

1

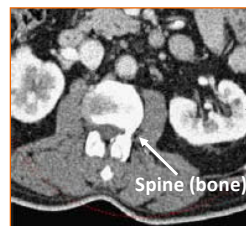
Dependence of x-ray attenuation on Z_{eff} , density and x-ray energy

x-ray attenuation

- x-ray attenuation μ depends on effective atomic number Z_{eff} , density ρ and x-ray energy E
- Introducing the mass attenuation coefficient $\left(\frac{\mu}{\rho}\right)(E, Z_{eff})$ we obtain

$$\mu = \rho \cdot \left(\frac{\mu}{\rho}\right)(E, Z_{eff})$$

- Different materials with different mass attenuation coefficients $\left(\frac{\mu}{\rho}\right)(E, Z_{eff})$ can have the same attenuation coefficient μ in a CT scan depending on the value of the mass density ρ !
- Example: Bones and iodine filled vessels in a CT scan with iodinated contrast agent



Kidney with iodinated contrast agent

Spine (bone)

2

Dependence of x-ray attenuation on Z_{eff} , density and x-ray energy

x-ray attenuation

- Two different absorption mechanisms dominate in the x-ray energy range of CT

- Photoelectric effect**

$$\left(\frac{\mu}{\rho}\right)_p(E, Z) \sim \frac{Z_{eff}^4}{E^3}$$

Dominates at low energies

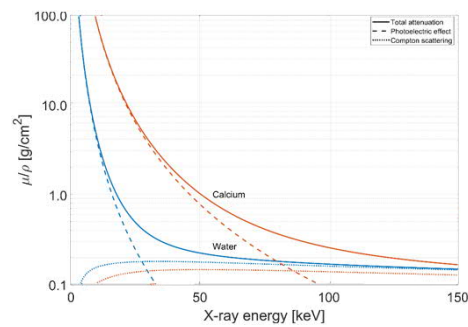
The more dominant the higher the atomic number Z_{eff}

- Compton effect**

$$\left(\frac{\mu}{\rho}\right)_c(E, Z) \sim Z_{eff} \cdot f_{KN}(E)$$

Dominates at high energies

$f_{KN}(E)$ (Klein-Nishina factor) is almost independent of E



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Dependence of x-ray attenuation on Z_{eff} , density and x-ray energy

x-ray attenuation

- The total mass attenuation coefficient is given by the sum of photoelectric effect and Compton effect

$$\left(\frac{\mu}{\rho}\right)(E, Z_{eff}) = a_p \left(\frac{\mu}{\rho}\right)_p(E, Z_{eff}) + a_c \left(\frac{\mu}{\rho}\right)_c(E, Z_{eff})$$

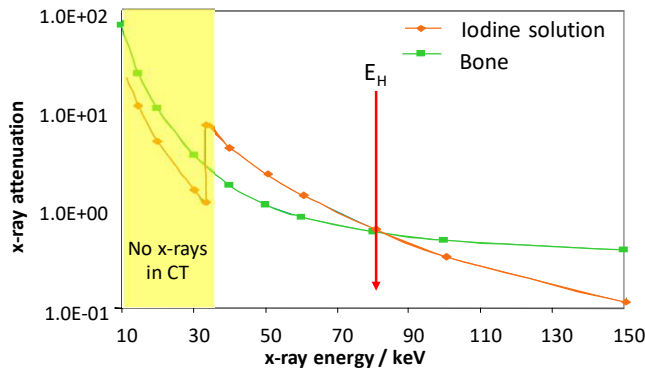
- The attenuation coefficient is then

$$\mu = \rho \cdot \left[a_p \left(\frac{\mu}{\rho}\right)_p(E, Z_{eff}) + a_c \left(\frac{\mu}{\rho}\right)_c(E, Z_{eff}) \right] = \rho \cdot \left[a_p \frac{Z_{eff}^4}{E^3} + a_c Z_{eff} \cdot f_{KN}(E) \right]$$

- a_p and a_c are constants for all energies and Z_{eff} 's !

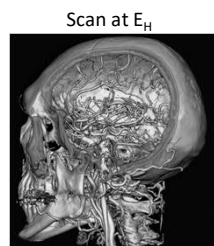
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Principle of material separation by CT-scans at two x-ray energies



Why do we need dual energy CT?

