

Reconstruction across imaging modalities: MRI

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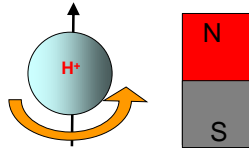


Introduction to Spins and Classical MRI Physics

Most MRI scans are looking at hydrogen nucleus.

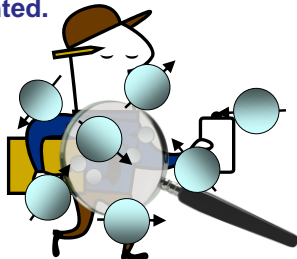
This is good: Body is mostly water → H₂O

Other nuclei are available in MRI: ¹³C, ¹⁹F, ²³Na, others

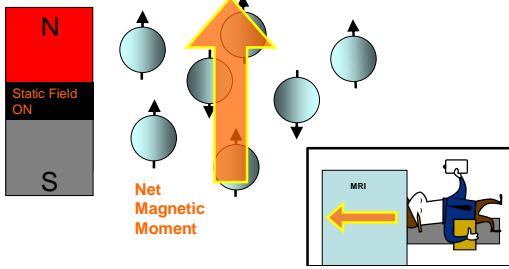


Spinning proton == SPIN
Spinning proton has a magnetic moment ::
Classical Description: Behaves as a bar magnet.

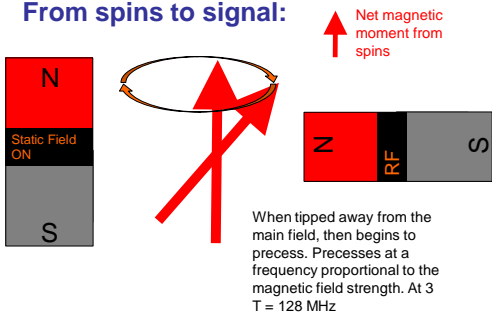
Sample outside the MRI magnet, spins randomly oriented.



Place sample in the MRI magnet...

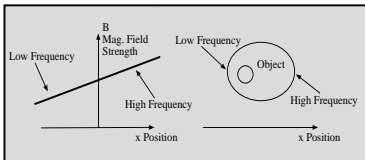


From spins to signal:

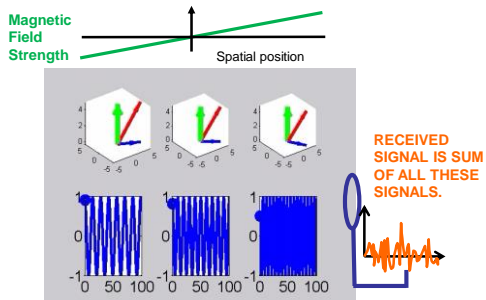


Localization in Space: Larmor Precession

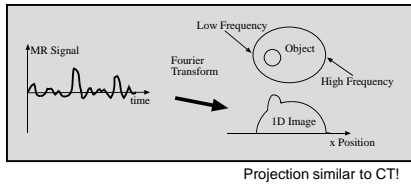
Since frequency is proportional magnetic field strength
 if we apply a magnetic field that varies with spatial position,
 the precession frequency varies with spatial position.



From Noll: fMRI Primer: http://fmri.research.umich.edu/documents/fmri_primer.pdf



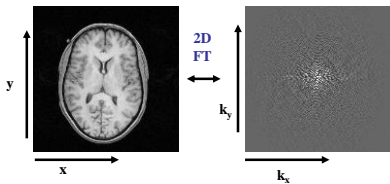
Fourier Image Reconstruction (1D)



From Noll: fMRI Primer: http://fmri.research.umich.edu/documents/fmri_primer.pdf

2D Imaging - 2D Fourier Transform

- Fourier encoding also works in 2 and 3 dimensions:

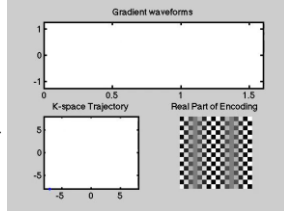


K-space

- We keep track of how much encoding (gradient amplitude times time it is on) by location in k-space:

$$k_x(t) = \gamma \int_0^t G_x(\tau) d\tau$$

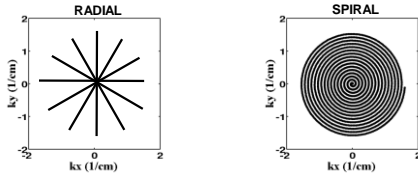
- This is similar to wave number or K-number, it captures the number of spatial cycles of intensity per unit distance
- Fill in k-space, then reconstruction by Fourier Transform



$$\gamma = 42.58 \text{ MHz/Tesla}$$

What about non-Cartesian Acquisitions?

- Instead of sampling regular k-space locations on a Cartesian grid, more efficient sampling might include:



Non-Cartesian:

Direct Reconstruction Approach

$$s(\vec{k}) = \int_{FOV} m(\vec{r}) e^{-i2\pi\vec{k}(t)\cdot\vec{r}} d\vec{r}$$

Signal Equation for MRI
NOTE: dr is evenly spaced pixels

- Can perform Inverse Fourier Transform, but must be careful

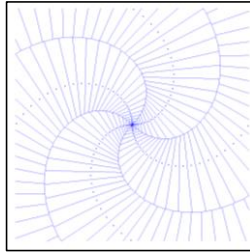
$$\hat{m}(\vec{r}) = \int_{\vec{k}} s(\vec{k}) e^{i2\pi\vec{k}(t)\cdot\vec{r}} d\vec{k}$$

IFT Reconstruction
NOTE: dk is NOT evenly spaced

- Have to take into account differences in sampling density in different areas of k-space, $d\vec{k}$

Sample Density Compensation

- Density compensation function (DCF) represents the differential area element for each sample.
- Can calculate DCF in many ways:
 - Voronoi area (shown)
 - Analytical formulation of gradient
 - Jacobian of time/k-space transformation
 - PSF optimization



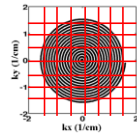
DCF Calculation via Voronoi diagram for 4 shot spiral, showing sampling locations and differential area elements

With DCF, now can perform recon

- Inverse Discrete Space Fourier Transform

$$\hat{m}(\vec{r}) = \sum_j w(\vec{k}_j) s(\vec{k}_j) e^{i2\pi\vec{k}_j \cdot \vec{r}} \quad w(\vec{k}_j) \text{ Density Compensation Function}$$

- But for large problems, we would like to use the Fast Fourier Transform (FFT).
 - k is still not equally sampled on **regular grid**: requirement for FFT!
- What are the options?

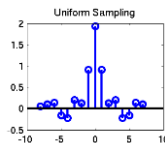


Simply **INTERPOLATE** onto regular grid and then use FFT?
 We will show that we can do better than that with **GRIDDING!**

Gridding: Fast, accurate, direct recon

- Steps in Gridding

1. Density compensate k-space data $w(\mathbf{k})s(\mathbf{k})$
2. Convolution with a fixed-width blurring kernel to fill in continuous sampling of k-space
3. Resample data at uniform Cartesian locations
4. Inverse FFT
5. Deapodization: Eliminate the effect of the fixed kernel interpolator.



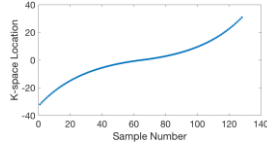
Convolution in k-space is multiplication in image space, so we can remove the effect of the convolution by dividing by the FT of the kernel in image space.

1D simulation from: Noll and Sutton, ISMRM Educational Session, 2003.

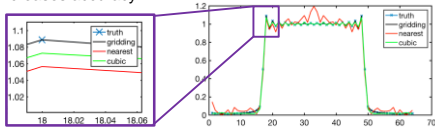
Jackson, et al. IEEE Trans Med Imaging. 1991;10(3):473-8.

Gridding is more accurate than interpolation

- 1D Example: Non-uniform spaced samples
- Compared to nearest neighbor interpolation and cubic spline
- Eliminating effect of interpolation drastically increases accuracy!



NRMSE:
 Nearest: 0.073
 Cubic: 0.0143
 Gridding: 6.7e-5



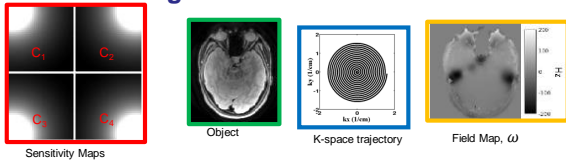
Inverse Problem Approach

Instead of direct inversion, inverse problem approach enables advanced image acquisition and reconstruction

- Variety of additional physics can be accommodated in the signal equation for MRI, enabling advanced acquisitions and reconstructions
 - Coil sensitivities, magnetic field inhomogeneity, k-space trajectory distortions, eddy currents, subject motion, R2* decay, ...
- Image regularization penalties can enable faster imaging while making high quality images from fewer samples
 - Total variation, compressed sensing, low rank, ...
- Do not need to know sample density compensation function for inverse problem



Signal Model for MRI



$$s_n(\vec{k}_j) = \sum_i \text{sinc}(\vec{k}_j \Delta r) C_n(\vec{r}_i) m(\vec{r}_i) e^{-i2\pi \vec{k}_j \cdot \vec{r}_i} e^{-i\omega(\vec{r}_i)t_j}$$

$$\mathbf{y}_n = \mathbf{s}_n + \epsilon_n = \mathbf{A} \mathbf{C}_n \mathbf{m} + \epsilon_n \quad \rightarrow \quad \mathbf{y} = \mathbf{A} \mathbf{m} + \epsilon$$



Inverse problem approach

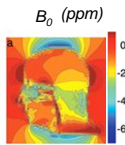
- In complex-valued MRI, noise is complex Gaussian.
- Makes statistics easier than other modalities
- Can use least squares approaches for image reconstruction $\hat{\mathbf{m}} = \arg \min_{\mathbf{m}} \|\mathbf{y} - \mathbf{A}\mathbf{m}\|_2^2$
- Can add regularization to help with the usually ill-conditioned problem, provides prior information on acceptable solutions, through some function $\mathcal{R}(\cdot)$

$$\hat{\mathbf{m}} = \arg \min_{\mathbf{m}} \|\mathbf{y} - \mathbf{A}\mathbf{m}\|_2^2 + \lambda \mathcal{R}(\mathbf{m})$$

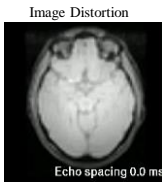
Example: Magnetic Field Inhomogeneity

Incorporation of the field inhomogeneity map into the inverse problem to correct for it.

Air/tissue interfaces cause most magnetic field disruptions, for example around sinuses.

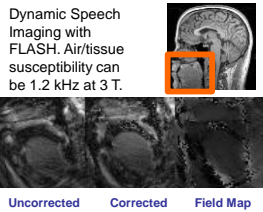
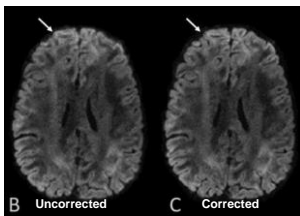


Air/Tissue interfaces can cause up to ~1 KHz off-resonance at 3 T



Distortion depends on K-space trajectory and bandwidth (how fast sample k-space).

Example: Magnetic Field Inhomogeneity



Dynamic Speech Imaging with FLASH. Air/tissue susceptibility can be 1.2 kHz at 3 T.

High Resolution (0.8 mm isotropic) diffusion MRI, b=1000 s/mm²

Holtrop and Sutton. J Medical imaging, 3(2): 023501 (2016). Sutton, et al. J Magn Reson Imaging, 32:1228 (2010)

Regularization and Constraints

- Image reconstruction is ill-conditioned problem
 - Sample only the minimum data (or less) that we need to keep scan time short
 - Push the spatial resolution higher → signal-to-noise lower
 - Non-ideal experimental conditions
 - Magnetic field map changed since measurement
 - Coil sensitivities changed
 - K-space trajectory deviations
- Must enforce prior information on the solution in order to achieve a high quality image

Types of Regularization/Constraints in MRI

- Not an exhaustive list, just main ones
- Energy penalty, reference image, Tikhonov
- Roughness penalty, first order derivative,
- TV - total variation
- Compressed sensing
 - Sparsity, Finite differences, DCT, Wavelets, ... thresholding
- Something to keep in mind: MRI images are complex valued – have magnitude and phase

Compressed Sensing

- CS takes advantage of k-space sampling patterns that cause **incoherent aliasing** from the undersampling.
 - Distributes aliasing energy around in an incoherent manner
 - Makes it noise-like
- Transforms images into a domain where they are **sparse**
- **Recovers sparse coefficients** in the background noise of aliasing – similar to denoising algorithms

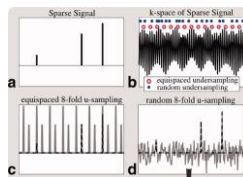


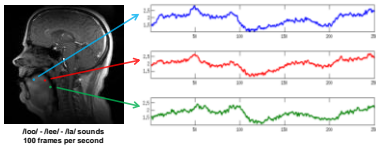
Figure from: Lustig, Donoho, Pauly.
Magn Reson Med, 58:1182 (2007)

<https://people.eecs.berkeley.edu/~mlustig/CS.html>

Low Rank through Partial Separability

- Not only are images typically sparse in finite difference or wavelet domains, dynamic timeseries data are typically low rank
- Low rank indicates strong spatial-temporal correlations in the data set.
- Simple example to play around with at: <http://go.illinois.edu/LowRank>

Partial Separability (PS) Model



- Leverages strong spatiotemporal correlation
- L^{th} order Partial Separability model:

$$d(\mathbf{k}, t) = \sum_{l=1}^L c_l(\mathbf{k}) \phi_l(t)$$

Z.-P. Liang, *IEEE-ISBI*, 2007.

Partial Separability Model

$$s(\mathbf{k}, t) = \sum_{l=1}^L c_l(\mathbf{k}) \phi_l(t) \Rightarrow \mathbf{c} = \begin{bmatrix} s(\mathbf{k}_1, t_1) & s(\mathbf{k}_1, t_2) & \dots & s(\mathbf{k}_1, t_M) \\ s(\mathbf{k}_2, t_1) & s(\mathbf{k}_2, t_2) & \dots & s(\mathbf{k}_2, t_M) \\ \vdots & \vdots & \ddots & \vdots \\ s(\mathbf{k}_N, t_1) & s(\mathbf{k}_N, t_2) & \dots & s(\mathbf{k}_N, t_M) \end{bmatrix}$$

rank(C) ≤ L, over any point set $\{(\mathbf{k}_m, t_n)\}_{m,n=1}^{M,N}$

$$\hat{\mathbf{C}} = \arg \min_{\mathbf{C}} \|\mathbf{H}(\mathbf{C} - \hat{\mathbf{C}})\|_2^2 \quad \text{s.t. rank}(\hat{\mathbf{C}}) = L$$

$$\hat{\mathbf{C}} = \arg \min_{\mathbf{C}} \|\mathbf{H}(\mathbf{C} - \hat{\mathbf{C}})\|_2^2 + \lambda \|\hat{\mathbf{C}}\|$$

Z.-P. Liang, *IEEE-ISBI*, 2007.

What you can achieve with PS-Sparse

Subject:

A female speaker of Mid-Atlantic American English

Spatial coverage:

$280 \times 280 \times 40 \text{ mm}^3$

Matrix size:

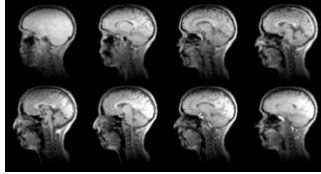
$128 \times 128 \times 8$

Spatial resolution:

$2.2 \times 2.2 \times 5.0 \text{ mm}^3$

Nominal Frame Rate:

166 frames per second!

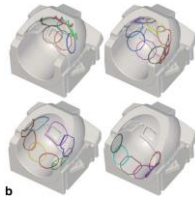


Carrier Phrase – "I said writing to you, I said riding to you" [1]

Fu, Barlaz, Holtrop, et al. Magn Reson Med 77(4): 1619, 2017. M. Barlaz et al, InterSpeech, 2015.

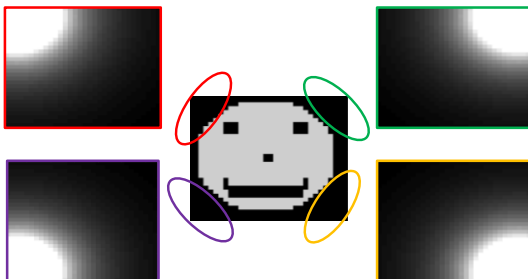
Speeding Up Acquisitions: Parallel Imaging

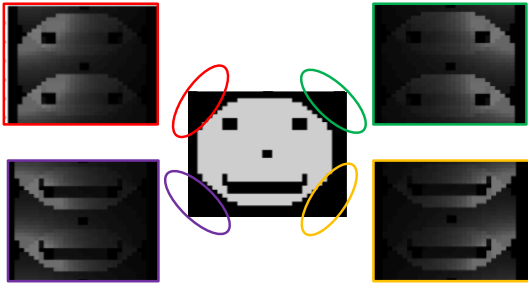
- Reconstruction approaches above did not leverage the use of multiple, smaller receiver coils
- Each coil is most sensitive to tissue in its local area
- Reduced spatial encoding requirements because aliasing signal may overlap region where there is low sensitivity for a coil
- Easily incorporated into cost function for sensitivity encoding (SENSE)



b

Kaza, Klose, and Lotze. J Magn Reson Imaging. 34:173 (2011)





Parallel Imaging Reconstruction

- SENSE – least squares estimation with coil sensitivities in image space

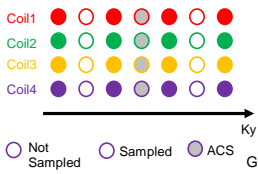
$$s_n(\vec{k}_j) = \sum_i \text{sinc}(\vec{k}_j \Delta r) C_n(\vec{r}_i) m(\vec{r}_i) e^{-i2\pi \vec{k}_j \cdot \vec{r}_i} e^{-i\omega(\vec{r}_i) t_j}$$

- GRAPPA – operates in k-space. Multiplication by coil sensitivities in image space is convolution in k-space. So, find the convolution kernel in k-space to fill in missing samples

Pruessmann, et al. Magn reson Med 42:952 (1999). Griswold, et al. Magn Reson Med 47: 1202 (2002)

GeneRalized Autocalibrating Partially Parallel Acquisitions (GRAPPA)

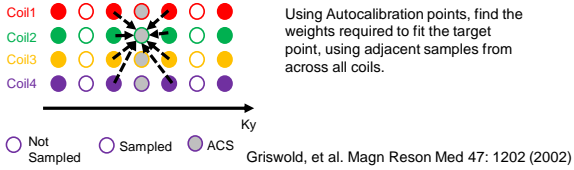
- Learn linear relationship between kernels of sampled points in k-space to "interpolate" those that were not sampled.



Griswold, et al. Magn Reson Med 47: 1202 (2002)

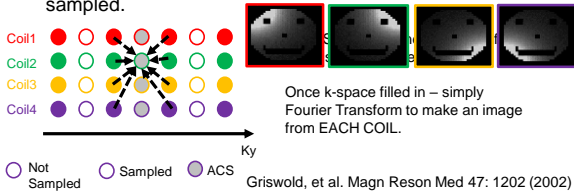
GeneRalized Autocalibrating Partially Parallel Acquisitions (GRAPPA)

- Learn linear relationship between kernels of sampled points in k-space to "interpolate" those that were not sampled.



GeneRalized Autocalibrating Partially Parallel Acquisitions (GRAPPA)

- Learn linear relationship between kernels of sampled points in k-space to "interpolate" those that were not sampled.



COMPUTATIONAL Challenge of Image Reconstruction

- Large matrix size
 - 1.25 mm isotropic data set
- Data Size:
 - 18 GB for 6.5 min scan
- Physics:
 - 3D non-Cartesian (Spiral) sampling
 - Parallel Imaging with 32 channel coil
 - Magnetic Field Inhomogeneity Correction
 - Motion-induced Phase Correction
- Reconstruction Time
 - 8 days for reconstruction running on workstation
- Graphics Processing Units (GPU)
 - 200 times faster: <1 hour.
 - Enables imaging resolutions not feasible before.



GTX 1080 Ti – \$700 (11 GB)
CORES: 3594
Boost Clock: 1582 MHz

IMPATIENT MRI:
Illinois Massively Parallel Acceleration Toolkit for Image reconstruction with ENhanced Throughput in MRI

PowerGrid –ISMRM
2016, p. 525



PowerGrid: A open source library for accelerated iterative magnetic resonance image reconstruction

Alex Ceranic^{1,2}, Joseph L. Holtrop^{1,2}, Giang Chau Ngo¹, Brent Leback³, Galen Arnold⁴, Mark Van Moer⁴, Genevieve Labadie^{2,5}, Jeffrey A. Fessler⁶, and Bradley P. Sutton^{1,2}

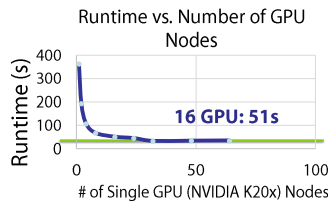
ISMRM 2016, p. 525

<http://mrfil.github.io/PowerGrid/>

- Enable leveraging of GPU and MPI in MRI reconstructions
- Using ISMRM RD – raw data format standard
- Translating MATLAB routines from IRT into C++ through Armadillo
- Packaging for easy use (Docker) – coming soon

Scale with PowerGrid

- How to use > 1 GPU?
- Message Passing Interface (MPI)
- Phase Corrected SENSE (pcSENSE) for Diffusion Imaging
 - 120 x 120 x 4 x 32 coils



Liu C, Moseley ME, Bammer R. Simultaneous phase correction and SENSE reconstruction for navigated multi-shot DWI with non-cartesian-k-space sampling. *MRM*. 2005;54(6):1412-1422.

K20x: 2688 cores, 732 MHz clock rate. 6 GB.

Push for Free, Open Source, Common Platforms for Image Reconstruction

- Advanced reconstructions are more complex than Fourier Transform, but enable significantly higher resolutions and shorter scan times.
- Image reconstructions can be specific for the sequence, MRI vendor platform, image reconstruction hardware, and can be difficult to reimplement from paper
- There is a growing effort at creating broad-based utilities to enable reproducibility, distribution, scaling, and impact
- Just a few listed here...
