

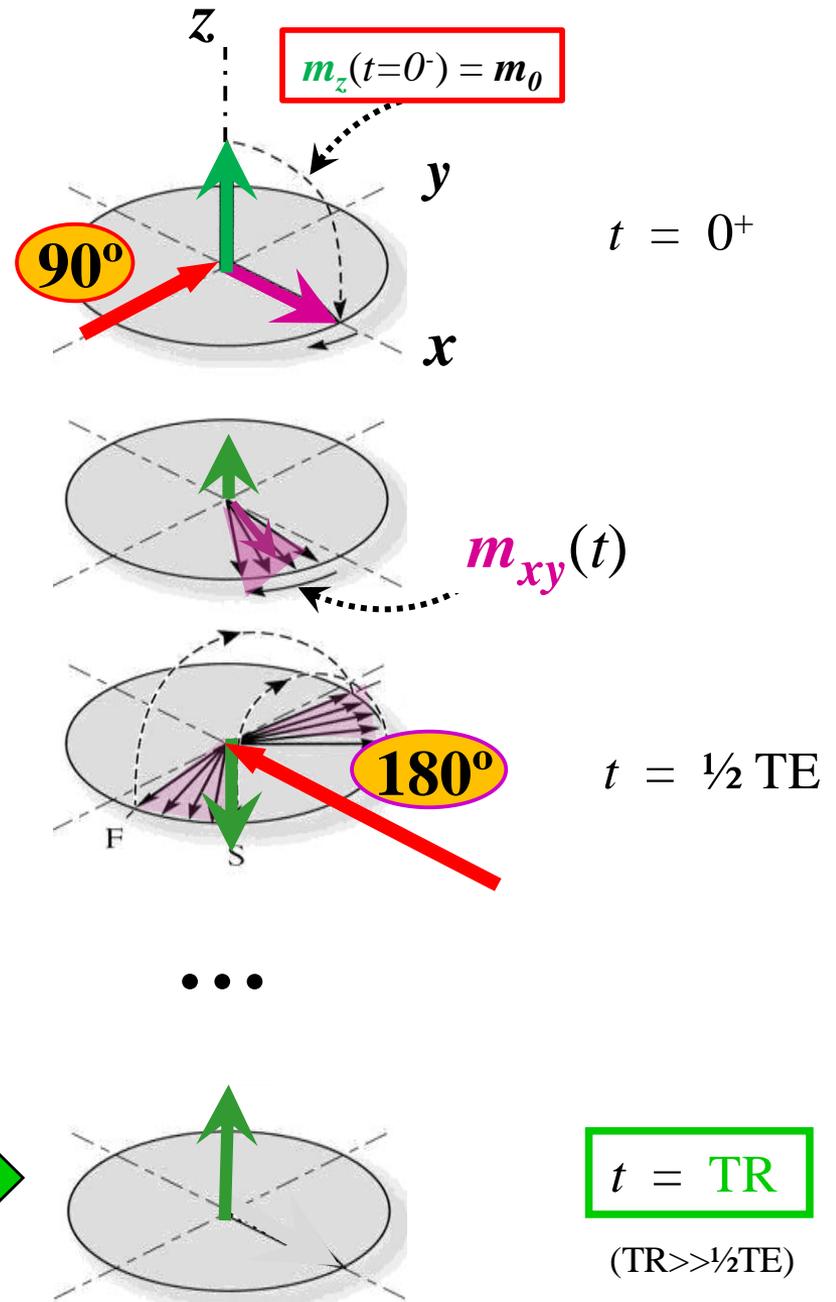
$$n=1$$

At end of first excitation, magnetization ( $m_z$ ) may not have fully recovered.

Depends on the tissue T1 and sequence TR.

This magnetization is passed on to the next excitation ( $n>1$ ).

$$m_z(t=TR^-) \sim m_0 \times (1 - e^{-TR/T1}) \rightarrow$$



# Spin-Echo

Generates an *Echo*

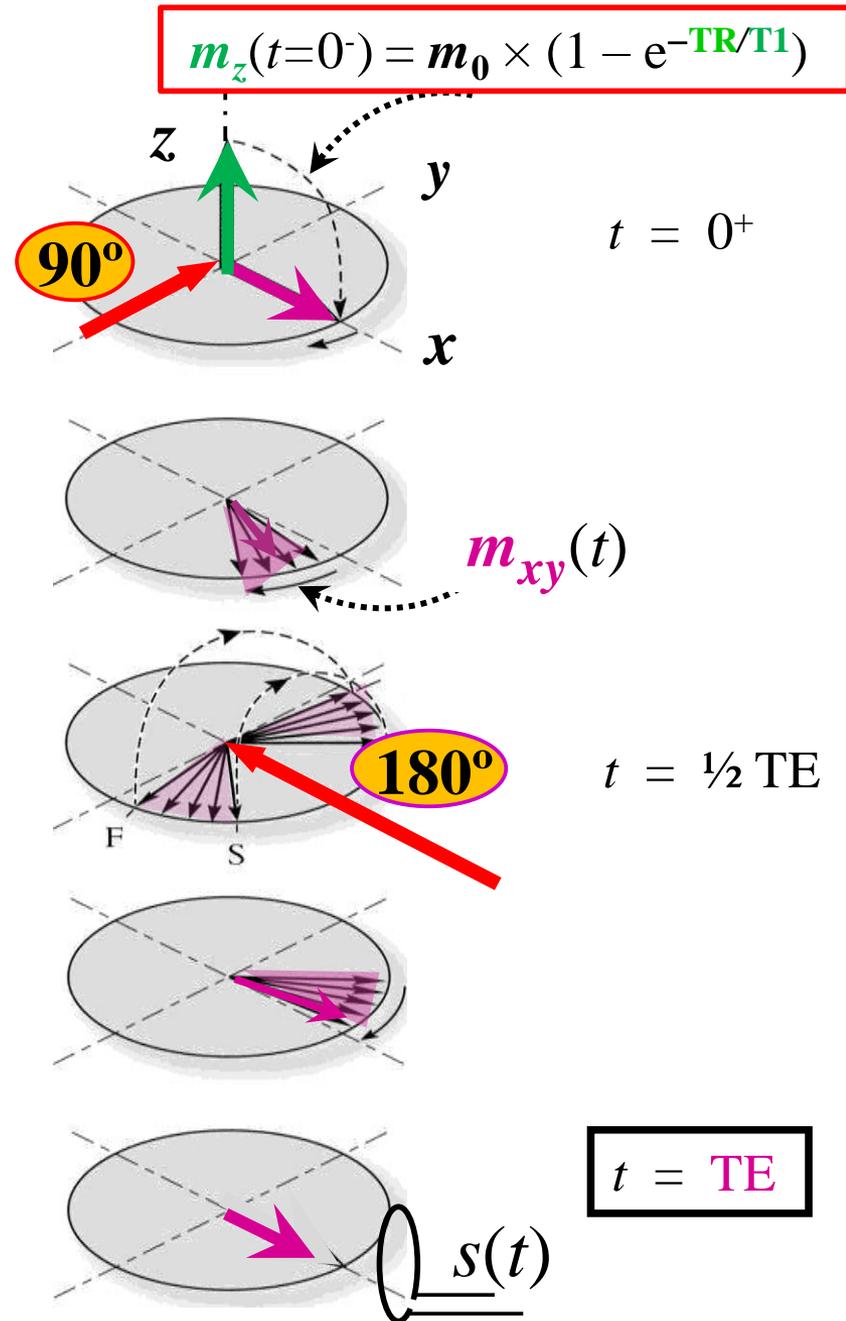
$$n > 1$$

After subsequent excitation, longitudinal magnetization tipped into transverse plane is a constant function of tissue T1 & TR.

Additional differences in the signal result of tissue T2-induced changes until measurement at TE.

$$m_z(t=0^-) \sim m_0 \times (1 - e^{-TR/T1})$$

$$m_{xy}(t=TE) \sim m_z(t=0^-) \times e^{-TE/T2}$$



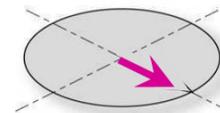
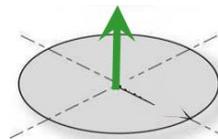
# MRI Signal Strength *at* $t = TE$ Depends on....

$$s(t = TE) \sim PD (1 - e^{-TR/T1}) e^{-TE/T2}$$

proton  
density

prior  
regrowth  
along **z-axis**

current  
dephasing  
in **x-y plane**



Inherent Parameters

**T1**

**T2**

PD

Operator Settings

**TR**

**TE**

time since  
previous  
excitation



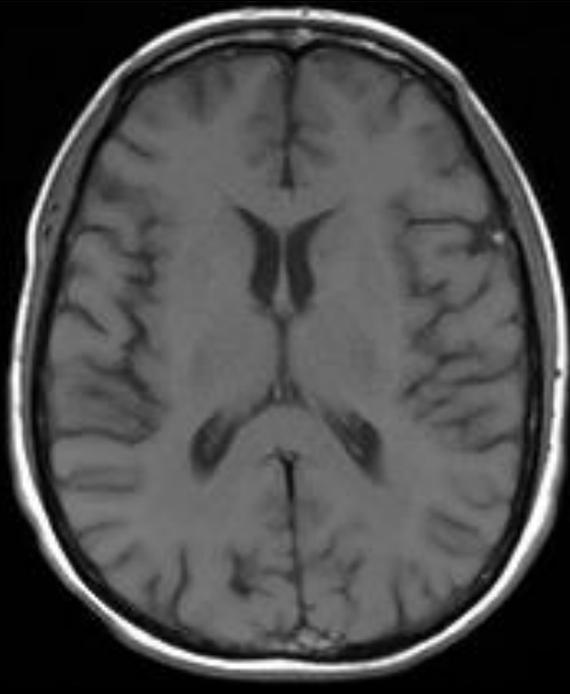
# Tissue Contrast-Weighting in SE

T1-*w*, T2-*w*, and PD-*w*

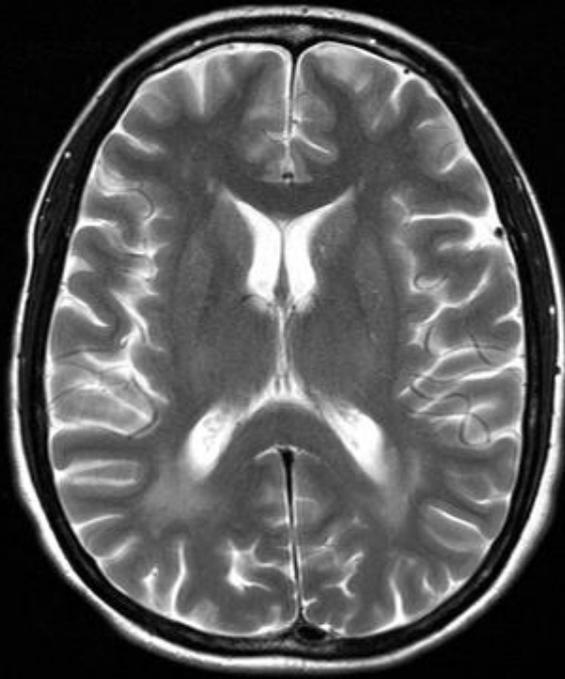
# Three Different MRI Contrast Weightings

---

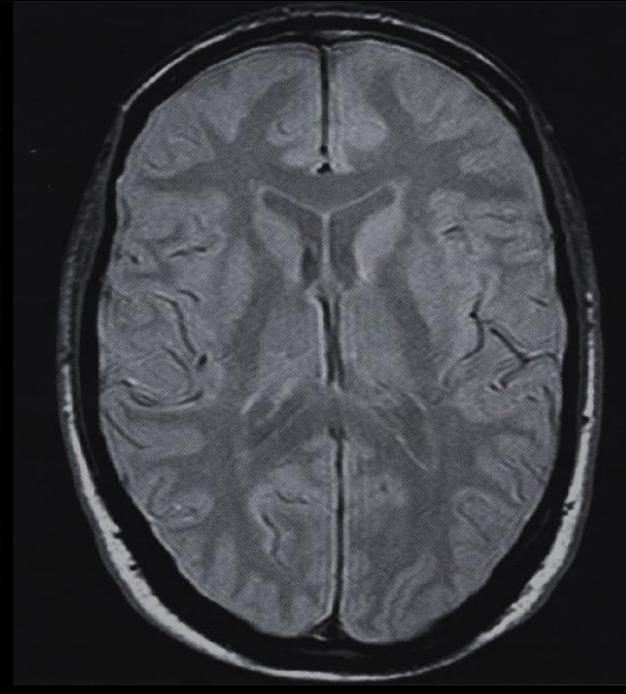
created by, and reflecting, three quite different physical processes