- Two different materials with different effective atomic number Z_{eff} and different density ρ can have the same x-ray attenuation at a certain x-ray energy E_H

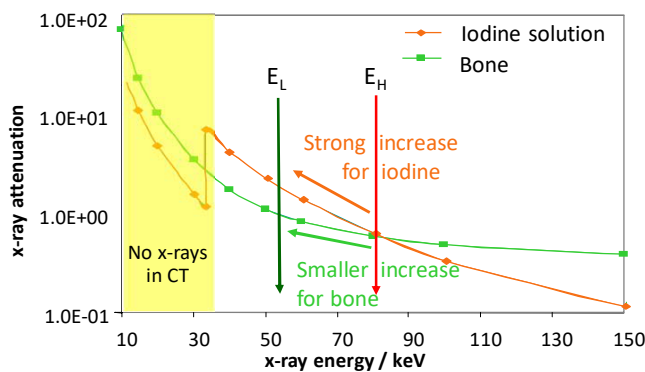


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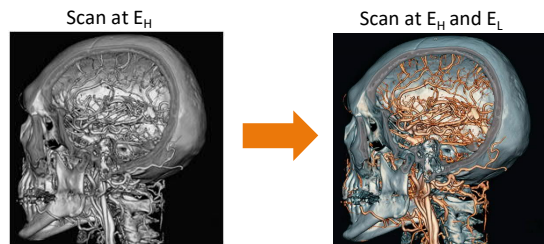
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Principle of material separation by CT-scans at two x-ray energies



Why do we need dual energy CT?

- Two different materials with different effective atomic number Z_{eff} and different density ρ can have the same x-ray attenuation at a certain x-ray energy E_H
- Increase of attenuation for lower x-ray energy depends on Z_{eff}
- Scan at two energies E_L and E_H : Change of x-ray attenuation from high to low x-ray energy characterizes the material



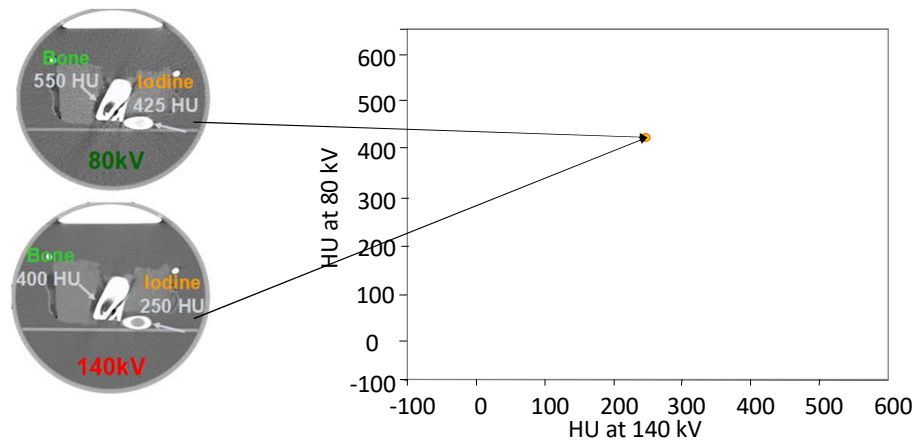
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Basic applications: differentiation of two materials A and B

Mixture of **blood + iodine** and **bone + bone marrow** in a phantom experiment, scan at 80kV and at 140kV

- **Iodine**: large CT-number difference in scans at two energies
- **Bone**: smaller CT-number difference in scans at two energies



