Quantitative Reconstruction in PET / CT and PET / MR

Georges El Fakhri, PhD, DABR
Gordon Center for Medical Imaging
Massachusetts General Hospital
Harvard Medical School

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Outline

- PET 101
- Tomography in medical imaging
- Projection imaging
- Sinogram
- Reconstruction-analytic
- Backprojection artifacts
- Reconstruction-iterative
- Comparison of analytic and iterative reconstructions
- Simultaneous PET-MR

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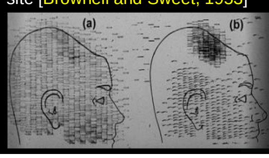
History of PET at MGH

The birthplace of Positron Emission Imaging was at MGH in 1952 in the Center for Radiological Sciences (Ancestor of the Gordon Center for Medical Imaging) where the first positron-imaging device was invented by Dr Gordon Brownell and used for the detection of brain tumors for neurosurgery by Dr Sweet (1953)

Coincidence (a) and "unbalance" (b) scans of a patient with recurring tumor (left) under previous operation site [Brownell and Sweet, 1953]



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What is tomography?

- Greek translation:
  - *tomos* means slice, section
  - *graph* means write
- 2-D representations of structures in a selected plane of a 3-D object
- Mathematical algorithms can be used to reconstruct the original 3-D object from the 2-D projections
- Used in medical imaging
  - SPECT and PET-Emission computed tomography
  - CT-Transmission computed tomography

http://science.howstuffworks.com/cat-scan1.htm

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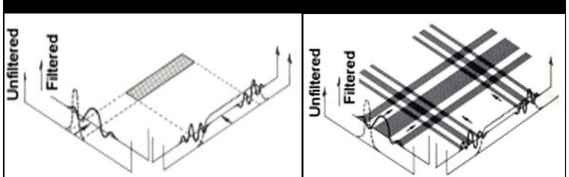
History of PET at MGH

MGH is also the birthplace of filtered backprojection that is stil widely used in PET and in CT. Dr David Chesler (Brownell Lab) presented the first results about filtered backprojection at the Meeting of Tomographic Imaging in Nuclear Medicine (1972).



So the PE & T of PET started at MGH!

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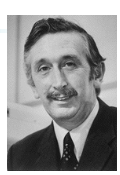
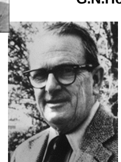
History

J. Radon



G.N.Hounsfield



A.M.Cormack

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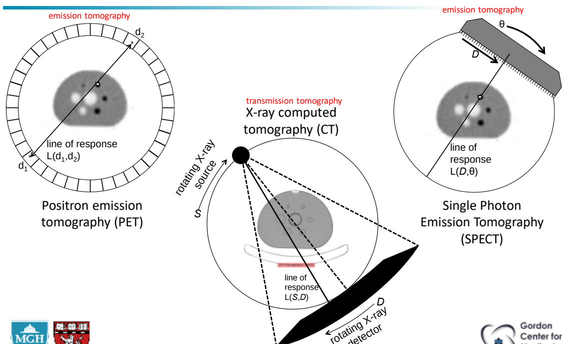
Why tomography over planar imaging?

10	10	10	30
10	20	10	40
10	10	10	30

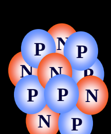
Contrast (Planar) = (40 – 30)/30 = 0.33
Contrast (Tomo) = (20 – 10)/10 = 1.00

Tomography in medical imaging



Gamma Ray Emission

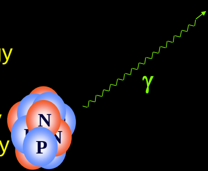
Unstable parent nucleus



Gamma Ray Emission

Nucleus drops to lower energy state.

Gamma ray carries away excess energy



Gamma Ray Emission

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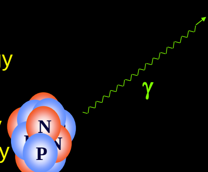
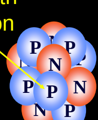
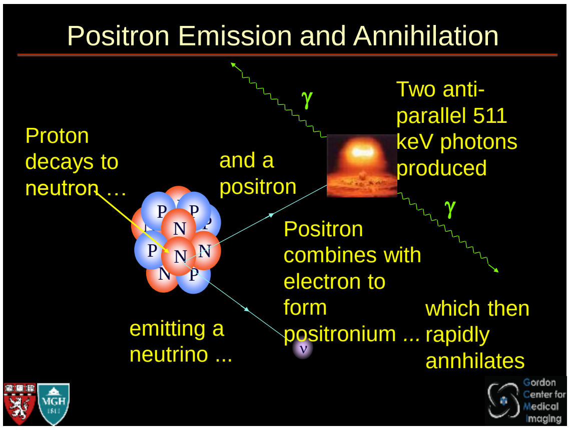


Positron Emission and Annihilation

Unstable parent nucleus with extra proton




Positron Emission and Annihilation
Proton decays to neutron …
and a positron
Two anti-parallel 511 keV photons produced
γ
γ
Positron combines with electron to form positronium …
which then rapidly annhilates
emitting a neutrino …
ν
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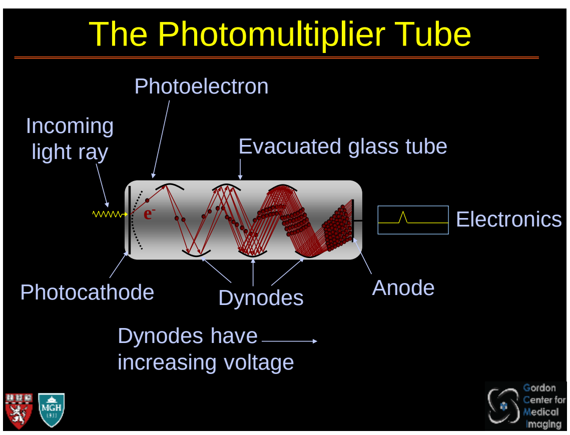
The Photomultiplier Tube
Photoelectron
Incoming light ray
Evacuated glass tube
Electronics
Photocathode
Dynodes
Anode
Dynodes have increasing voltage
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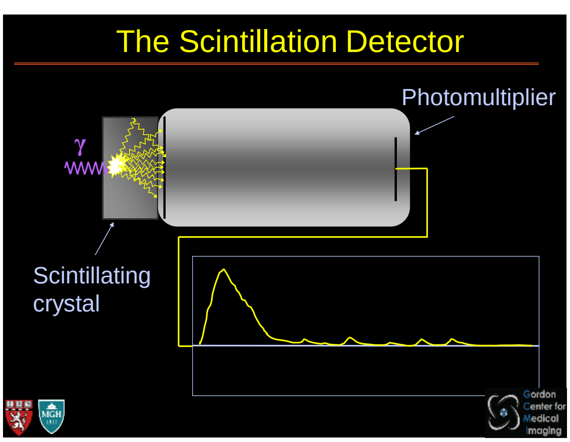
The Scintillation Detector
Photomultiplier
γ
Scintillating crystal
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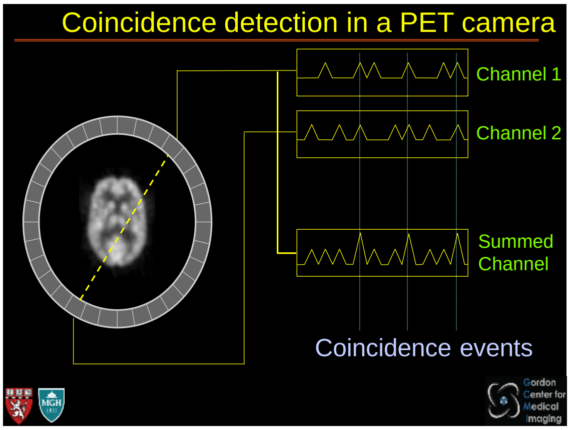
Coincidence detection in a PET camera
Channel 1
Channel 2
Summed Channel
Coincidence events
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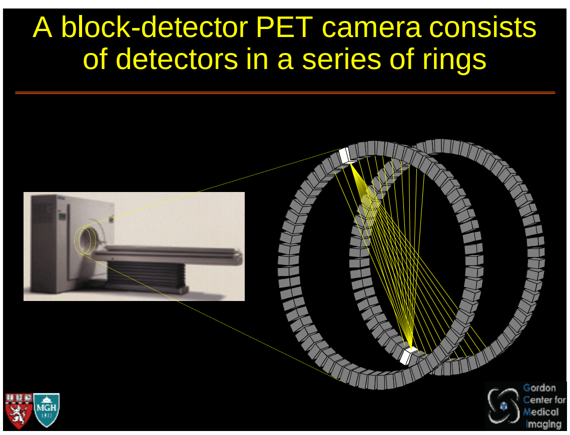
A block-detector PET camera consists of detectors in a series of rings
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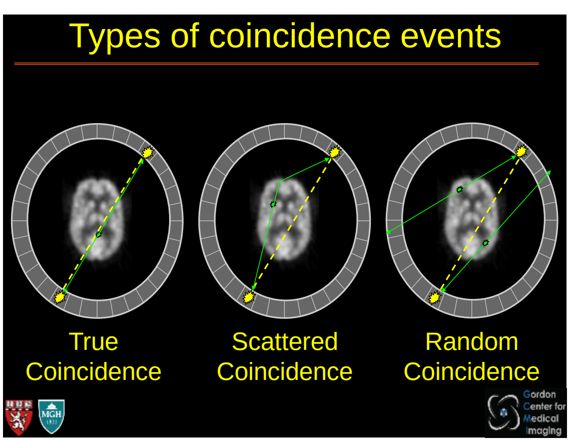
Types of coincidence events
True Coincidence
Scattered Coincidence
Random Coincidence
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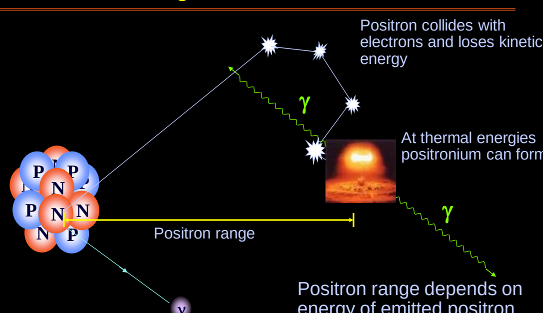
Example: Typical Whole Body PET

Type of coincidence	Percentage (%)
Raw	100
Trues	38
Randoms	34
Scattered	28
Multiple	7

Electronic collimation and intrinsic resolution

- Resolution depends on size of detector elements
- Resolution does not change much between the detectors

Positron range and intrinsic resolution



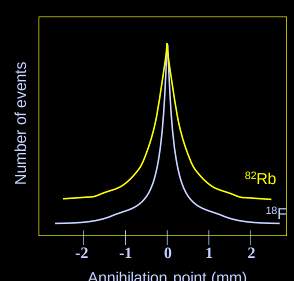
Positron range depends on energy of emitted positron

Positron range and intrinsic resolution (2)

Isotope	Maximum positron energy (MeV)	Mean positron energy (MeV)	Range in water FHWM (mm)
^{18}F	0.64	0.25	0.10
^{11}C	0.96	0.39	0.19
^{13}N	1.19	0.49	0.28
^{15}O	1.70	0.74	0.50
^{82}Rb	3.15		1.6

Positron range and intrinsic resolution (3)

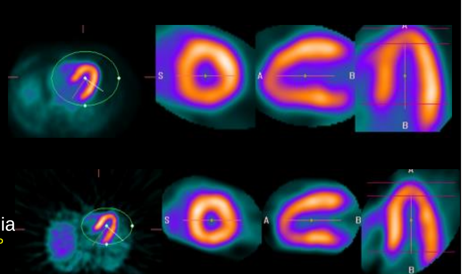
Approximate annihilation distribution for ^{18}F and ^{82}Rb in water



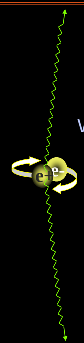
Positron range and intrinsic resolution (4)

^{82}Rb
GE-DST, OSEM

^{13}N-ammonia
Scanditronix, FBP



Photon non co-linearity & intrinsic resolution



Momentum is conserved.

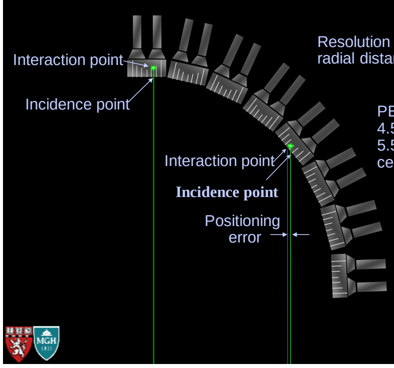
What happens to the momentum of the positronium?

The annihilation photons must carry it away - so they are not exactly co-linear.

Angular uncertainty ~ 0.5 degrees

Positional uncertainty for 1m PET scanner ~ 2 mm

Depth of interaction and intrinsic resolution

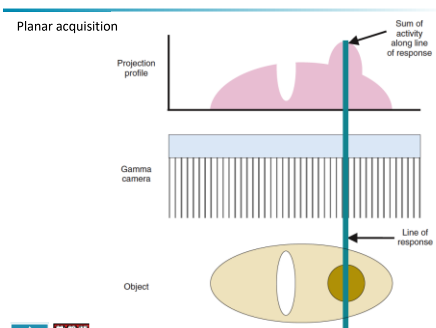


Resolution degrades as the radial distance increases

PET-CT:
4.5 mm at the center
5.5 mm at 10 cm from center

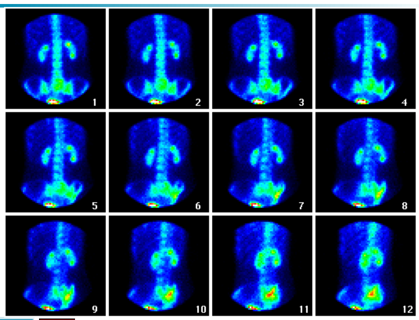
Most clinical tomographs have spatial resolution in 4-6 mm range

General concepts of tomography acquisition

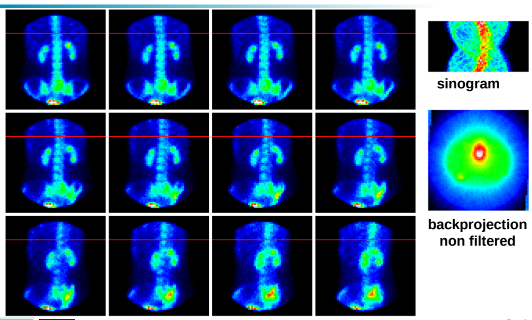


Cherry, S. R.; Sorenson, J. A.; Phelps, M. E. Physics in Nuclear Medicine; Elsevier, 2012.

Projection imaging

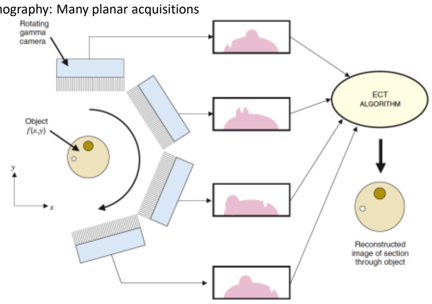


Projection imaging



General concepts of tomography acquisition

Tomography: Many planar acquisitions



Cherry, S. R.; Sorenson, J. A.; Phelps, M. E. Physics in Nuclear Medicine; Elsevier, 2012.

SPECT scanners

PET scanners

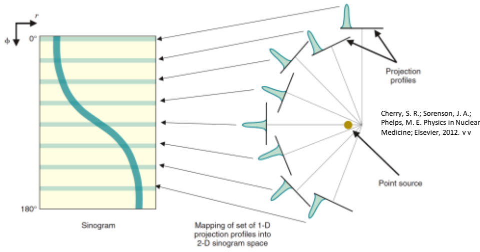
Biograph (Siemens)

Discovery (GE)

Gemini (Philips)

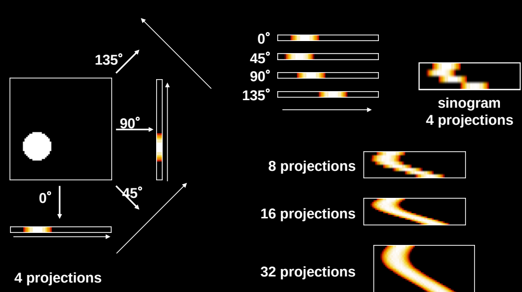
CT scanners

Displaying projection data: Sinogram



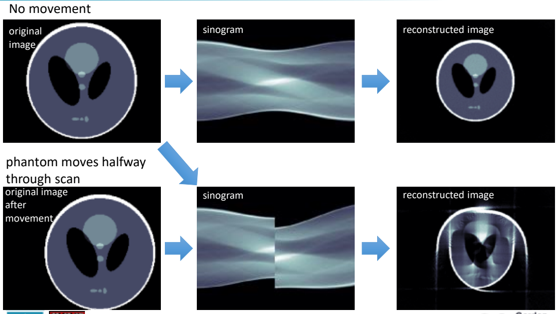
A sinogram is a representation of the projection data in a 2D matrix. Each slice will have its own 2D sinogram.

Tomography: sinogram



Sinograms are useful for detecting patient motion

No movement



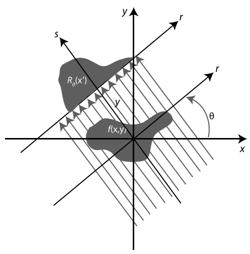
Scanner coordinate system

- Object space (x,y) to scanner space (r,s):

$$r = x\cos\theta + y\sin\theta$$

$$s = y\cos\theta - x\sin\theta$$

- Explains how radioactivity at location (x,y) contributes to signal recorded at location r acquired at rotation angle θ



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Foundation of backprojection: Radon transform

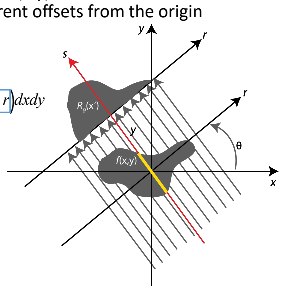
- An integral transform that takes f(x,y) and defines it as line integrals through f(x,y) at different offsets from the origin

Radon transform:

$$R(r,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta\left(\boxed{x\cos\theta + y\sin\theta - r}\right)dxdy$$

Backprojection:

$$b(x,y) = \int_{0}^{\pi} R(r,\theta)\big|_{r = x\cos\theta + y\sin\theta}\, d\theta$$



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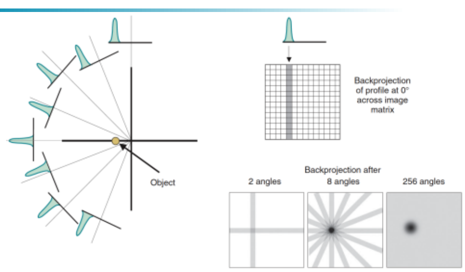
Simple backprojection



Cherry, S. R.; Sorenson, J. A.; Phelps, M. E. Physics in Nuclear Medicine; Elsevier, 2012.

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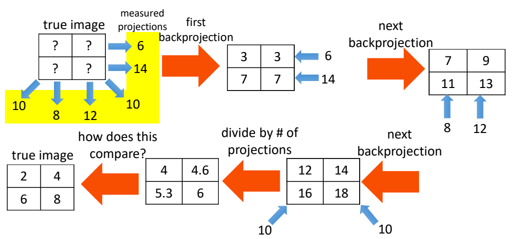
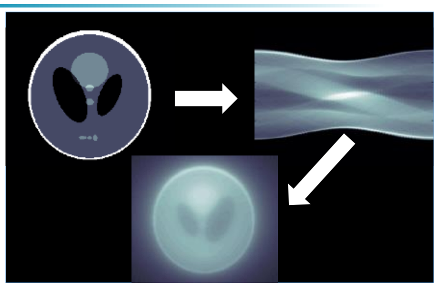
How does simple backprojection work?



Simple backprojection with no filtering

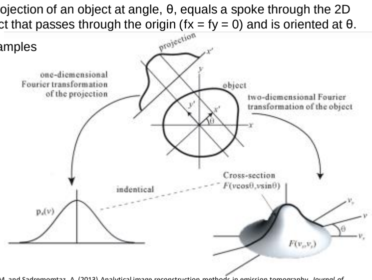


Simple backprojection results in blurring that is mathematically explained as:

$f'(x,y) = f(x,y) * (1/r)$ Often referred to as 1/r blurring.

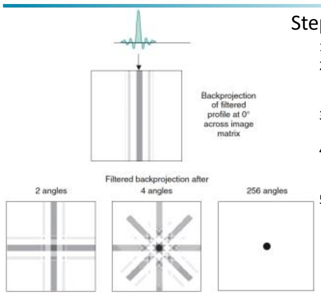
Central slice theorem

- To solve the 1/r blurring problem of simple backprojection we can use the central slice theorem
- The Fourier transform of a projection of an object at angle, θ, equals a spoke through the 2D Fourier transform of the object that passes through the origin (fx = fy = 0) and is oriented at θ.
- Simple backprojection oversamples in Fourier space



Asl, M. and Sadremomtaz, A. (2013) Analytical image reconstruction methods in emission tomography. *Journal of Biomedical Science and Engineering*, **6**, 100-107. doi: 10.4236/jbise.2013.61013.

Steps for filtered backprojection (FBP)



Cherry, S. R.; Sorenson, J. A.; Phelps, M. E. Physics in Nuclear Medicine; Elsevier, 2012.

Steps for FBP

1. Acquire projection images
2. Compute the 1D Fourier transform of each profile (convert to spatial frequency domain)
3. Apply the filter in the frequency domain
4. Compute the inverse Fourier transform to convert back to spatial domain
5. Perform backprojection

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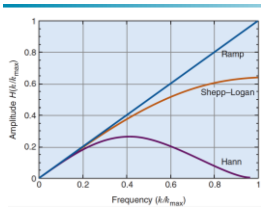
Filtered Backprojection: Filters



Cherry, S. R.; Sorenson, J. A.; Phelps, M. E. Physics in Nuclear Medicine; Elsevier, 2012.

- Filtering is used to remove the 1/r blurring found in simple backprojection
  - Ramp is simplest
- Others are used to remove noise artifacts at high frequencies
  - Shepp-Logan
  - Hann
- Filtering is performed in spatial frequency space following a Fourier transform

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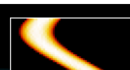
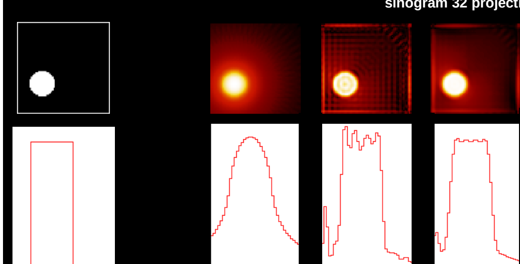
Tomography: filtering



sinogram 32 projections



Initial Image | No filtering | ramp | Hanning

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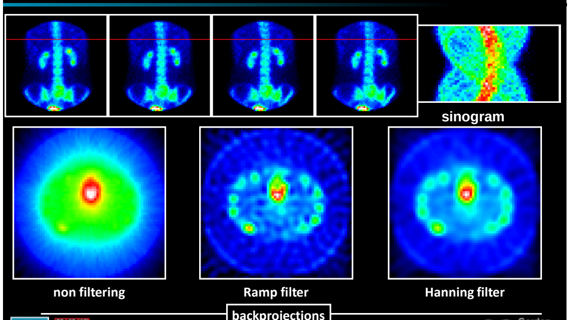
Filtered Backprojection: Equation form

Steps for FBP

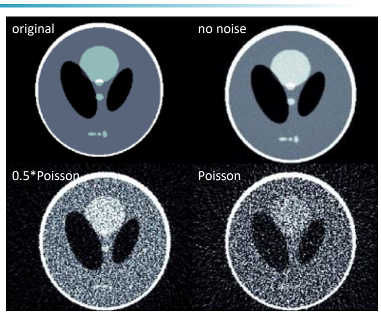
1. **Acquire projection images** $p(r,\theta)$
2. **Compute the 1D Fourier transform of each profile (convert to frequency domain)**
3. **Apply the filter in the frequency domain**
4. Compute the inverse Fourier transform to convert back to spatial domain
5. **Perform backprojection**
6. **Then apply this procedure and sum over all projections**

$$f(x,y) = \int_0^{\pi} d\theta \left[\int_0^{\infty} d\omega \, |\omega| \, P(\omega) \, e^{2\pi i \omega r} \right]_{r = x\cos\theta + y\sin\theta}$$

Tomography : reconstruction of 1 slice

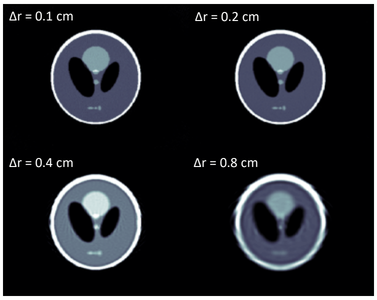


Factors affecting image quality: Noise

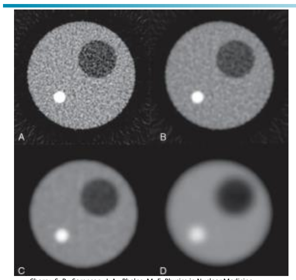


Factors affecting image quality: Acquisition sampling

Low projection sampling resolution can cause blurring and aliasing artifacts

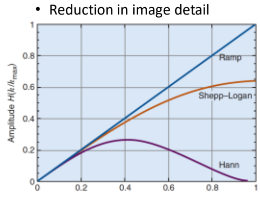


Factors affecting image quality: Reconstruction filter



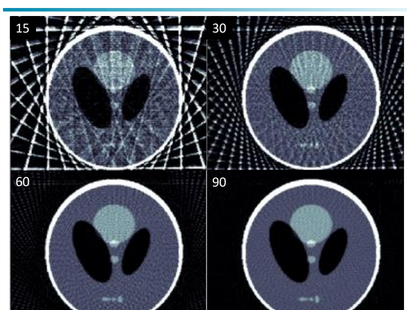
Cherry, S. R.; Sorenson, J. A.; Phelps, M. E. Physics in Nuclear Medicine; Elsevier, 2012.

- Reduction of cutoff frequency:
  - Increase blurring
  - Reduction in noise
  - Reduction in image detail



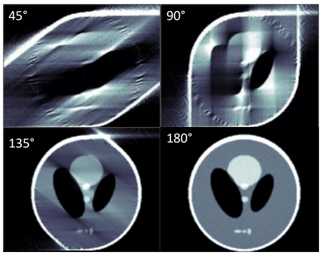
Cherry, S. R.; Sorenson, J. A.; Phelps, M. E. Physics in Nuclear Medicine; Elsevier, 2012.

Factors affecting image quality: Angular sampling

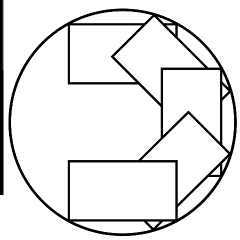


- Reduction in acquired projection angles:
- Decrease acquisition time
- Increase spoke-like artifacts

Factors affecting image quality: Angular sampling range

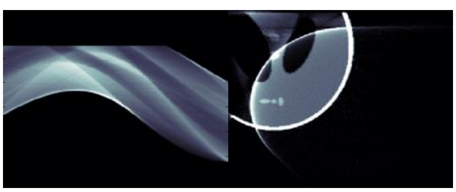


- Full 180° angular sampling is needed



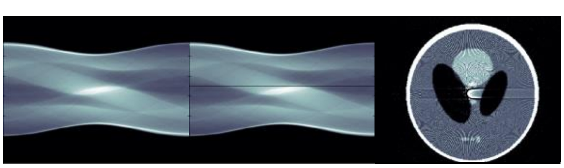
Factors affecting image quality: Full object coverage

- Incomplete coverage of the object during some or parts of the scan can lead to artifacts

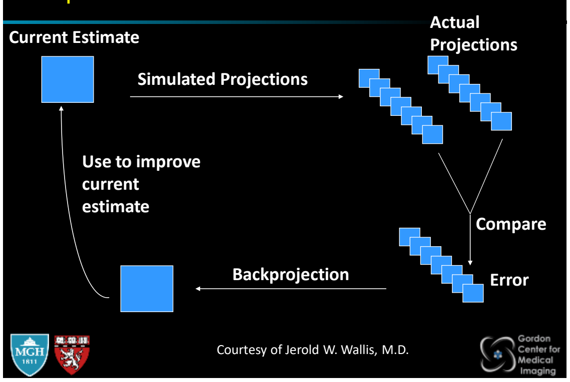


Factors affecting image quality: Missing detector

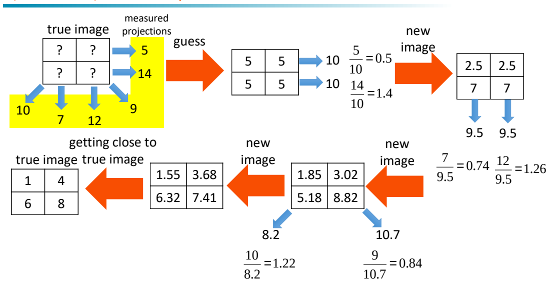
- Instrumentation failure can cause artifacts due to missing data



Steps of Iterative reconstruction of 1 slice



Maximum likelihood expectation maximization (MLEM): example



It's an iterative procedure

Maximum likelihood expectation maximization (MLEM): Equation form

$$f_i^{(n+1)} = f_i^{(n)} \times \frac{1}{\sum_j a_{ij}} \sum_j \frac{g_j}{\sum_k a_{kj} f_k^{(n)}} a_{ij}$$

The current image estimation: $f_i^{(n)}$

The probability that activity emitted in voxel i is detected by detector j: a_{ij}

a_{ij} can contain physical information such as effects of spatial resolution, scatter, attenuation and other characteristics of the detection process

Actual measured projection: g_j

Current forward projection estimate: $\sum_k a_{kj} f_k^{(n)}$

Therefore, the ratio of the measured projection to the current projection is: $\frac{g_j}{\sum_k a_{kj} f_k^{(n)}}$

Current backprojection of this ratio is: $\sum_j \frac{g_j}{\sum_k a_{kj} f_k^{(n)}} a_{ij}$

Which then acts upon the current estimate $f_i^{(n)}$ to form our new estimate $f_i^{(n+1)}$

Steps of maximum likelihood expectation maximization (MLEM): equation form

$$f_i^{(n+1)} = f_i^{(n)} \times \frac{1}{\sum_j a_{ij}} \sum_j \frac{g_j}{\sum_k a_{kj} f_k^{(n)}} a_{ij}$$

1. The first $f_i^{(0)}$ is a guess and is typically uniform.

2. Forward project: Simulate the projection measurement from the previous estimate $\sum_k a_{kj} f_k^{(n)}$

3. Compare the forward projected estimate to the actual measured projection $\frac{g_j}{\sum_k a_{kj} f_k^{(n)}}$

4. Next, update (improve) our estimated image using the current information $f_i^{(n+1)} = f_i^{(n)} \times \frac{1}{\sum_j a_{ij}} \sum_j \frac{g_j}{\sum_k a_{kj} f_k^{(n)}} a_{ij}$

5. Repeat this until convergence is reached!

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Maximum-likelihood, expectation maximization algorithm (MLEM)

- Correct for Poisson noise

- Positivity guaranteed

- Slow compared to FBP

- Acceleration of the process by the "Ordered Subsets (OSEM) approach": projections are divided into subsets, which are updated at each iteration

- Noise at high iteration numbers (approximation of a continuous function by a pixelated one)

- Noise can be reduced greatly by convolving the noisy image estimate with a gaussian kernel (regularization)

- How to define when to stop?

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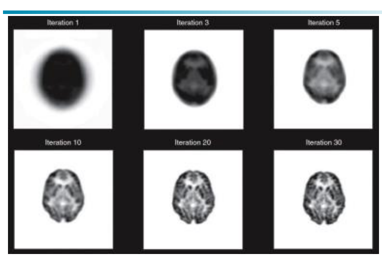
How do we know when to stop?



Cherry, S. R.; Sorenson, J. A.; Phelps, M. E. Physics in Nuclear Medicine; Elsevier, 2012.

- Low frequencies are reconstructed first
- As iterations increase image detail is recovered and so is noise
- Too few iterations: no image detail and lack of convergence
- Too many iterations: image is noisy
- Solution: assure proper convergence and remove noise with a gaussian filter

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Ordered subset expectation maximization

- Solution to improve MLEM: Ordered-subset expectation maximization (OSEM)
- At each step, project and backproject at only some angles (i.e. a subset)
- Perform the steps in an ordered way to include all angles
- Data start to converge even before the 1st iteration is complete
- Convergence achieved in 3 - 10 iterations
- Much quicker than MLEM

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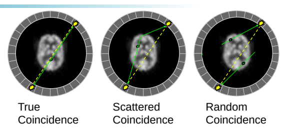
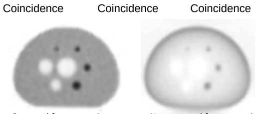
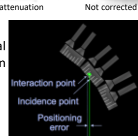
Iterative reconstruction can model the reality of emission tomography

- Attenuation
- Positron range
- Noncollinearity of photons (PET)
- Deadtime
- Scatter coincidences
- Random coincidences
- Physics of crystal: size, intercrystal scatter and penetration
- Noise





crystal penetration



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Iterative vs. FBP

- Advantages of iterative methods:
  - The results must be better because the correct physics is included in the reconstruction: The reconstruction algorithm "knows" the physics
  - Attenuation correction
  - Reduction of streak artifact
  - Overall quality
- Disadvantages of iterative methods (MLEM)
  - Slow convergence to the desired solution (e.g. tens - hundreds of iterations)
  - Computationally demanding - number of iterations and inclusion of the physics

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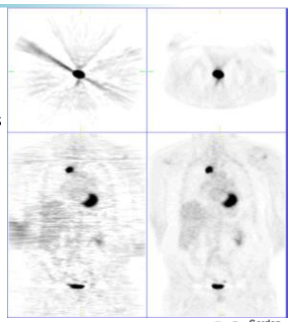
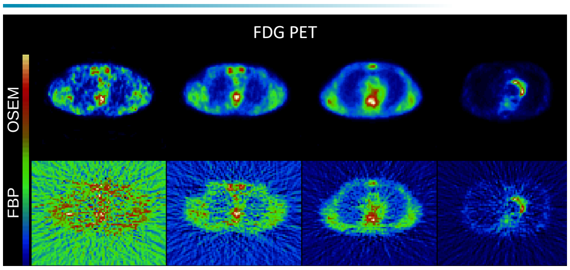
OSEM vs FBP

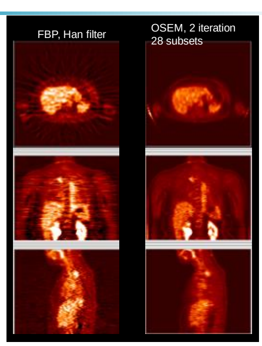
- Filtered Back-Projection
  - Fast
  - Robust
  - Subject to noise & streaks
- OSEM
  - Almost as fast
  - Handles noise & streaks



Analytic vs iterative reconstructions

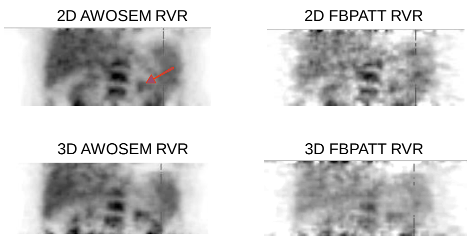


Analytic vs iterative reconstructions



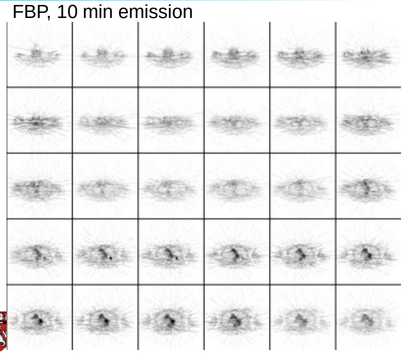
Analytic vs iterative reconstructions

Coronal slices



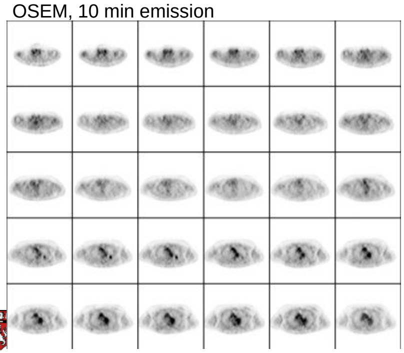
Analytic vs iterative reconstructions

FBP, 10 min emission



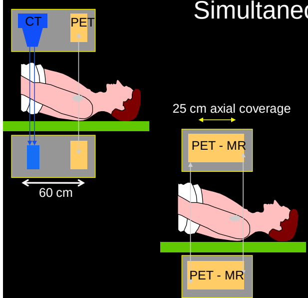
Analytic vs iterative reconstructions

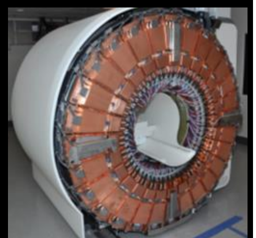
OSEM, 10 min emission



Integrated Whole-Body PET-MR

Simultaneous PET – MR





G. El Fakhri, Ph.D.

Methods: Motion Corrected OSEM

- List-mode MLEM reconstruction algorithm with motion modeled in the system matrix:

$$a_{ni}(f)=\sum_{j=1}^{M} a_{nj}\times m_{ji}(f)$$

Static system matrix

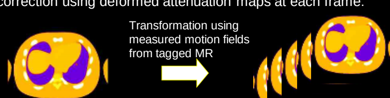
Motion interpolation matrix (i.e., contribution of pixel i in the ref. frame to pixel j in the deformed frame)

Number of counts in list-mode

Motion dependent system matrix

$$\rho_i^{(a+1)}=\frac{\rho_i^{(a)}}{\sum_{f=1}^{F} s_i(f)}\sum_{n=1}^{N}\frac{a_{ni}(f_n)}{\sum_{j=1}^{M} a_{nj}(f_n)\rho_j^{(a)}}$$

- Attenuation correction using deformed attenuation maps at each frame:



Attenuation map in the reference frame

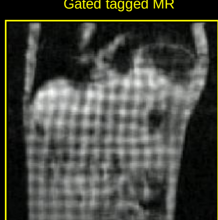
Attenuation maps in the deformed frames

Ouyang J., Petibon Y., El Fakhri G.

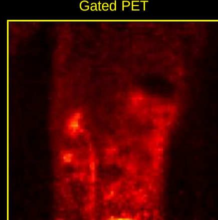
Primate Results: Acquisition

- Motion Correction with *Primate* in simultaneous PET-MR

Gated tagged MR



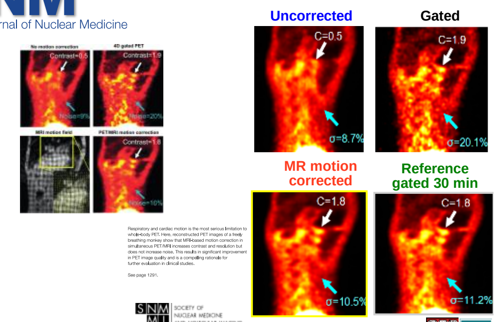
Gated PET



Chun et al. J. Nucl. Med. 2012

Nonhuman Primate Results (2/3)

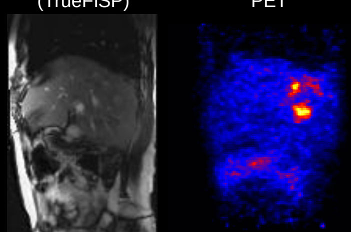
JNM
The Journal of Nuclear Medicine



Chun S.Y., Reese T., Guerin B., Catana C., Zhu X., Alpert N., El Fakhri G. Tagged MR-based Motion Correction in Simultaneous PET-MR. *JNM* 2012;

Liver patient study (1/3)

Cine MRI (TrueFISP) | Respiratory Gated PET



Respiratory motion amplitude in the dome of the liver (~0.7-1.5cm).

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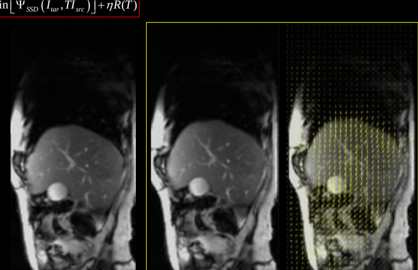
Petibon, Huang, Ouyang and El Fakhri.
Relative role of MR-based motion in WB PET-MR. Med. Phys., 2014 41

Initial results in hepatic lesions (2)

- Estimated Motion via B-spline non-rigid image registration

$$\hat{T} = \arg\min\left[\Psi_{SSD}\left(I_{tar}, TI_{src}\right)\right] + \eta R(T)$$



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Reference : End-inspiration (I_{src}) End-exhalation (I_{tar})

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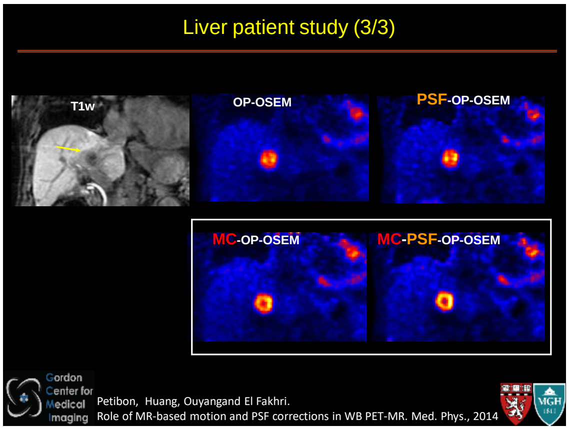

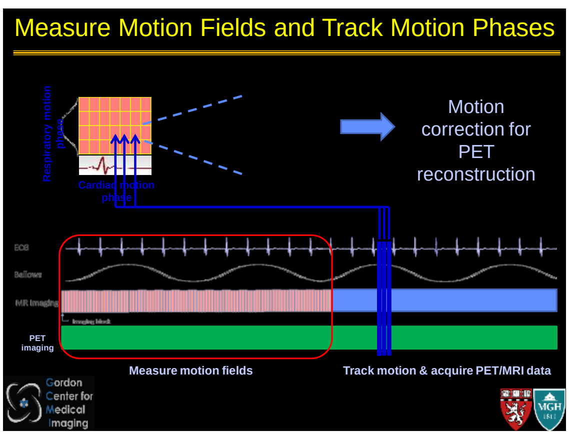

Summary

- PET 101
- Tomography in medical imaging
- Projection imaging
- Sinogram
- Reconstruction-analytic
- Backprojection artifacts
- Reconstruction-iterative
- Comparison of analytic and iterative reconstructions
- Simultaneous PET-MR

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Quantitative Reconstruction in PET / CT and PET / MR

THANK YOU!

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