T1



T2



PD

# T2-w – Long TR to Minimize T1 Differences

For T2-w  
imaging

Long TR:  
eliminate  
T1 impact

TE at mid-T2:  
maximize  
contrast

$$S(t \sim TE) \sim PD \frac{(1 - e^{-TR/T1})}{1} e^{-TE/T2}$$



prior  
regrowth  
along *z*-axis



current  
de-phasing  
in *x-y* plane

T2-w



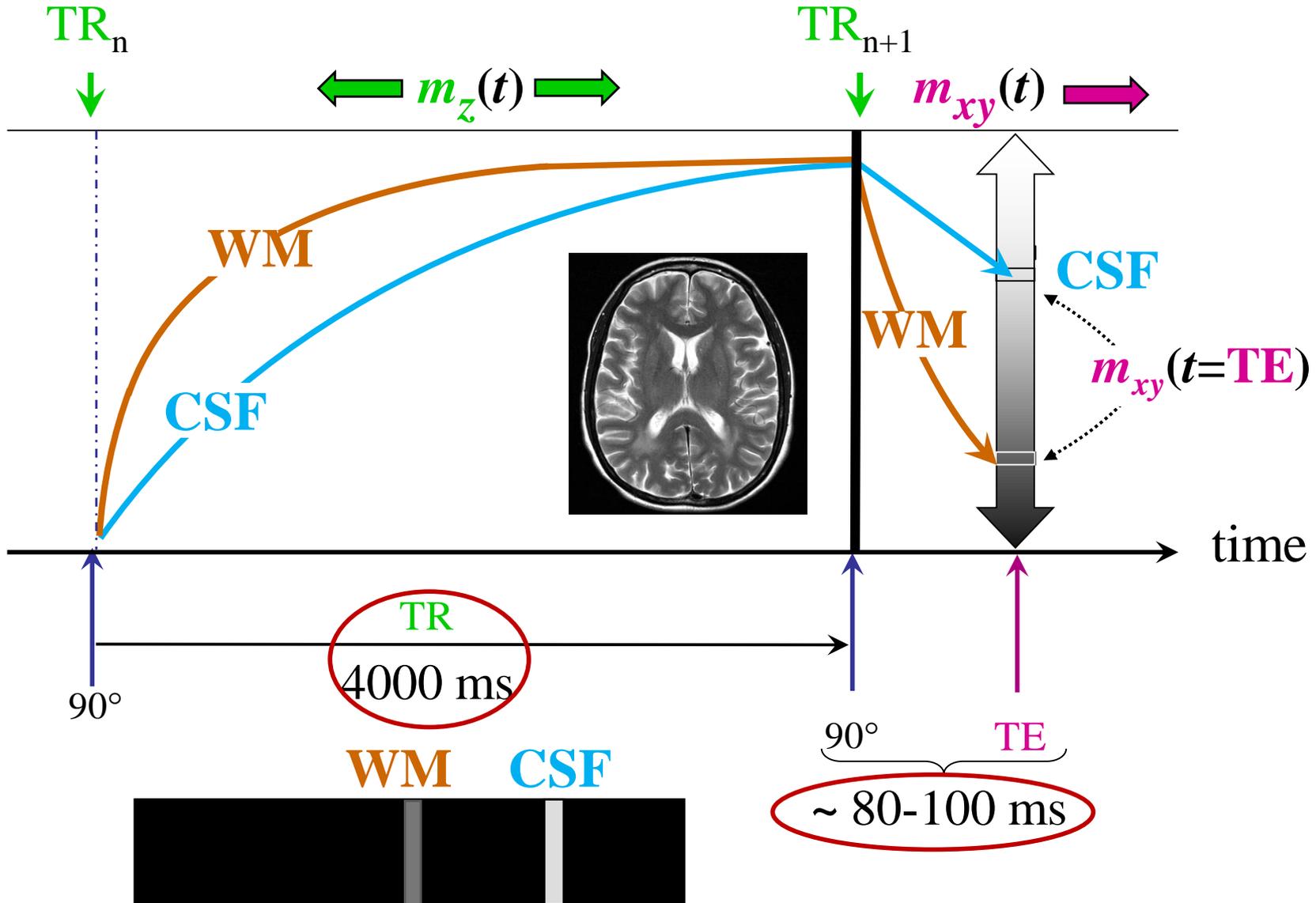
*Long-T2 tissues bright*

WM = White Matter

CSF = Cerebral Spinal Fluid

# T2-w – Long TR to Minimize T1 Differences

TE ~ mid-T2 to maximize T2 contrast



# T1-weighted – Short TE to Minimize T2 Changes

TR ~ T1<sub>av</sub>  
to maximize  
contrast

Short TE  
to minimize  
T2 differences

$$S(t \sim TE) \sim PD (1 - e^{-TR/T1}) \frac{e^{-TE/T2}}{1}$$

proton  
density

prior  
regrowth  
along z-axis

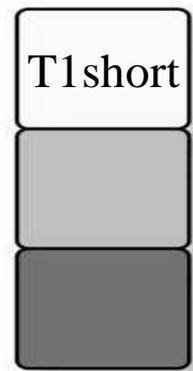
current  
de-phasing  
in x-y plane

T1-w



WM

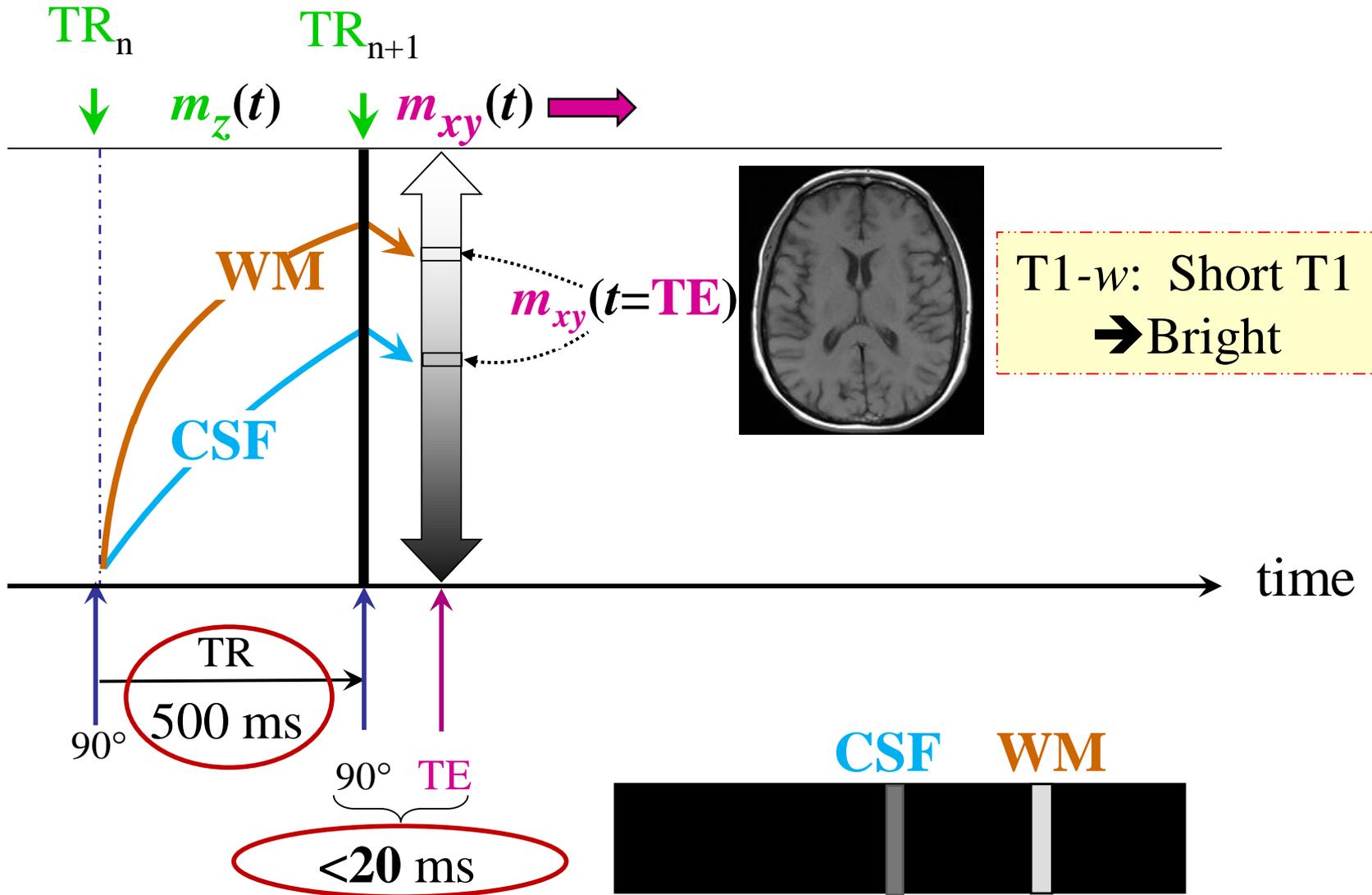
CSF



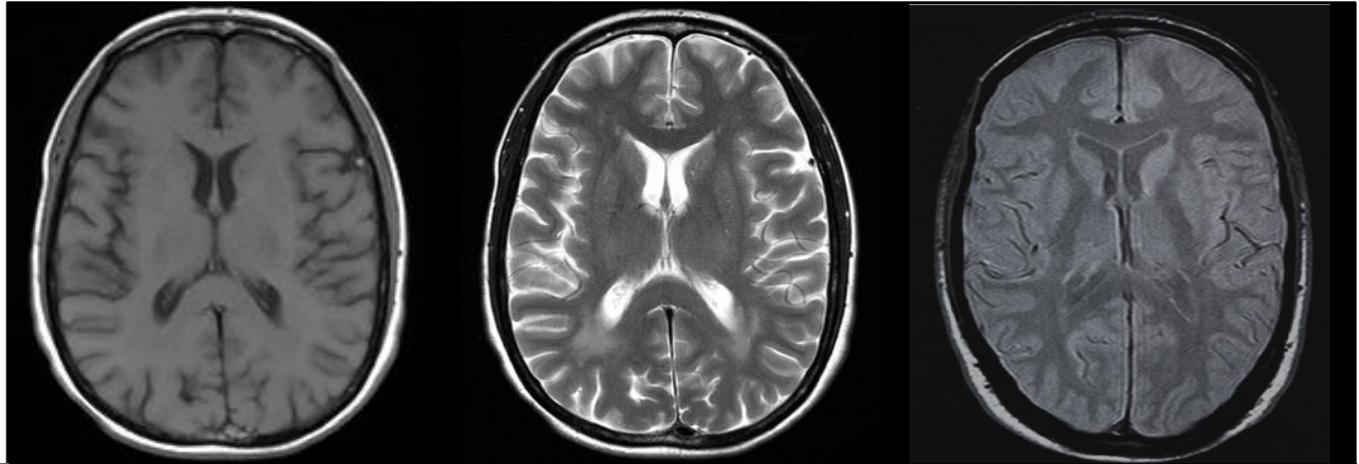
*For T1-w: Short-T1 tissues are Bright*

# T1-weighted – Short TE to Minimize T2 Changes

TR  $\sim$  T1<sub>avg</sub> to maximize T1 contrast



# PD-, T1-, & T2-Weighted Spin-Echo Images (1.5 T)



Contrast	T1-weighted	T2-weighted	PD-weighted
TR (ms)	Mid $\sim T1_{avg}$ 300-700	Long $\geq 2,000$	Long $\geq 2,000$
TE (ms)	Short $\leq 20$	Mid $\sim T2_{avg}$ 60-150	Short $\leq 20$
Bright	Short T1	Long T2	High PD
SNR	Middle	Lower	Best

# Spin-Echo / Spin-Warp in 2D

# Signal Modulation in 2D Spatial Encoding

---

- Signal in 2D plane after slice excitation (rotating frame)

$$S \propto \iint S(x, y) \cdot dx \cdot dy$$

- What happens if we turn on our *linear gradients*?

$$S(G_x, G_y) \propto \iint S(x, y) \cdot \exp[-2\pi i \gamma \int (G_x \cdot x + G_y \cdot y) d\tau] \cdot dx \cdot dy$$

- This equation can be expressed in the form of a 2D FT of  $S(x, y)$

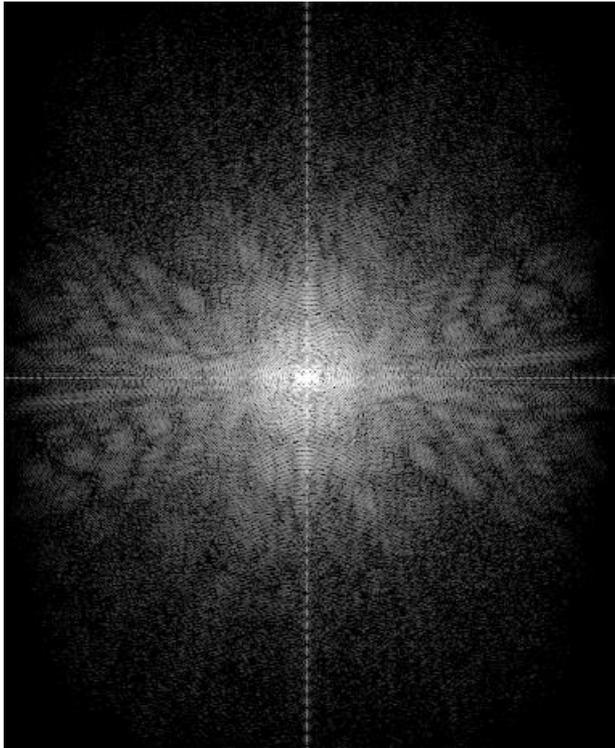
$$S(k_x, k_y) \propto \iint S(x, y) \cdot \exp[-2\pi i (k_x \cdot x + k_y \cdot y)] \cdot dx \cdot dy$$

$$\text{with } k_x(t) = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y(t) = \gamma \int_0^t G_y(\tau) d\tau$$

# Fourier Transform of MRI Data in $k$ -Space to Real Space

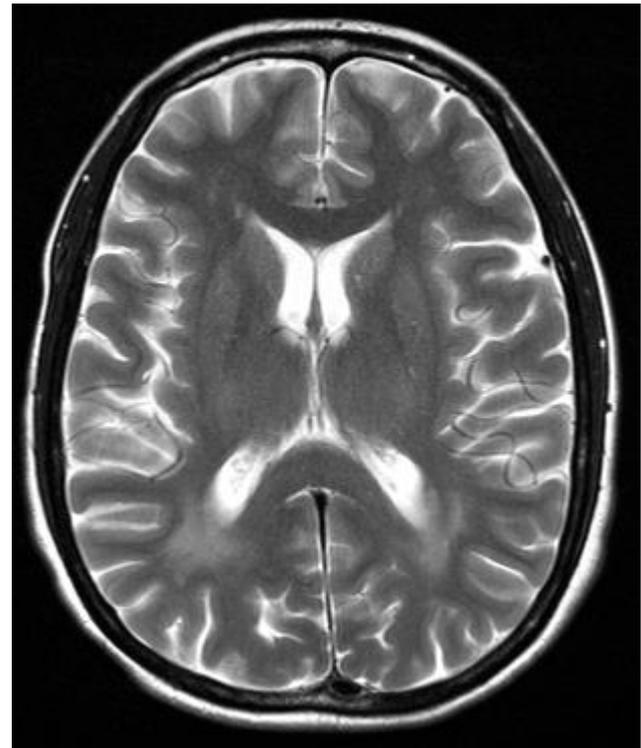
$$S(k_x, k_y)$$

Complex  $k$ -space



$$|S(x, y)| \sim |\text{FT}\{S(k_x, k_y)\}|$$

Real-space



# 2D Spatial Encoding

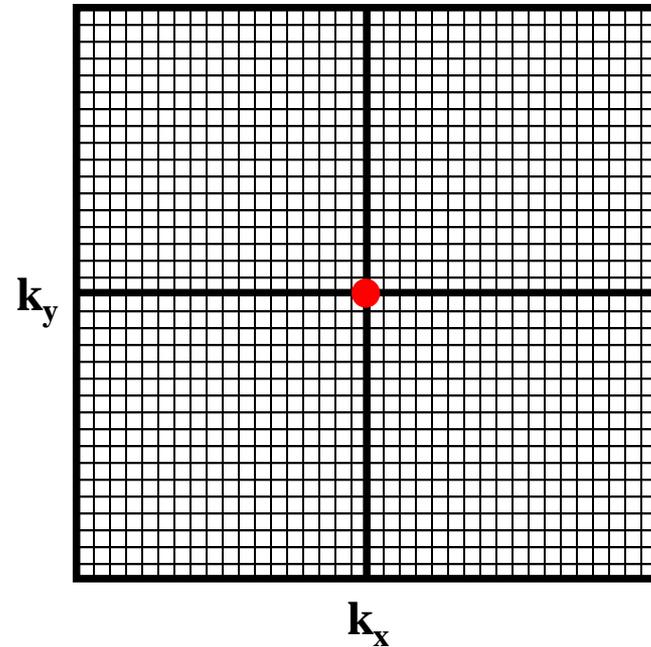
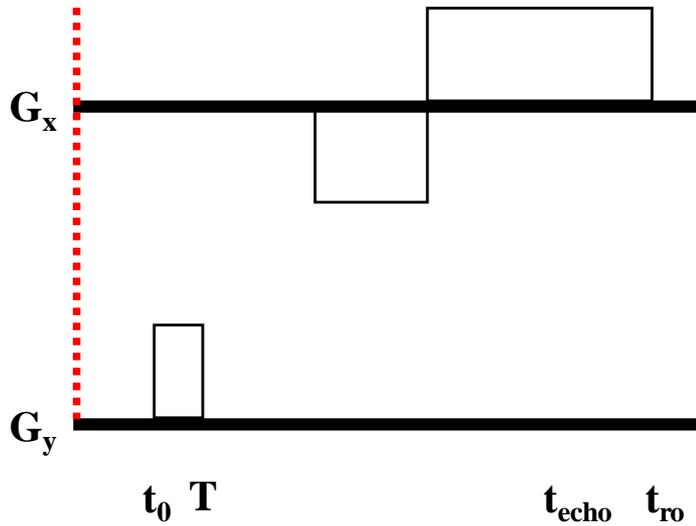
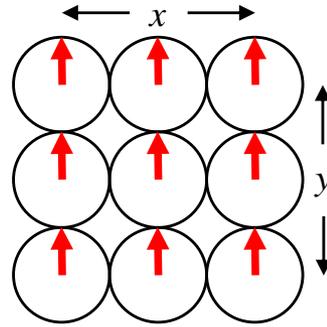
---

$$k_x(t) = \gamma \int^t G_x(\tau) d\tau \quad \text{and} \quad k_y(t) = \gamma \int^t G_y(\tau) d\tau$$

- So, what does this mean?
  - Gradient **ON**: spins precess at different rates as function of location
  - Gradient **OFF**: accumulated phase as a function of location encoded into signal
  - Received signal phase modulated as function of spatial frequency state
  - ‘Spatial frequency’ = *area under gradient-time curve*.
  - Careful coordination of how/when gradients turn on/off facilitates
    - navigation through k-space
    - ability to sample  $S(k_x, k_y)$  over necessary k-space domain
- Let’s walk through an example ...

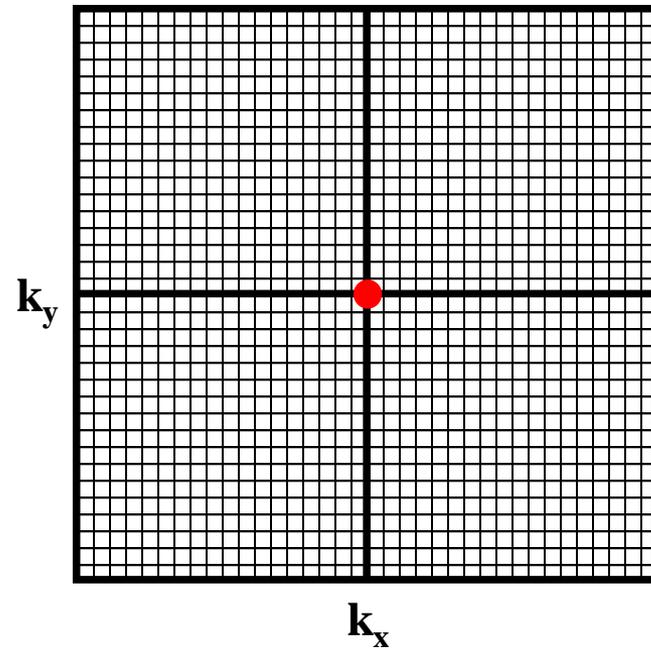
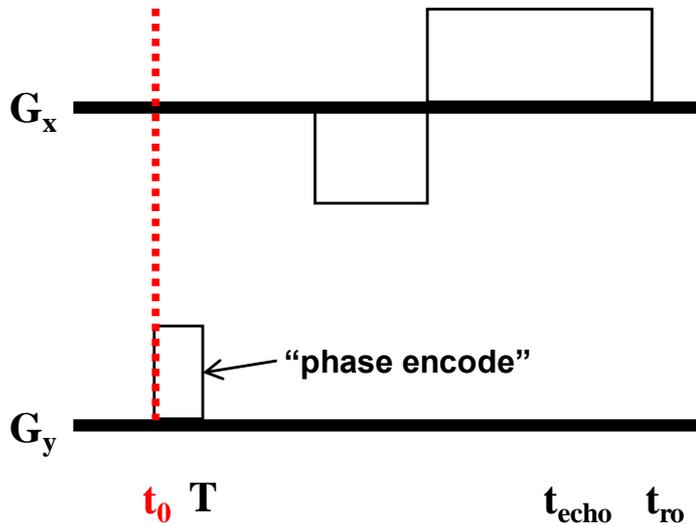
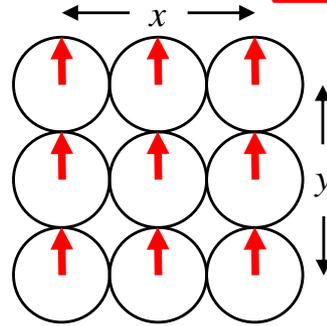
# 2D Spatial Encoding

$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



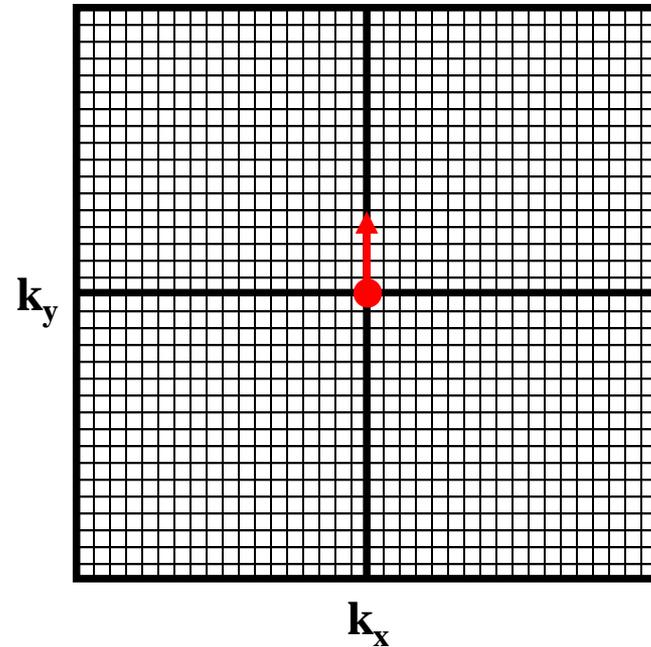
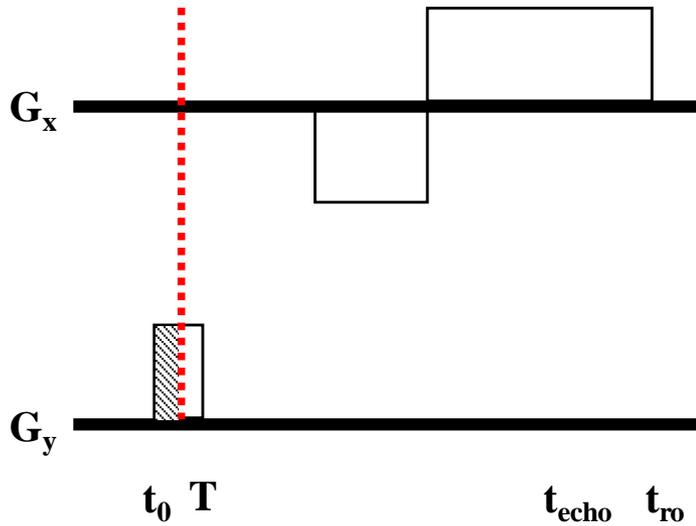
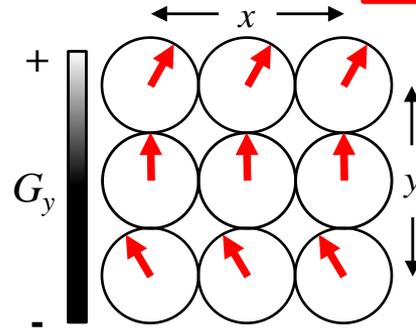
# 2D Spatial Encoding

$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



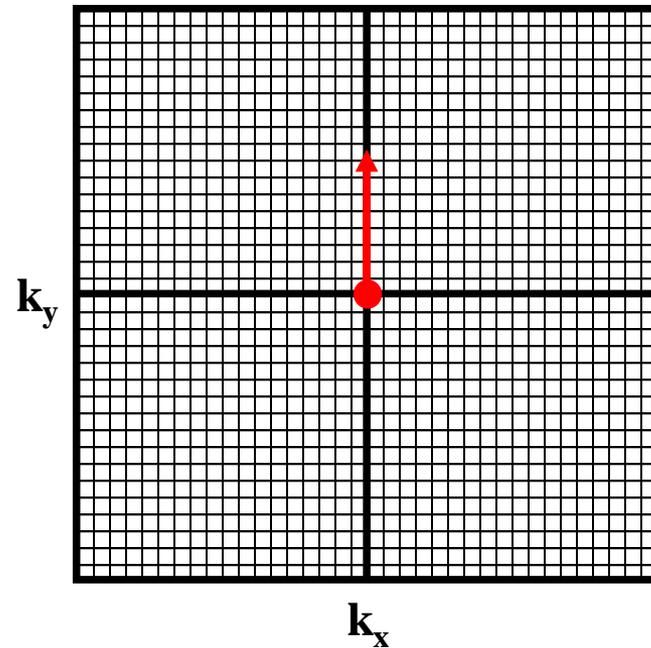
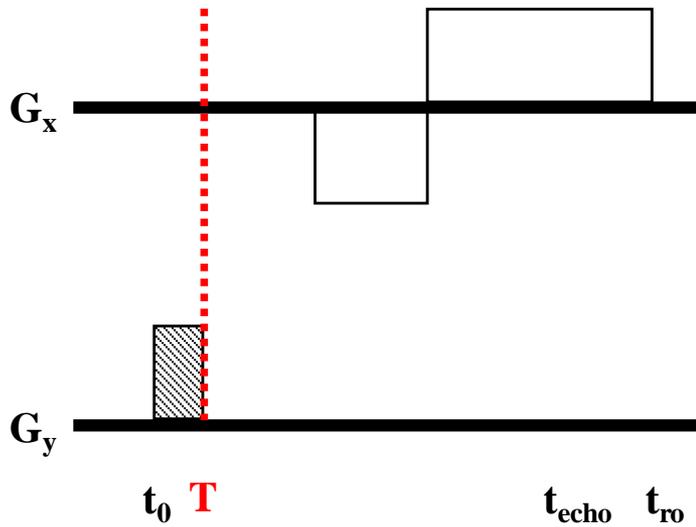
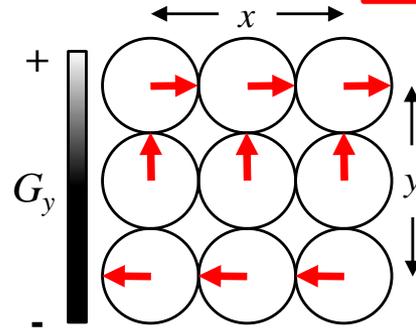
# 2D Spatial Encoding

$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



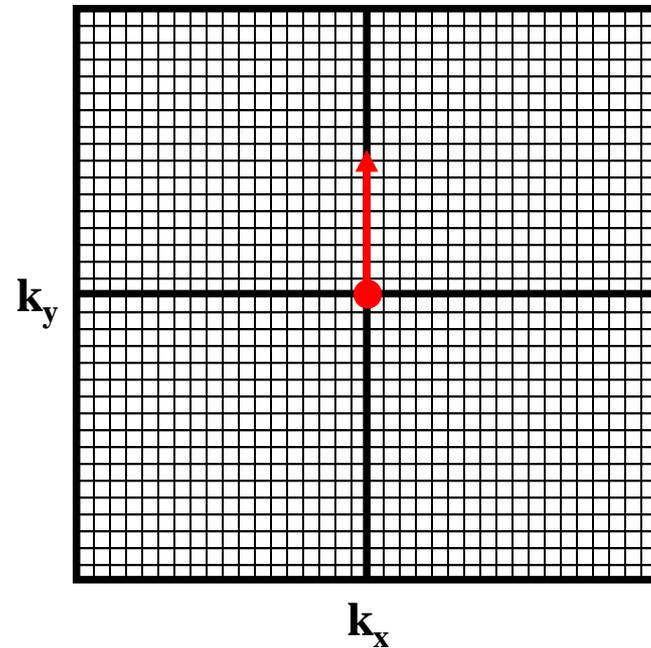
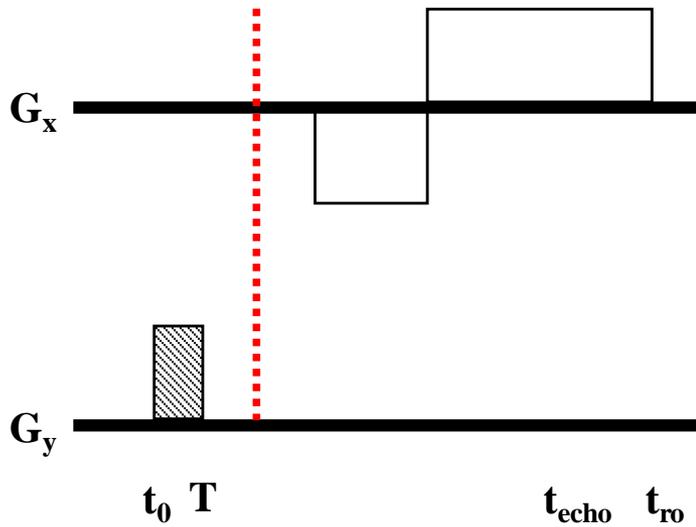
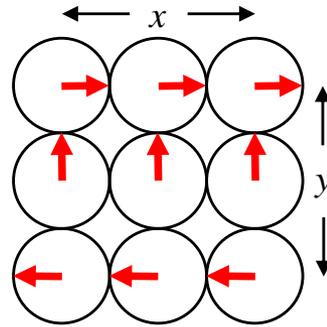
# 2D Spatial Encoding

$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



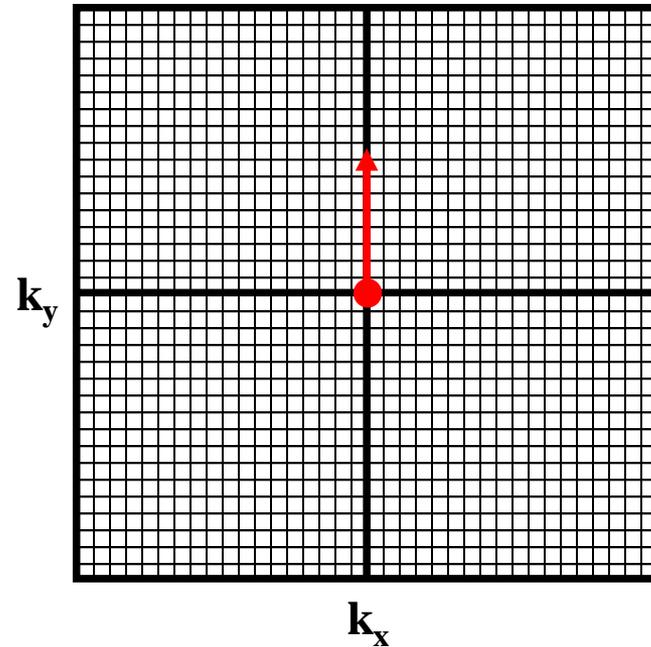
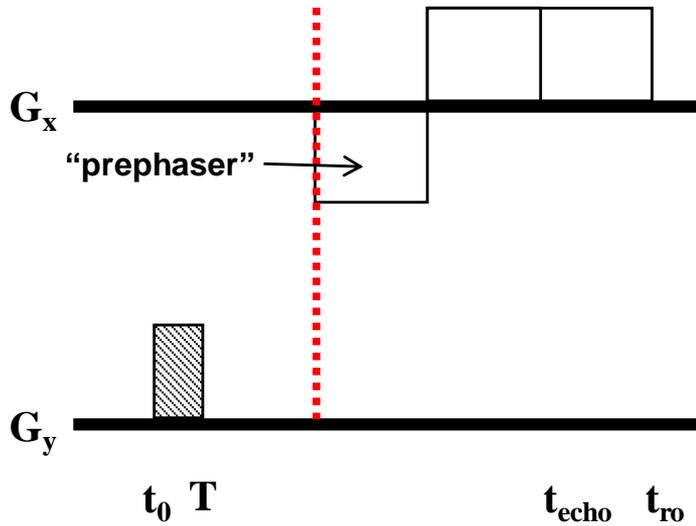
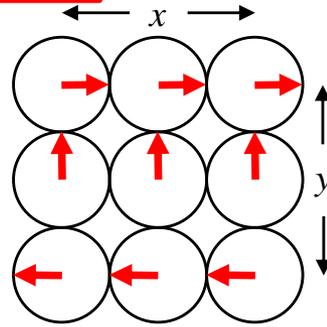
# 2D Spatial Encoding

$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



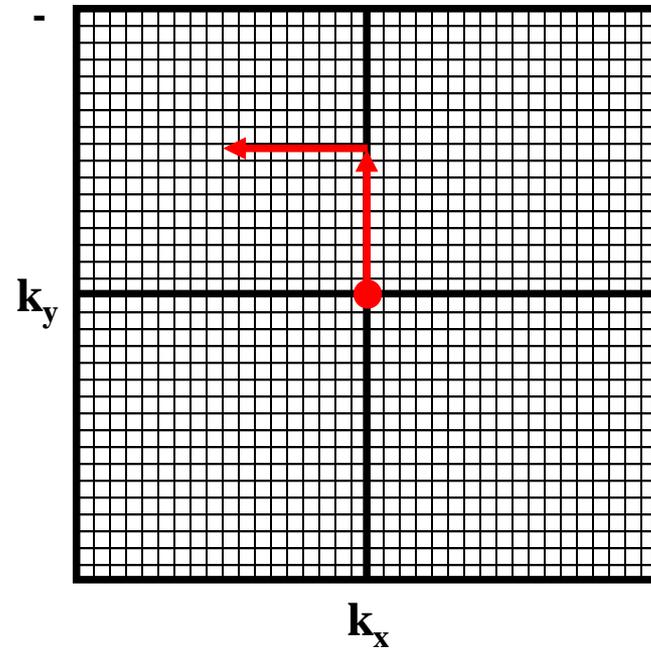
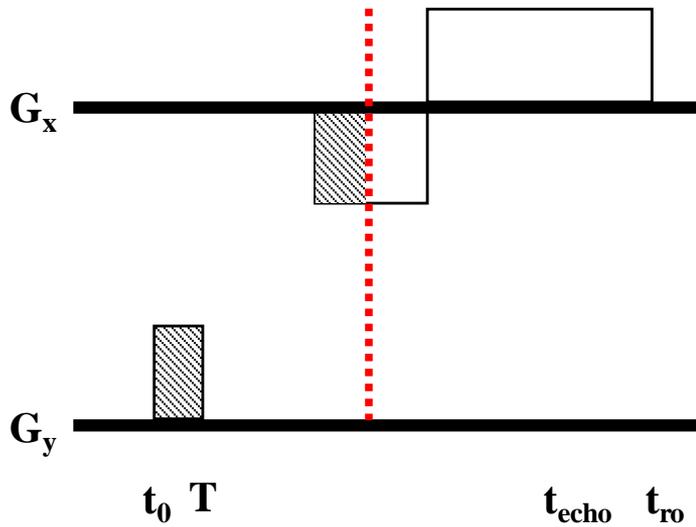
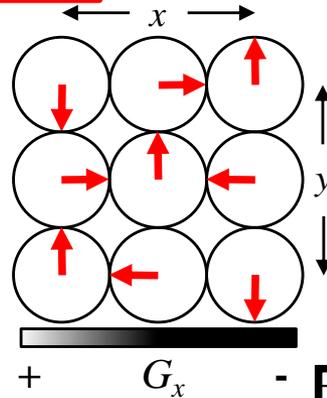
# 2D Spatial Encoding

$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



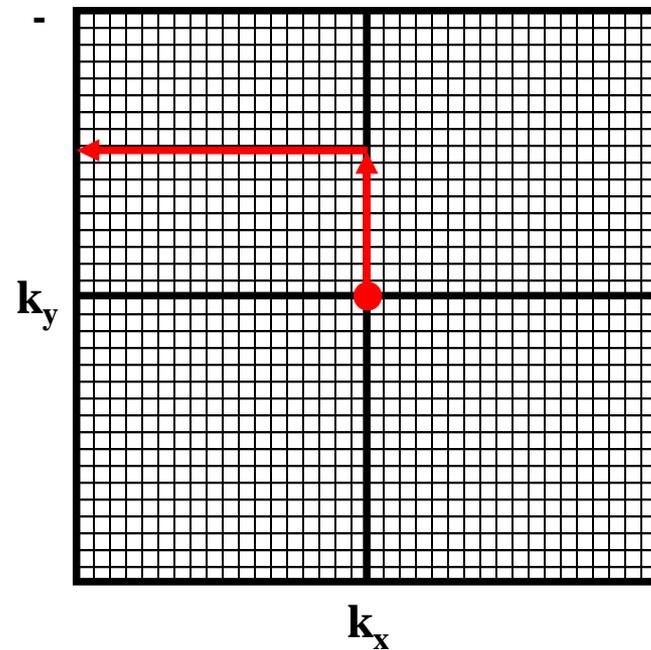
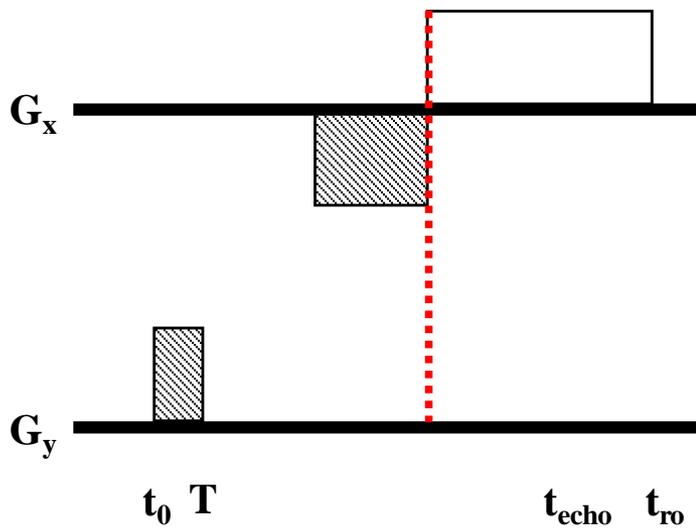
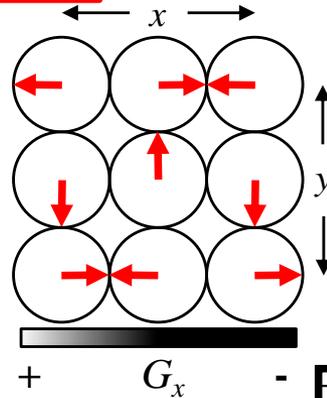
# 2D Spatial Encoding

$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



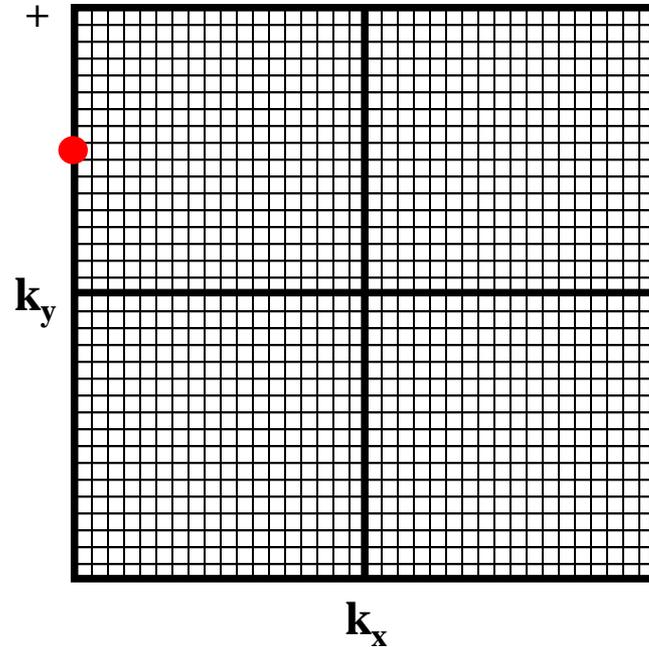
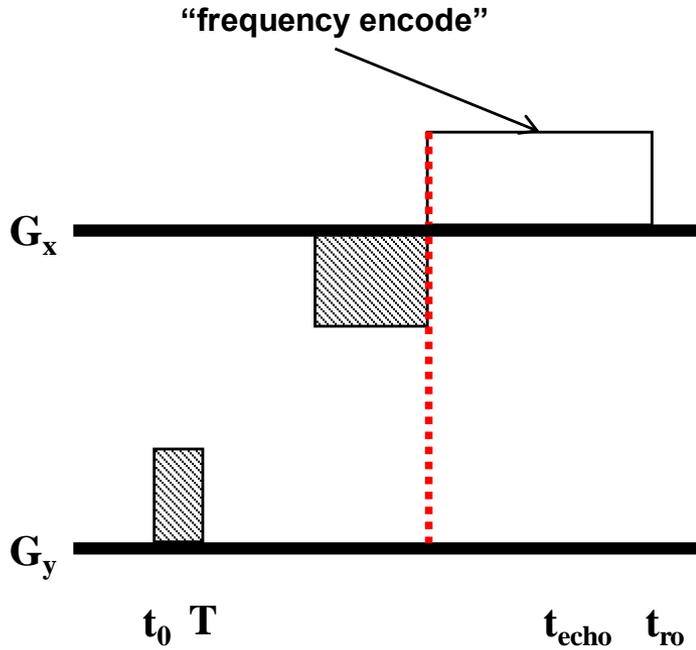
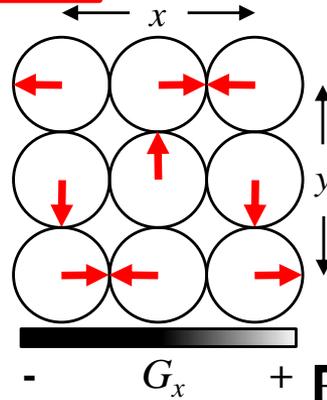
# 2D Spatial Encoding

$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



# 2D Spatial Encoding

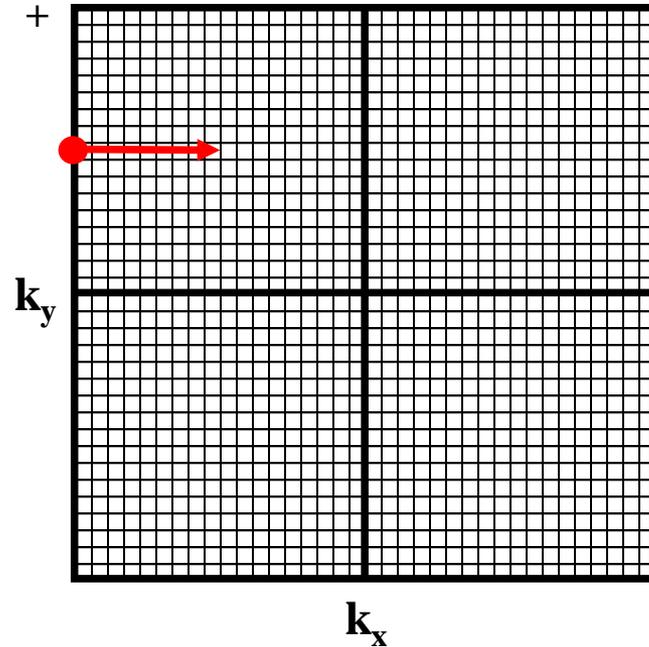
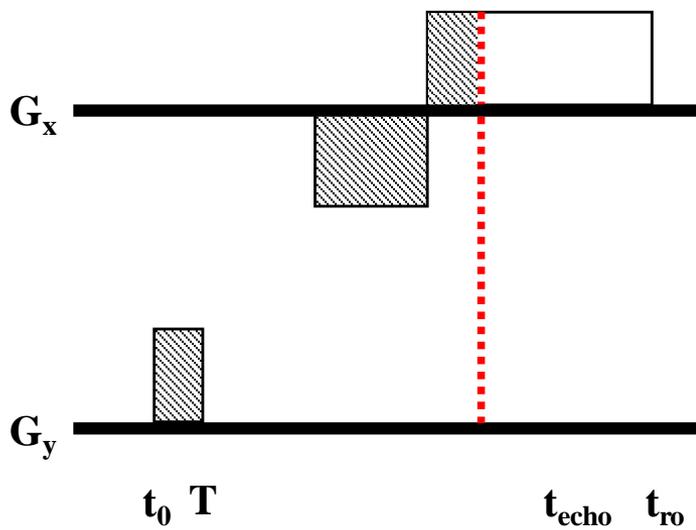
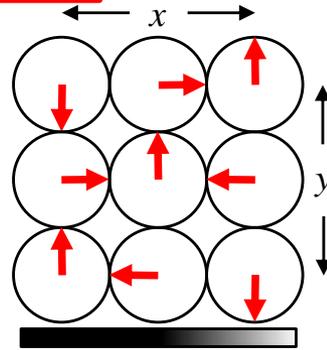
$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



(READOUT PERIOD)

# 2D Spatial Encoding

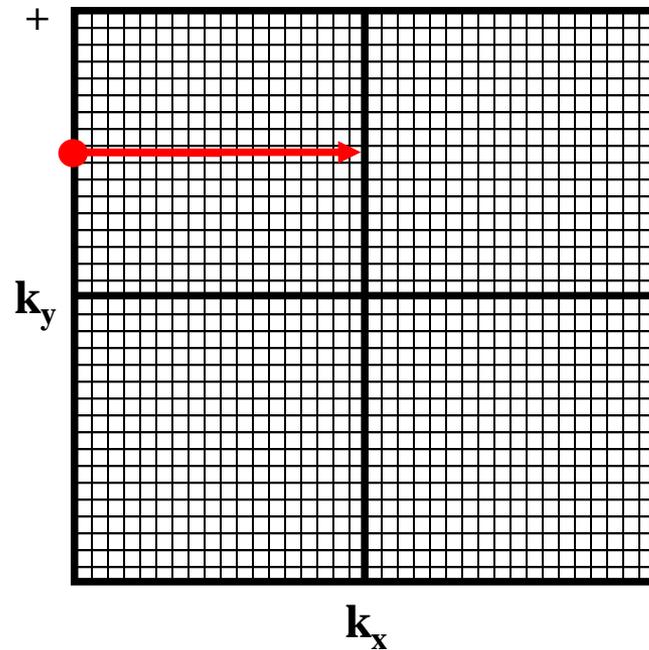
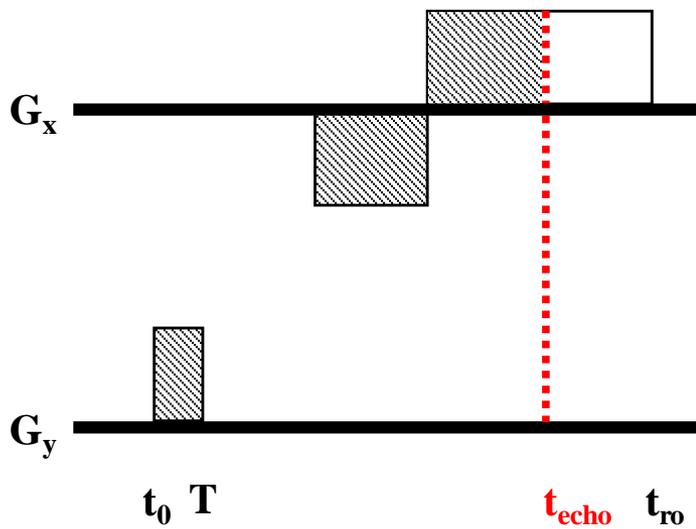
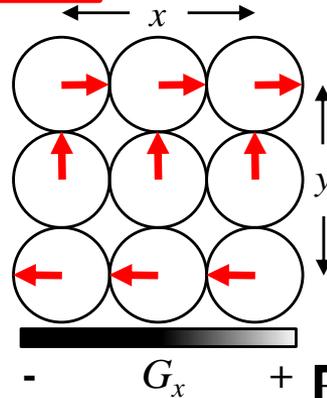
$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



(READOUT PERIOD)

# 2D Spatial Encoding

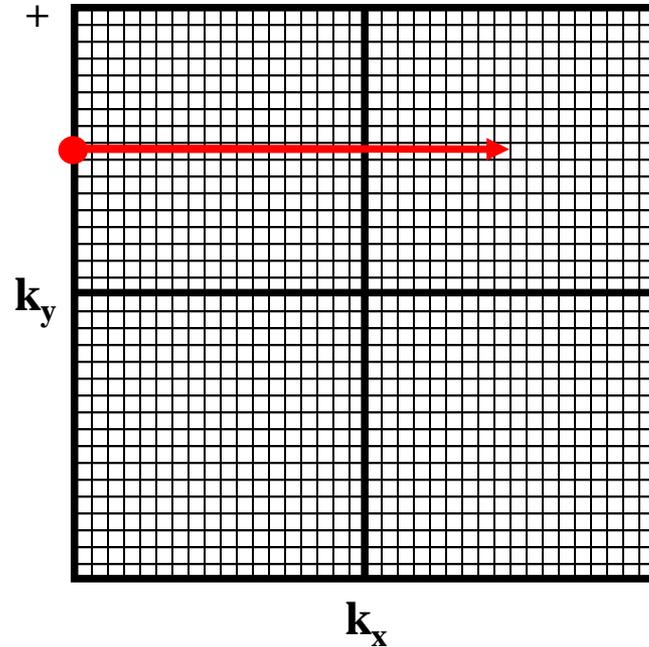
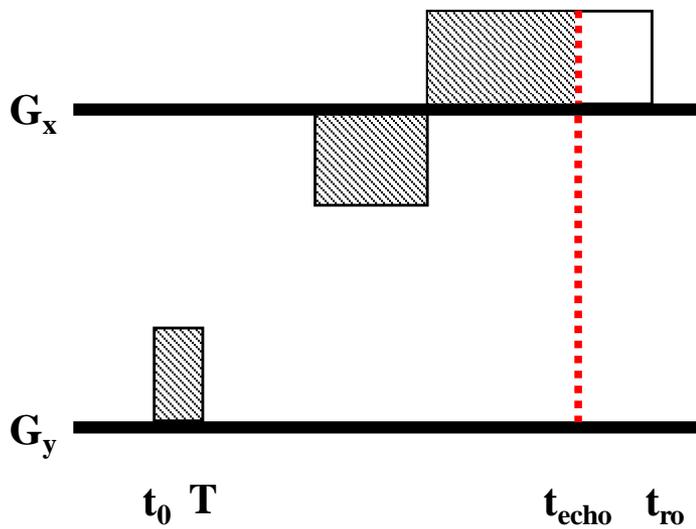
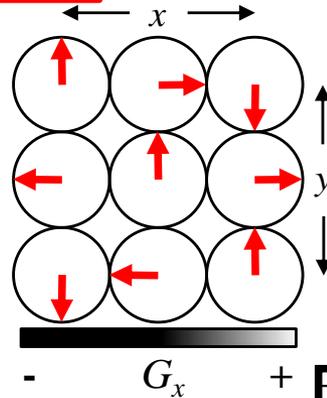
$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



(READOUT PERIOD)

# 2D Spatial Encoding

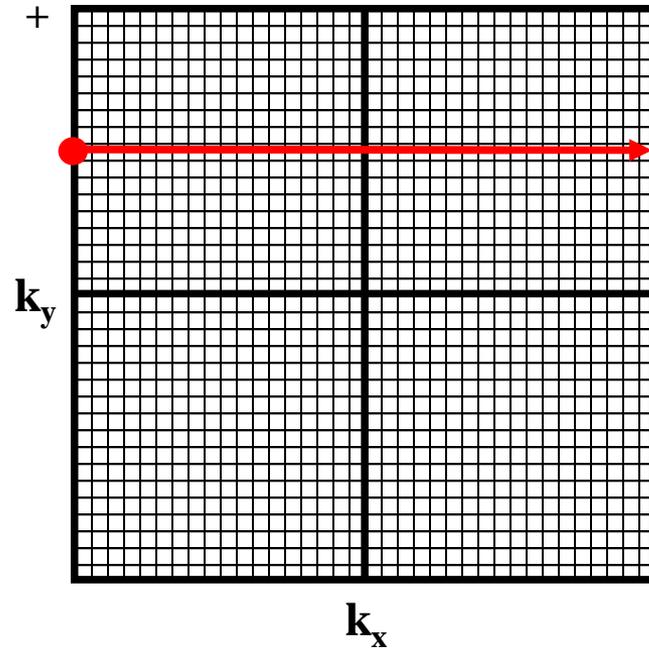
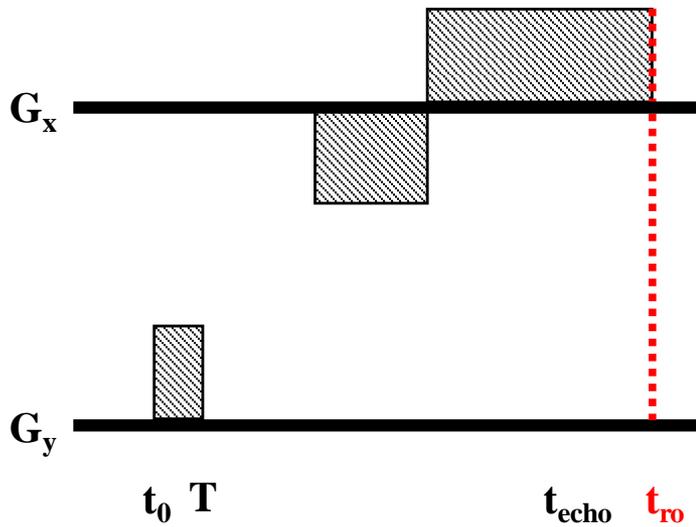
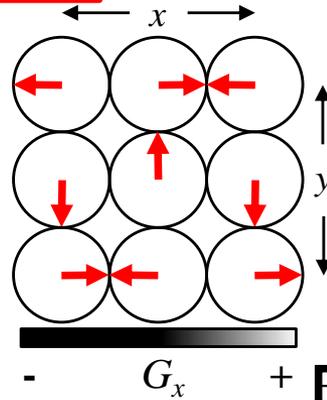
$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



(READOUT PERIOD)

# 2D Spatial Encoding

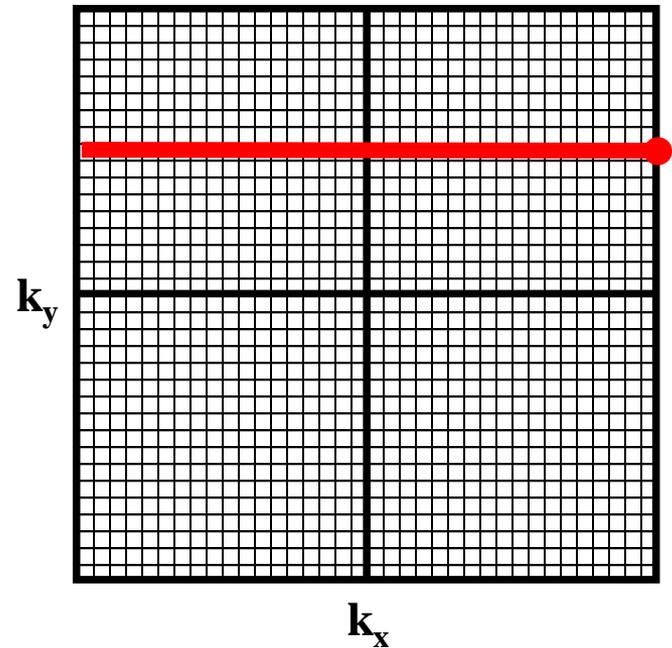
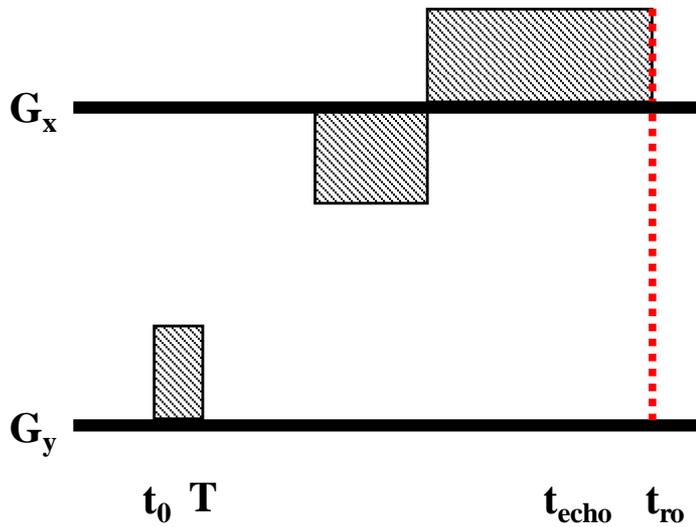
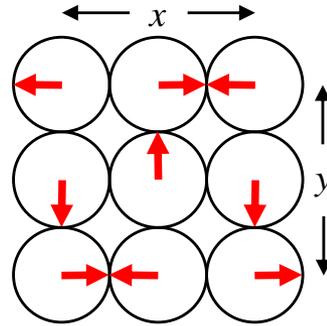
$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



(READOUT PERIOD)

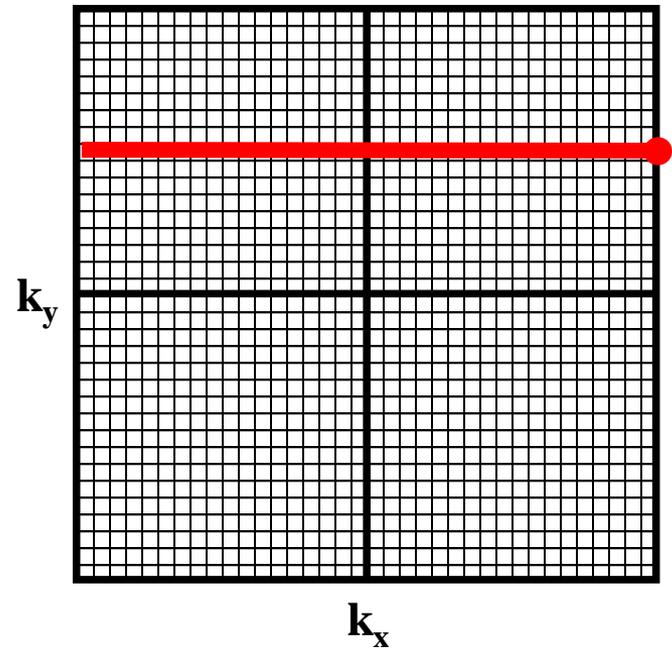
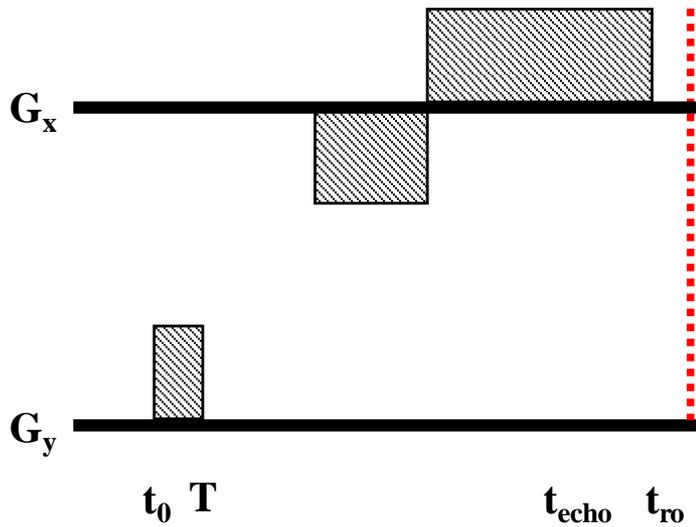
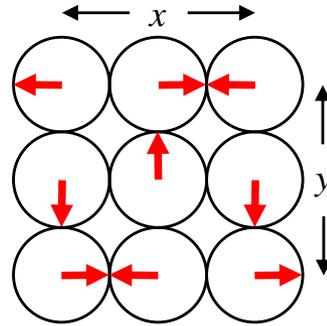
# 2D Spatial Encoding

$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



# 2D Spatial Encoding

$$k_x = \gamma \int_0^t G_x(\tau) d\tau \quad \text{and} \quad k_y = \gamma \int_0^t G_y(\tau) d\tau$$



# Fourier Transform of MRI Data in $k$ -Space to Real Space

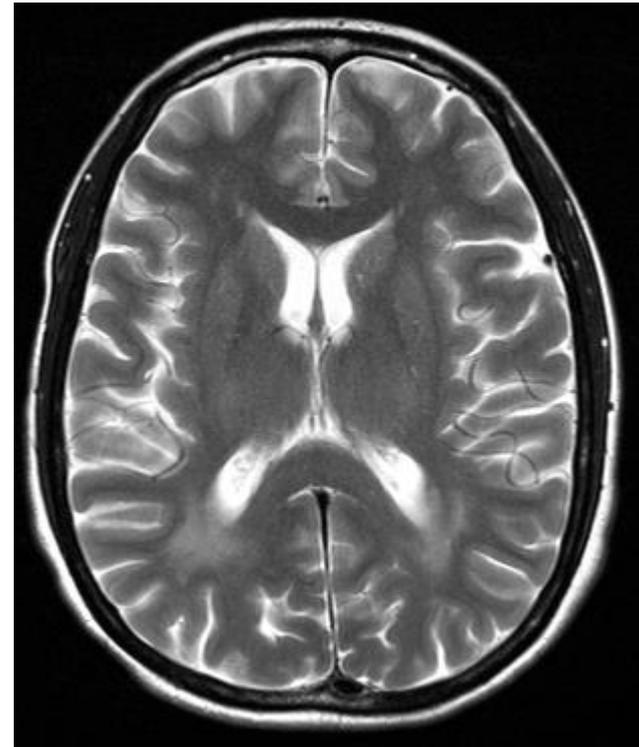
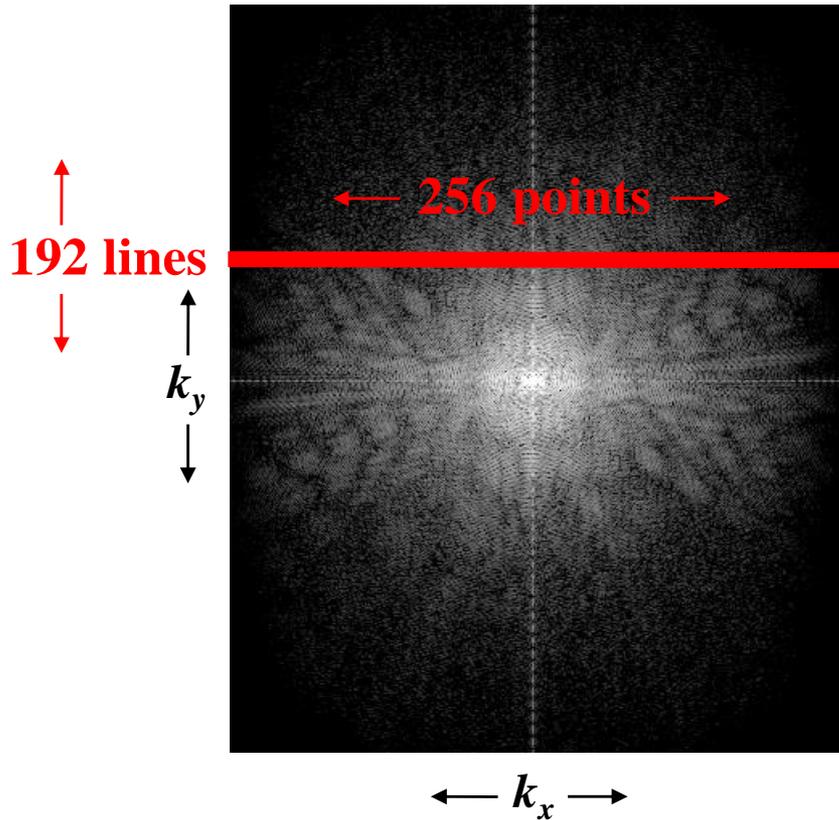
(e.g.,  $256 \times 192$  matrix)

$$S(k_x, k_y)$$

Complex  $k$ -space

$$|S(x, y)| \sim |\text{FT}\{S(k_x, k_y)\}|$$

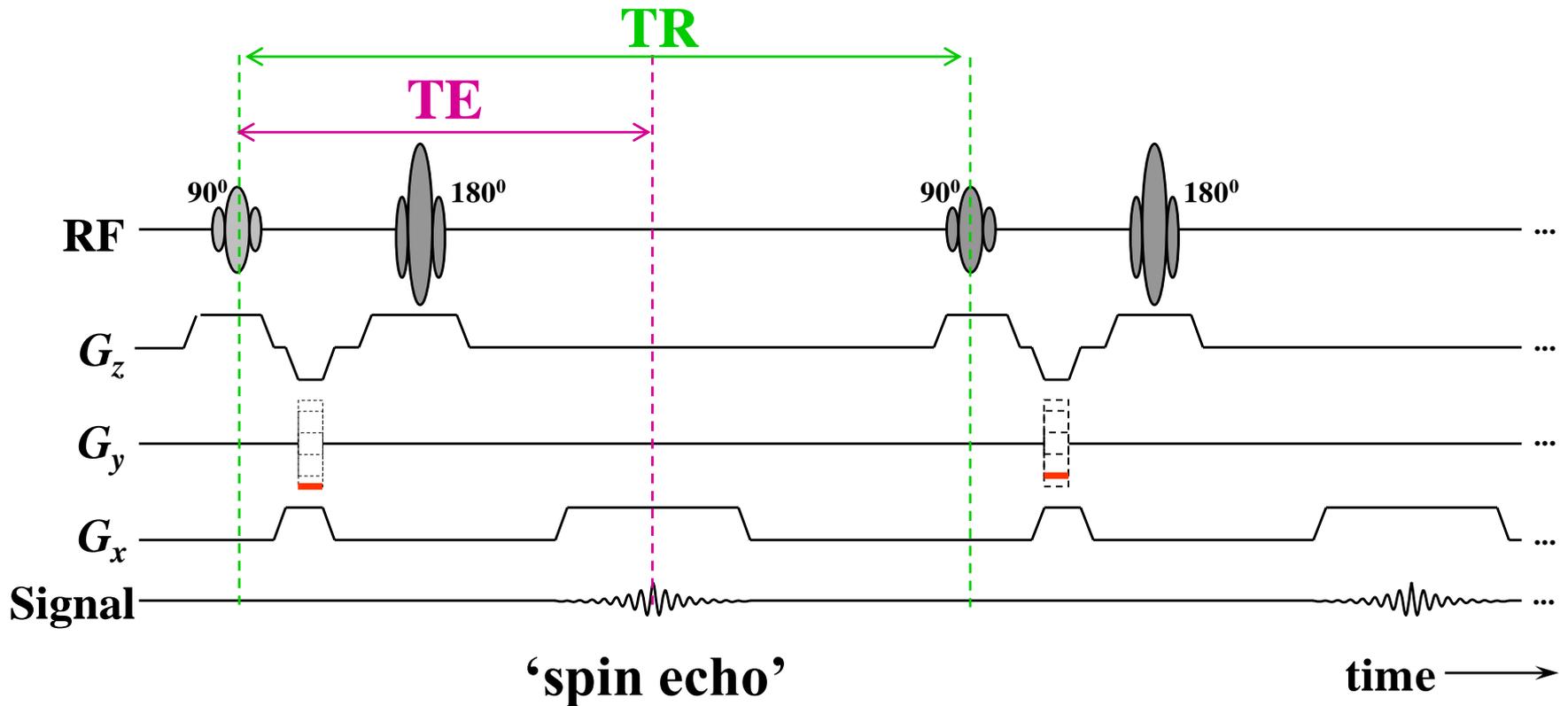
Real-space



$F$

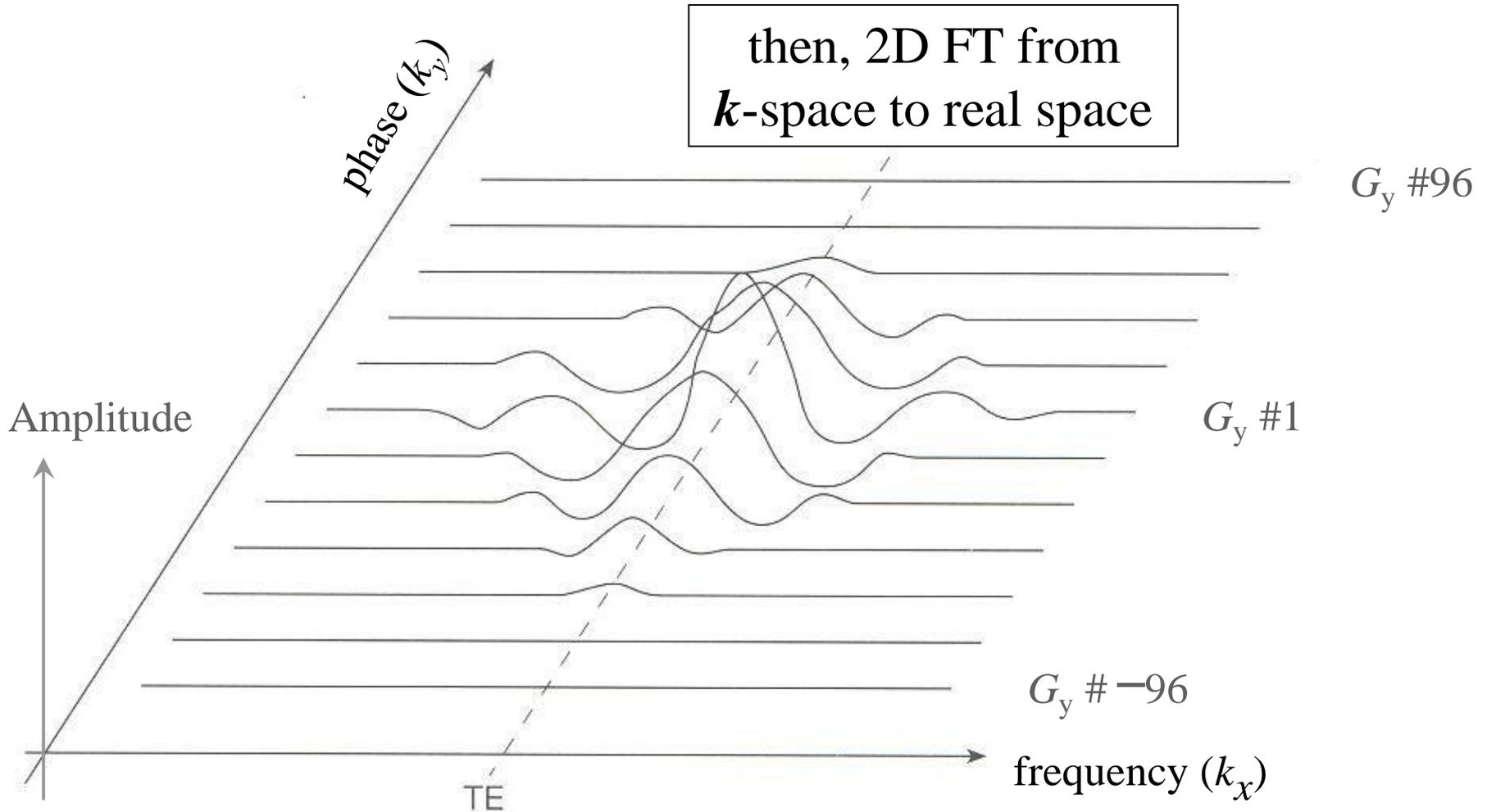
# 2D Spin-Echo Sequence

$$s(t = TE) \sim PD (1 - e^{-TR/T1}) e^{-TE/T2}$$



# Creation of an MRI 'Image' in 2D $k$ -Space

each line in  $k$ -space from data obtained during one  $M_{xy}(TE)$  readout;  
different lines for different phase-encode gradient strength

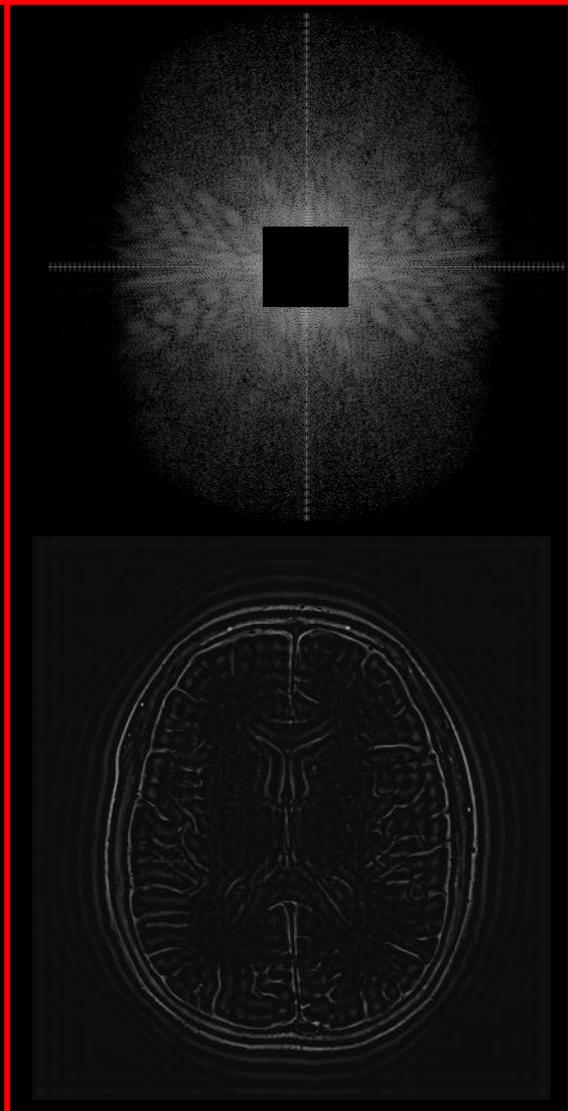
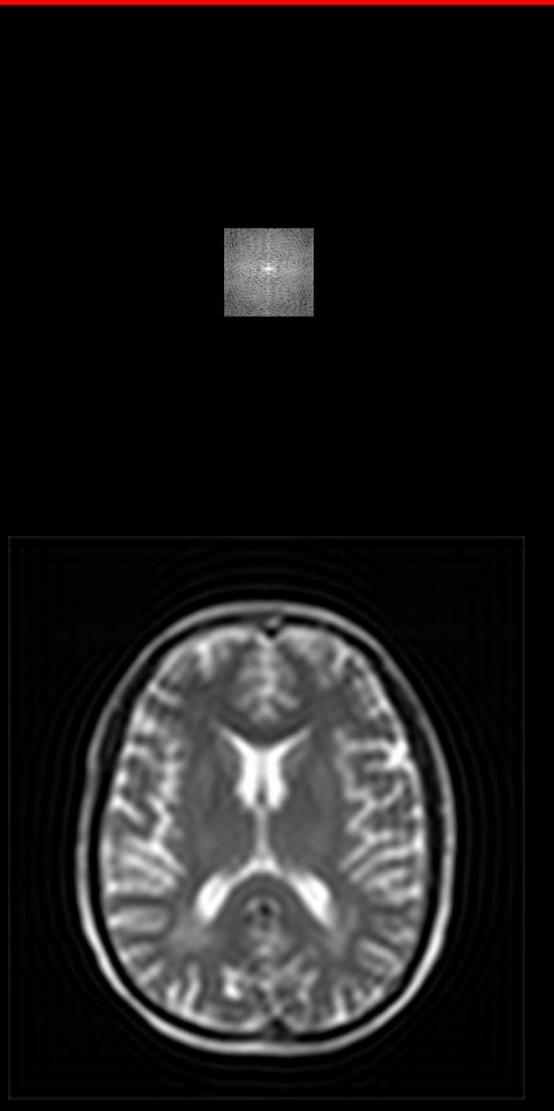
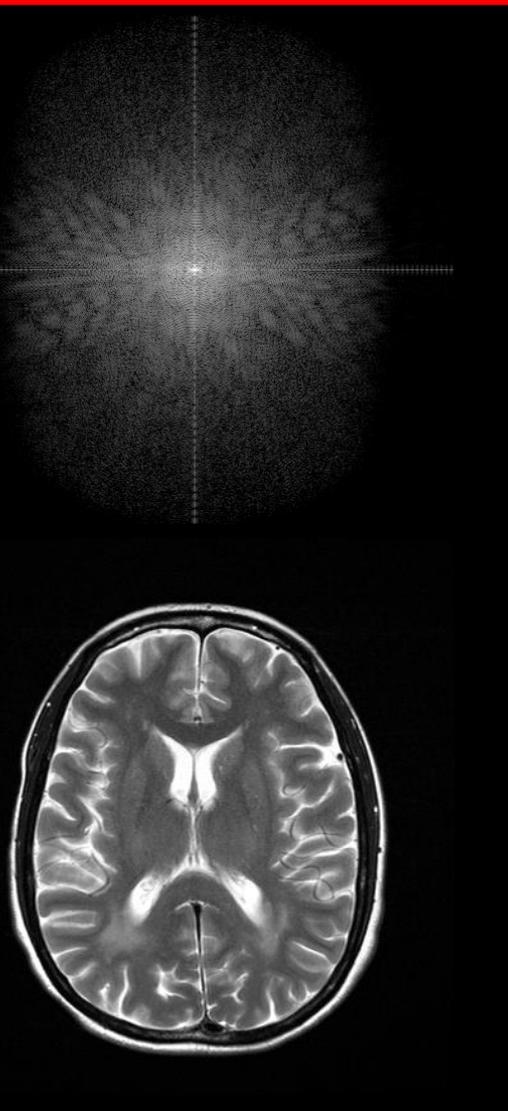


# Fourier Transform from Parts of $k$ -Space to Real Space

Fully sampled  $k$ -space

Center of  $k$ -space  
(SNR, contrast)

Outer  $k$ -space  
(edges, resolution)



$F$

# Fourier Transform from Parts of $k$ -Space to Real Space

Spike in  $k$ -space  
(pattern function of location & intensity)

RF Contamination  
(door seal, device in-room, etc)

Periodic  $k$ -space Errors  
(motion, gradient error, etc)

'spike'

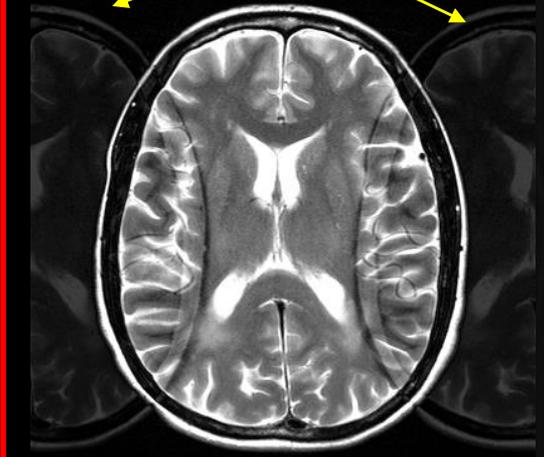
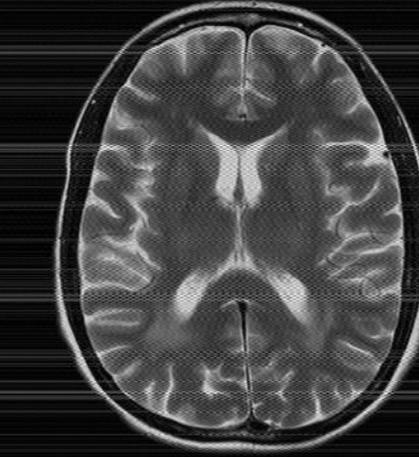
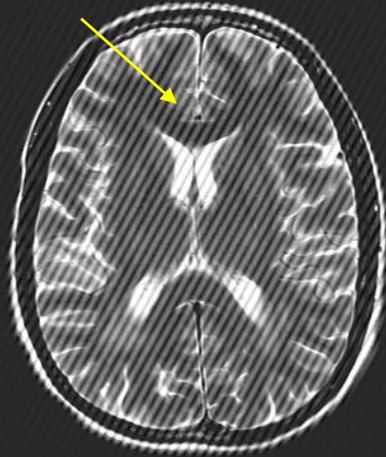
signal leakage

$N/2$  PE errors

line artifacts

RF artifacts

FOV/2 'ghosts'



$F$

# Some Important Topics Not Addressed Today

3D, 4D Imaging

Gradient-Recalled Echo Acquisition

Inversion Recovery (FLAIR and STIR)

Fat Saturation & Suppression

Fast Imaging (e.g., EPI)

Dynamic Contrast Techniques

MR Angiography & Flow

MRI-Elastography

Image SNR Considerations

Image QA, and ACR Accreditation

MRI Safety