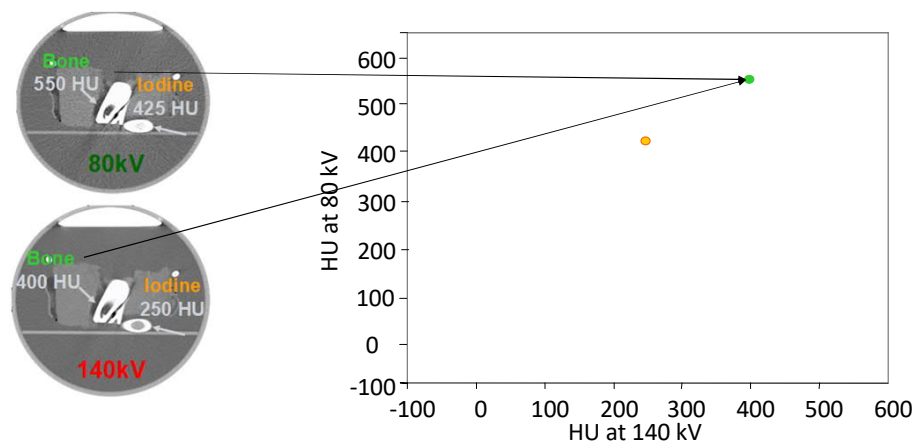
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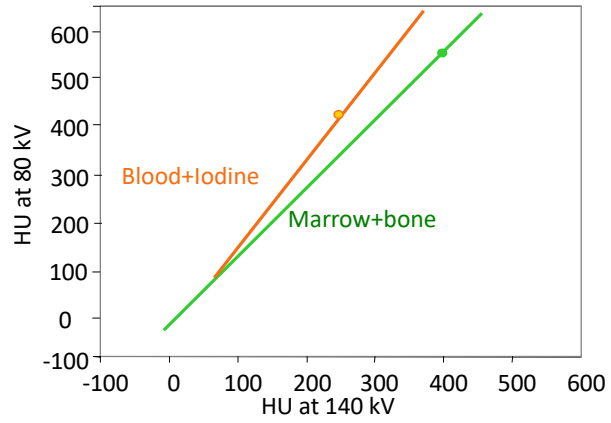
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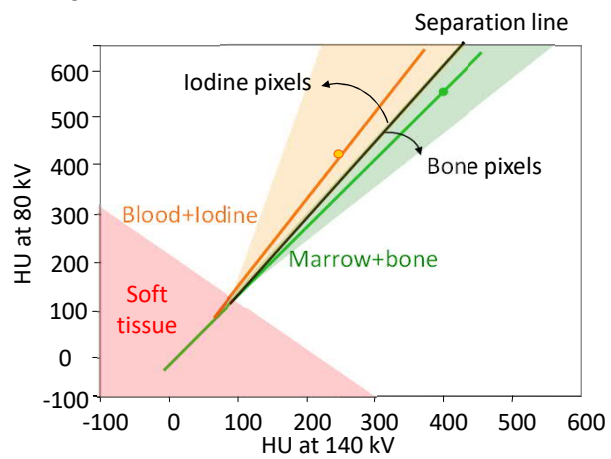
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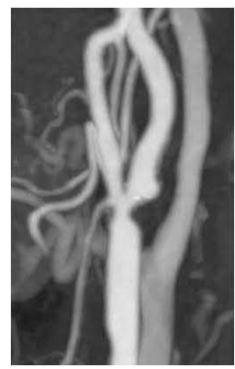
- **Iodine**: large CT-number difference in scans at two energies
- **Bone**: smaller CT-number difference in scans at two energies



CTA



DSA

Dual energy CT
Plaque removal

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Courtesy of PUMC, Beijing, China

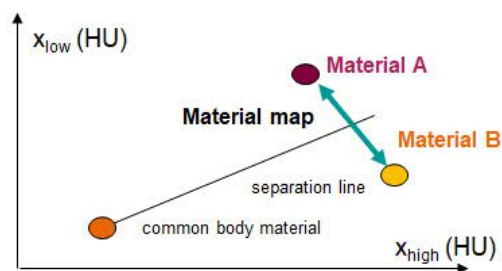
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Basic applications: differentiation of two materials A and B

Differentiation of two materials:

an image pixel is either **material A** or **material B**



Clinically relevant applications

- Automatic bone removal (iodine versus calcium)
- Characterization of kidney stones (uric acid versus calcium)
- Characterization of gout (uric acid)

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Basic applications: imaging of electron density and effective atomic number

Computation of ρ - and Z_{eff} - images

- The attenuation coefficient is

$$\mu = \rho \cdot \left[a_p \frac{Z_{\text{eff}}^4}{E^3} + a_c Z_{\text{eff}} \cdot f_{KN}(E) \right] = \frac{a_p}{E^3} \cdot \rho Z_{\text{eff}}^4 + \underbrace{a_c f_{KN}(E)}_{\text{Known parameters}} \cdot \rho Z_{\text{eff}}$$

- