Quantitative Reconstruction in PET/CT and PET/MR

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Outline

• PET 101
• Tomography in medical imaging
• Projection imaging
• Sinogram
• Reconstruction-analytic
• Backprojection artifacts
• Reconstruction-iterative
• Comparison of analytic and iterative reconstructions
• Simultaneous PET-MR

History of PET at MGH

The birthplace of Positron Emission Imaging was at MGH in 1952 in the Center for Radiological Sciences (Ancestor of the Gordon Center for Medical Imaging) where the first positron-imaging device was invented by Dr Gordon Brownell and used for the detection of brain tumors for neurosurgery by Dr Sweet (1953).

Coincidence (a) and “unbalance” (b) scans of a patient with recurring tumor (left) under previous operation site [Brownell and Sweet, 1953]
What is tomography?

- Greek translation:
  - tomos means slice, section
  - graph means write
- 2-D representations of structures in a selected plane of a 3-D object
- Mathematical algorithms can be used to reconstruct the original 3-D object from the 2-D projections
- Used in medical imaging
  - SPECT and PET-Emission computed tomography
  - CT-Transmission computed tomography

History of PET at MGH

MGH is also the birthplace of filtered backprojection that is still widely used in PET and in CT. Dr David Chesler (Brownell Lab) presented the first results about filtered backprojection at the Meeting of Tomographic Imaging in Nuclear Medicine (1972).

History
Why tomography over planar imaging?

Contrast (Planar) = \(\frac{40 - 30}{30} = 0.33\)
Contrast (Tomo) = \(\frac{20 - 10}{10} = 1.00\)

Tomography in medical imaging

Gamma Ray Emission

Unstable parent nucleus
Nucleus drops to lower energy state. Gamma ray carries away excess energy.

Unstable parent nucleus with extra proton. Positron emission and annihilation.
Positron Emission and Annihilation

Proton decays to neutron ...
and a positron emitting a neutrino ...
Positron combines with electron to form positronium ... rapidly annhilates
Two anti-parallel 511 keV photons produced

The Photomultiplier Tube

Incoming light ray
Photocathode
Dynodes
Evacuated glass tube
Anode
Dynodes have increasing voltage
Photoelectron
Electronics

The Scintillation Detector

Scintillating crystal
Photomultiplier

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Coincidence detection in a PET camera

A block-detector PET camera consists of detectors in a series of rings

Types of coincidence events

- True Coincidence
- Scattered Coincidence
- Random Coincidence
Example: Typical Whole Body PET

<table>
<thead>
<tr>
<th>Type of coincidence</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw</td>
<td>100</td>
</tr>
<tr>
<td>Trues</td>
<td>38</td>
</tr>
<tr>
<td>Randoms</td>
<td>34</td>
</tr>
<tr>
<td>Scattered</td>
<td>28</td>
</tr>
<tr>
<td>Multiple</td>
<td>7</td>
</tr>
</tbody>
</table>

Electronic collimation and intrinsic resolution

Detector

Field of View

- Resolution depends on size of detector elements
- Resolution does not change much between the detectors

 Positron range and intrinsic resolution

Positron collides with electrons and loses kinetic energy
At thermal energies, positronium can form
Positron range depends on energy of emitted positron
### Positron range and intrinsic resolution (2)

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Maximum positron energy (MeV)</th>
<th>Mean positron energy (MeV)</th>
<th>Range in water FHWM (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{18}$F</td>
<td>0.64</td>
<td>0.25</td>
<td>0.10</td>
</tr>
<tr>
<td>$^{11}$C</td>
<td>0.96</td>
<td>0.39</td>
<td>0.19</td>
</tr>
<tr>
<td>$^{13}$N</td>
<td>1.19</td>
<td>0.49</td>
<td>0.28</td>
</tr>
<tr>
<td>$^{15}$O</td>
<td>1.70</td>
<td>0.74</td>
<td>0.50</td>
</tr>
<tr>
<td>$^{82}$Rb</td>
<td>3.15</td>
<td>1.6</td>
<td></td>
</tr>
</tbody>
</table>

### Positron range and intrinsic resolution (3)

Approximate annihilation distribution for $^{18}$F and $^{82}$Rb in water

![Approximate annihilation distribution](image)

### Positron range and intrinsic resolution (4)

- $^{82}$Rb
  - GE-OST, OSEM
- $^{13}$N-ammonia
  - Scanditronix, FBP
Momentum is conserved.
What happens to the momentum of the positronium?
The annihilation photons must carry it away - so they are not exactly co-linear.
Angular uncertainty ~ 0.5 degrees
Positional uncertainty for 1m PET scanner ~ 2 mm

Depth of interaction and intrinsic resolution

Resolution degrades as the radial distance increases
PET-CT: 4.5 mm at the center
5.5 mm at 10 cm from center
Most clinical tomographs have spatial resolution in 4-6 mm range

General concepts of tomography acquisition
Projection imaging

General concepts of tomography acquisition

Tomography: Many planar acquisitions
A sinogram is a representation of the projection data in a 2D matrix. Each slice will have its own 2D sinogram.

Sinograms are useful for detecting patient motion.
Scanner coordinate system

- Object space \((x, y)\) to scanner space \((r, s)\):
  \[
  r = x \cos q + y \sin q \\
  s = y \cos q - x \sin q
  \]
- Explains how radioactivity at location \((x, y)\) contributes to signal recorded at location \(r\) acquired at rotation angle \(\theta\)

Foundation of backprojection: Radon transform

- An integral transform that takes \(f(x, y)\) and defines it as line integrals through \(f(x, y)\) at different offsets from the origin

  \[
  R(r, q) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \left( x \cos q + y \sin q - r \right) dx dy
  \]

  Radon transform:

  \[
  b(x, y) = R(r, q) \bigg|_{r=x \cos q + y \sin q}
  \]

  Backprojection:

  \[
  b(x, y) = R(r, q) \bigg|_{r=x \cos q + y \sin q} dq
  \]

Simple backprojection
How does simple backprojection work?

Simple backprojection with no filtering

Simple backprojection results in blurring that is mathematically explained as:

\[ f'(x, y) = f(x, y) \cdot \left(1/r\right) \]

Often referred to as 1/r blurring.

Central slice theorem

- To solve the 1/r blurring problem of simple backprojection we can use the central slice theorem.
- The Fourier transform of a projection of an object at angle, \( \theta \), equals a spoke through the 2D Fourier transform of the object that passes through the origin (\( f_x = f_y = 0 \)) and is oriented at \( \theta \).
- Simple backprojection oversamples in Fourier space.
Steps for filtered backprojection (FBP)

1. Acquire projection images
2. Compute the 1D Fourier transform of each profile (convert to spatial frequency domain)
3. Apply the filter in the frequency domain
4. Compute the inverse Fourier transform to convert back to spatial domain
5. Perform backprojection

Filtered Backprojection: Filters

- Filtering is used to remove the $1/r$ blurring found in simple backprojection
- Ramp is simplest
- Others are used to remove noise artifacts at high frequencies
  - Shepp-Logan
  - Hann
- Filtering is performed in spatial frequency space following a Fourier transform

Tomography: filtering

Initial Image

No filtering

ramp

Hanning

sinogram 32 projections
Filtered Backprojection: Equation form

Steps for FBP
1. Acquire projection images \( p(r) \)
2. Compute the 1D Fourier transform of each profile (convert to frequency domain)
3. Apply the filter in the frequency domain
4. Compute the inverse Fourier transform to convert back to spatial domain
5. Perform backprojection
6. Then apply this procedure and sum over all projections

\[
f(x, y) = \int_0^d \int_0^d P(e^{2\pi j r}) e^{j \pi r} d\rho d\phi
\]

Tomography: reconstruction of 1 slice

Factors affecting image quality: Noise
Factors affecting image quality: Acquisition sampling

Low projection sampling resolution can cause blurring and aliasing artifacts

Factors affecting image quality: Reconstruction filter

- Reduction of cutoff frequency:
  - Increase blurring
  - Reduction in noise
  - Reduction in image detail

Factors affecting image quality: Angular sampling

- Reduction in acquired projection angles:
  - Decrease acquisition time
  - Increase spoke-like artifacts
Factors affecting image quality: Angular sampling range

• Full 180° angular sampling is needed

Factors affecting image quality: Full object coverage

• Incomplete coverage of the object during some or parts of the scan can lead to artifacts

Factors affecting image quality: Missing detector

• Instrumentation failure can cause artifacts due to missing data
Maximum likelihood expectation maximization (MLEM): example

Maximum likelihood expectation maximization (MLEM): Equation form

\[ f_i^{(n+1)} = f_i^{(n)} \times \frac{1}{a_{ij}} \frac{g_j}{s_i f_k^{(n)}} a_{kj} \]

The current image estimation: \( f_i^{(n)} \)

The probability that activity emitted in voxel i is detected by detector j: \( a_{ij} \)

\( a_{ij} \) can contain physical information such as effects of spatial resolution, scatter, and attenuation and other characteristics of the detection process.

Actual measured projection: \( g_j \)

Current forward projection estimate: \( s_i f_k^{(n)} \)

Therefore, the ratio of the measured projection to the current projection is:

\[ \frac{g_j}{s_i f_k^{(n)}} \]

Current backprojection of this ratio is:

\[ \frac{g_j}{s_i a_{ij} f_k^{(n)}} a_{kj} \]

Which then acts upon the current estimate \( f_i^{(n)} \) to form our new estimate \( f_i^{(n+1)} \)

It’s an iterative procedure.
Steps of maximum likelihood expectation maximization (MLEM): equation form

\[ f^{(n+1)}_i = f^{(n)}_i \sum_j a_{ij} s_j g_j f^{(n)}_k \]

1. The first \( f^{(0)}_i \) is a guess and is typically uniform.
2. Forward project: Simulate the projection measurement from the previous estimate
3. Compare the forward projected estimate to the actual measured projection
4. Next, update (improve) our estimated image using the current information
5. Repeat this until convergence is reached!

Maximum-likelihood, expectation maximization algorithm (MLEM)

- Correct for Poisson noise
- Positivity guaranteed
- Slow compared to FBP
- Acceleration of the process by the "Ordered Subsets (OSEM) approach": projections are divided into subsets, which are updated at each iteration
- Noise at high iteration numbers (approximation of a continuous function by a pixelated one)
- Noise can be reduced greatly by convolving the noisy image estimate with a gaussian kernel (regularization)
- How to define when to stop?

How do we know when to stop?

- Low frequencies are reconstructed first
- As iterations increase, image detail is recovered and so is noise
- Too few iterations: no image detail and lack of convergence
- Too many iterations: image is noisy
- Solution: assure proper convergence and remove noise with a gaussian filter
Ordered subset expectation maximization

- Solution to improve MLEM: Ordered-subset expectation maximization (OSEM)
- At each step, project and backproject at only some angles (i.e. a subset)
- Perform the steps in an ordered way to include all angles
- Data start to converge even before the 1st iteration is complete
- Convergence achieved in 3 - 10 iterations
- Much quicker than MLEM

Iterative reconstruction can model the reality of emission tomography

- Attenuation
- Positron range
- Noncollinearity of photons (PET)
- Deadtime
- Scatter coincidences
- Random coincidences
- Physics of crystal: size, intercrystal scatter and penetration
- Noise

Iterative vs. FBP

- Advantages of iterative methods:
  - The results must be better because the correct physics is included in the reconstruction: The reconstruction algorithm "knows" the physics
  - Attenuation correction
  - Reduction of streak artifact
  - Overall quality
- Disadvantages of iterative methods (MLEM)
  - Slow convergence to the desired solution (e.g. tens - hundreds of iterations)
  - Computationally demanding - number of iterations and inclusion of the physics
OSEM vs FBP

- Filtered Back-Projection
  - Fast
  - Robust
  - Subject to noise & streaks
- OSEM
  - Almost as fast
  - Handles noise & streaks

Analytic vs iterative reconstructions

Analytic vs iterative reconstructions
Analytic vs iterative reconstructions

Coronal slices

2D AWOSEM RVR

2D FBPATT RVR

3D AWOSEM RVR

3D FBPATT RVR

Analytic vs iterative reconstructions

FBP, 10 min emission

Analytic vs iterative reconstructions

OSEM, 10 min emission
Methods: Motion Corrected OSEM

- List-mode MLEM reconstruction algorithm with motion modeled in the system matrix:

\[ a_m(f) = \sum_{j=1}^{M} a_{m,j}(f) \]

Motion correction means a contribution of pixel \( n \) in the ref frame to pixel \( j \) in the deformed frame.

- Attenuation correction using deformed attenuation maps at each frame:

\[ \rho_j(f) = \sum_{n=1}^{N} a_n(f) a_{m,n}(f) \rho_n(f) \]

- Transformation using measured motion fields from tagged MR:

Ouyang J., Petibon Y., El Fakhri G.

Primate Results: Acquisition

- Motion Correction with *Primate* in simultaneous PET-MR:

Gated tagged MR  Gated PET

Nonhuman Primate Results (2/3)

Chun S.Y., Reese T., Guerin B., Catana C., Zhu X., Alpert N., El Fakhri G.
Tagged MR-based Motion Correction in Simultaneous PET-MRI. JNM; 2012;

Liver patient study (1/3)

Cine MRI
(TrueFISP)
Cine MRI
(TrueFISP)
Respiratory Gated PET

Respiratory motion amplitude in the dome of the liver (~0.7-1.5cm).

Initial results in hepatic lesions (2)

• Estimated Motion via B-spline non-rigid image registration

\[ \hat{\eta} = \arg \min_{\eta} (I_{\text{tar}} - I_{\text{src}})^T \eta (I_{\text{tar}} - I_{\text{src}}) \]
Liver patient study (3/3)

[Image of liver patient study]


Measure Motion Fields and Track Motion Phases

[Diagram of motion fields and tracking]

Motion correction for PET reconstruction

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THANK YOU!