THE DEPARTMENT OF BIOMEDICAL ENGINEERING

Getting More Image Quality with Less Data in X-Ray CT

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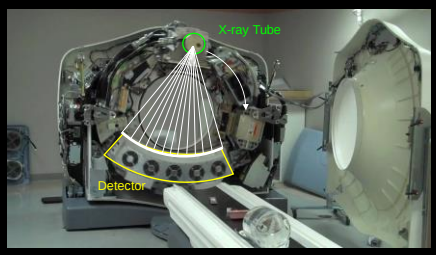
August 30, 2018

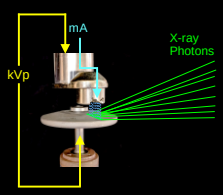
CT Hardware Basics

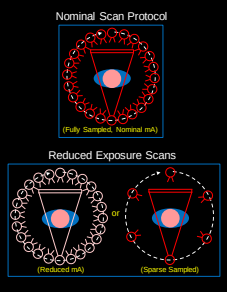


(image Credit) https://www.youtube.com/watch?v=bg0iNhw2ARw

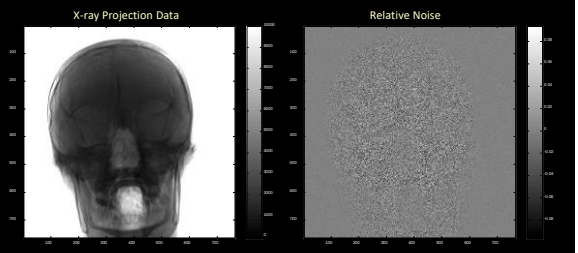
X-ray Tube Physics and Dose Reduction

Number of photons ∝ mA





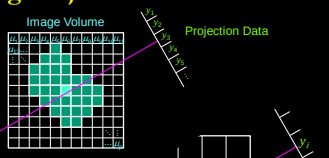
Noise in X-ray Projection Data



Modeling Projection Data

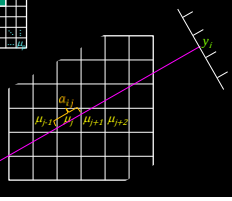
Discrete-Discrete Parameterization

$$\bar{y}_i = I_0 \exp\left(-\sum_{j=1}^{p} a_{ij}\mu_j\right)$$

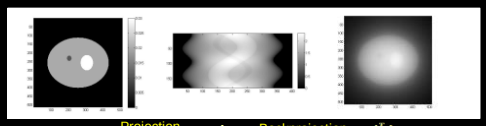


System matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1p} \\ \vdots & \ddots & \vdots & & \vdots \\ a_{i1} & \cdots & a_{ij} & \cdots & a_{ip} \\ \vdots & & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nj} & \cdots & a_{np} \end{bmatrix}$$



Mathematical Forward Model



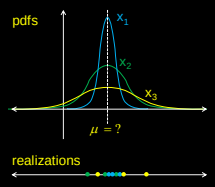
Mean measurements as a function of parameters

$$\bar{y}(\mu) = I_0 \exp(-\mathbf{A}\mu)$$

How to deal with unequal measurement noise (Simple Estimation Problem)

3 Random Variables
Different std dev ($\sigma_1, \sigma_2, \sigma_3$)

Best way to estimate μ?

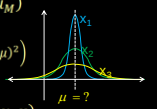


Maximum Likelihood Estimation

Find the parameter values most likely to be responsible for the observed data.

Likelihood Function $L(\boldsymbol{x};\boldsymbol{\mu}) = p(x_1, x_2, \dots, x_N|\mu_1, \mu_2, \dots, \mu_M)$

$$L(\boldsymbol{x};\mu) = p(\boldsymbol{x}|\mu) = \prod_{i=1}^{3} p(x_i|\mu) \quad p(x_i|\mu) = \frac{1}{\sqrt{2\pi\sigma_i^2}}\exp\left(-\frac{1}{2\sigma_i^2}(x_i-\mu)^2\right)$$



Maximum Likelihood Objective Function $\hat{\boldsymbol{\mu}} = \operatorname{argmax} L(\boldsymbol{x};\boldsymbol{\mu})$

$$\frac{\partial}{\partial\mu}\log L(\boldsymbol{x};\mu) = \sum_{i=1}^{3}\frac{1}{\sigma_i^2}(x_i-\mu) \quad \xrightarrow[\frac{\partial}{\partial\mu}=0]{} \quad \hat{\mu} = \frac{\sum_{i=1}^{3}\frac{x_i}{\sigma_i^2}}{\sum_{i=1}^{3}\frac{1}{\sigma_i^2}}$$

Maximum Likelihood Estimation for CT

Log transformed data case (e.g. $l = -\log\left[\frac{y}{I_0}\right]$)

$$\bar{l}(\mu) = \boldsymbol{A}\mu \qquad P_{l_i} = \frac{1}{\sqrt{2\pi\sigma_i^2}}\exp\left(-\frac{1}{2\sigma_i^2}(l_i-\bar{l}_i(\mu))^2\right)$$

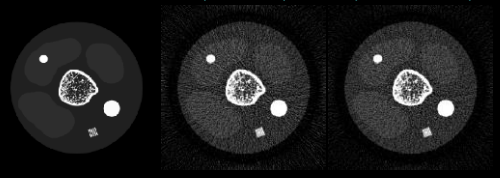
Likelihood-based Objective $\log L(y;\mu) = -\sum_i^N \frac{1}{2}\log(2\pi\sigma_i^2) - \sum_i^N \frac{1}{2\sigma_i^2}(l_i-[\mathbf{A}\mu]_i)^2$

$$\cong \boxed{-[l-\mathbf{A}\mu]^T\mathbf{D}\left\{\frac{1}{\sigma_i^2}\right\}[l-\mathbf{A}\mu] = -\|l-\mathbf{A}\mu\|^2_{\Sigma^{-1}}}$$

Solve for μ $\hat{\mu} = \operatorname{argmax}\log L(y;\mu) = \left[\mathbf{A}^T\mathbf{D}\left\{\frac{1}{\sigma_i^2}\right\}\mathbf{A}\right]^{-1}\mathbf{A}^T\mathbf{D}\left\{\frac{1}{\sigma_i^2}\right\}l$

Statistical vs Nonstatistical Reconstruction

Ground Truth | Nonstatistical Method (FBP, ART, etc.) | Statistical Method (Maximum-Likelihood)



Statistical methods weigh important of individual data points
BUT noise control requires additional information

Additional Information through Regularization

Integrating information via a change in the objective function

$$\hat{\mu} = \operatorname{argmax} \log L(y;\mu) - \beta R(\mu)$$

Penalized-Likelihood Estimation

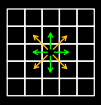
Choices of regularization $R(\mu)$

- Local smoothness
- Edge-preservation
- Prior images
- Patches/dictionaries/learned regularization

Regularization of Local Image Properties

Pairwise Penalty

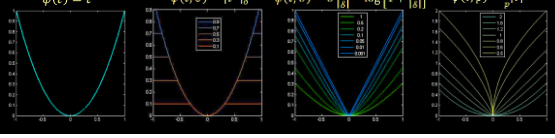
$$R(\mu) = \sum_j \sum_{k \in N} w_{jk} \psi(\mu_j - \mu_k)$$



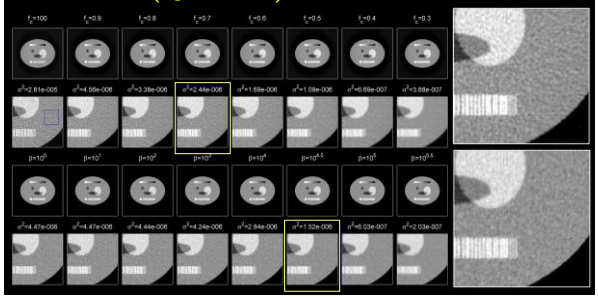
$$k_{1st} = \begin{bmatrix} & -1 & \\ -1 & 4 & -1 \\ & -1 & \end{bmatrix} \quad k_{2nd} = \begin{bmatrix} \frac{-1}{\sqrt{2}} & -1 & \frac{-1}{\sqrt{2}} \\ -1 & 4+\frac{4}{\sqrt{2}} & -1 \\ \frac{-1}{\sqrt{2}} & -1 & \frac{-1}{\sqrt{2}} \end{bmatrix}$$

Penalty Choices

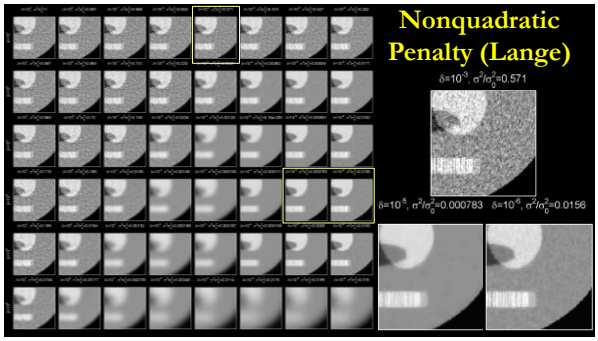
Quadratic $\psi(t) = t^2$ | Truncated Quadratic $\psi(t;\delta) = |t^2|_\delta$ | Lange $\psi(t;\delta) = \delta\left[\frac{|t|}{\delta} - \log\left[1 + \frac{|t|}{\delta}\right]\right]$ | P-norm $\psi(t;p) = \frac{1}{p}|t|^p$



FBP vs (Quadratic) Penalized-Likelihood



Nonquadratic Penalty (Lange)



Regularization using Prior Images

Knowledge of anatomy
- Target locations
- Previous imaging studies
  - Post-treatment assessments
  - Monitoring disease progression
  - Dynamic studies

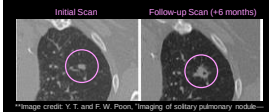
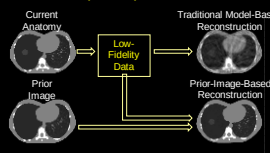


**Image credit: Y. T. and F. W. Poon, "Imaging of solitary pulmonary nodule—a clinical review," Quant. Im. in Med. and Surg. 3(6), Dec. 2013

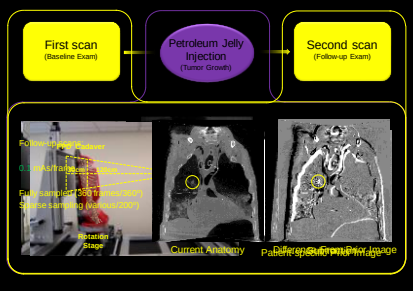
Prior Image Registration Penalized-Likelihood Estimation (PIRPLE)



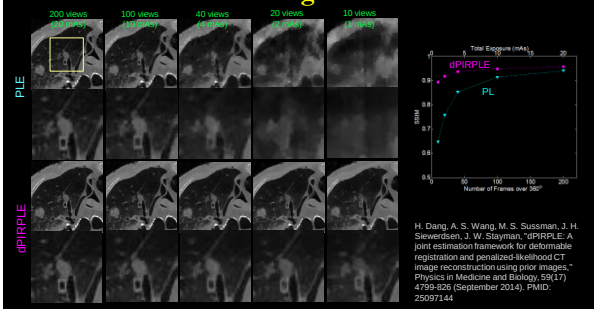
$$\{\hat{\mu}, \hat{\lambda}\} = \operatorname{argmax} \log L(y;\mu) - \beta_R R(\mu) - \beta_P \|\mu - \mathbf{W}(\lambda)\mu_P\|_1$$

Prior Image Registration Penalty Term

Prior Image Regularization Experiments

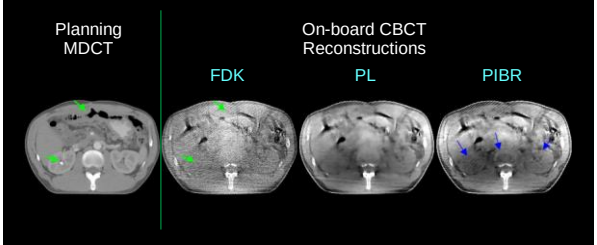


Dose reduction in Lung Nodule Surveillance



H. Dang, A. S. Wang, M. S. Sussman, J. H. Siewerdsen, J. W. Stayman, "dPIRPLE: A joint estimation framework for deformable registration and penalized-likelihood CT image reconstruction using prior images," Physics in Medicine and Biology, 59(17) 4799-826 (September 2014), PMID: 25097144

Prior Images for Image Quality Improvement



H. Zhang, G. J. Gang, J. Lee, J. Wong, J. W. Stayman, "Integration of prior CT into CBCT reconstruction for improved image quality via reconstruction of difference: first patient studies," Proc. SPIE Medical Imaging Orlando, FL, March 2017, 10132, 1013211-1-6.

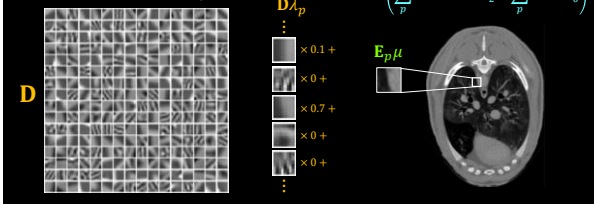
Regularization using Patches/Dictionaries

General/learned knowledge of image features
- Need a dictionary of features/patches
- Sparse representations, linear combinations of few patches

Objective Function

$$\{\hat{\mu}, \hat{\lambda}\} = \operatorname{argmax} \log L(y; \mu)$$

$$-\beta\left(\sum_p \|\mathbf{E}_p\mu - \mathbf{D}\lambda_p\|_2^2 + \sum_p \nu_p \|\lambda_p\|_0\right)$$



Images adapted from: Q Xu, H Yu, X Mou, L Zhang, J Hsieh, G Wang, "Low-dose x-ray CT reconstruction via dictionary learning," *IEEE Trans. Medical Imaging*, 31(9), September 2012.

Dictionary Methods & Few View Reconstructions

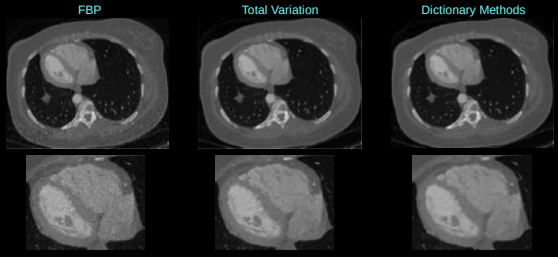


Image credits: Q Xu, H Yu, X Mou, L Zhang, J Hsieh, G Wang, "Low-dose x-ray CT reconstruction via dictionary learning," *IEEE Trans. Medical Imaging*, 31(9), September 2012.

Additional Information in the Forward Model

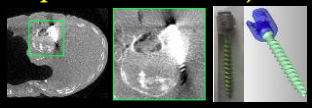
$$\bar{y}(\mu) = I_0 \exp(-\mathbf{A}\mu)$$

Forward model has many simplifications in physics…
Model ignores
- Scatter
- Spectral effects
- Focal spot blur
- Detector blur

Object model may have constraints
- Nonnegativity
- Known element/components in the field-of-view

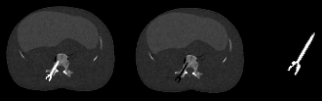
Modeling Known Components in the Object

Implants
- Susceptible to metal artifacts
- Strongest near the device
- Region-of-interest is near implant



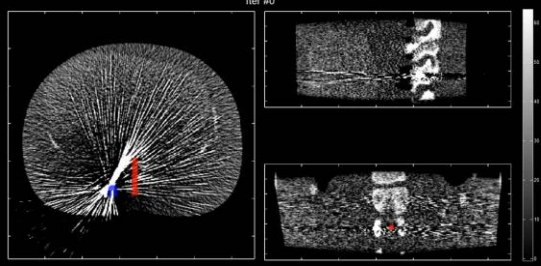
Include a component model in explicitly in the object model

$$\mu(\mu_{\text{anatomy}}, \lambda) = \mathbf{D}\{\mathbf{W}(\lambda)m\}\mu_{\text{anatomy}} + \mathbf{W}(\lambda)\mu_{\text{implant}}$$

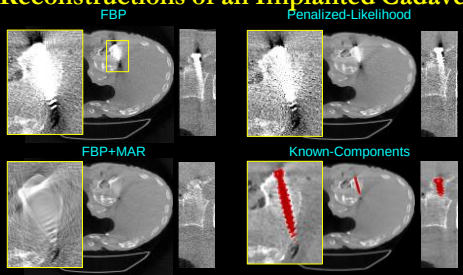


J. W. Stayman, Y. Otake, J. L. Prince, J. H. Siewerdsen, "Model-based Tomographic Reconstruction of Objects containing Known Components," IEEE Trans. Medical Imaging, 31(10), 1837-1848 (October 2012). PMID: 22614574

Known Component Reconstruction Iterations

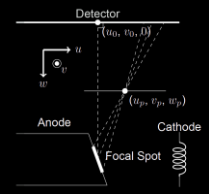


Reconstructions of an Implanted Cadaver

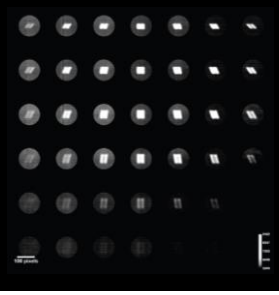


S. Xu, A. Uneri, A. J. Khanna, J. H. Siewerdsen, J. W. Stayman, "Polyenergetic known-component CT reconstruction with unknown material compositions and unknown x-ray spectra," Physics in Medicine and Biology, 62(8), 3352-74 (April 2017)

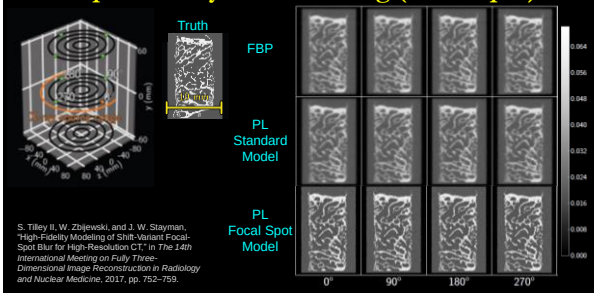
Improving the Physical Model (Focal Spot Blur)



$$\bar{y}_i = \sum_{k=1}^{K} b_{ik} \exp\left(-\sum_{j=1}^{p} a_{kj}\mu_j\right)$$



Improved Physical Modeling (Focal Spot)



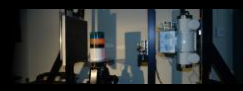
S. Tilley II, W. Zbijewski, and J. W. Stayman, "High-Fidelity Modeling of Shift-Variant Focal-Spot Blur for High-Resolution CT," in *The 14th International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine*, 2017, pp. 752–759.

CT Reconstruction Summary

- Advanced Reconstruction Aims
  - Dose reduction, improved image quality
- CT Forward Model
  - Nonlinear, but often linearized
  - Measurement statistics are important (SNR varies widely)
  - Advanced physical modeling permits image quality improvements
- CT Regularization Strategies
  - Standard smoothness and edge-preservation
  - Use of prior images (e.g., sequential studies)
  - Generalized dictionary methods including machine learning
- Other Objective Function Modifications
  - Additional constraints on the object (e.g., known components)

Acknowledgements

Advanced Imaging Algorithms & Instrumentation Laboratory



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Funding/Support
NIH: U01EB014964, R01EB025470, R21CA219608, R21EB014964, KL2TR001077, R01CA112163
Academic-Industry Partnerships: Siemens, Varian Medical Systems, Elekta AB, Carestream Health
Research Partnership/Agreements: Philips Healthcare, Canon Medical

This work was supported, in part, by the above grants. The contents of this presentation are solely the responsibility of the authors and do not necessarily represent the official view of Johns Hopkins, the NIH, or other organizations.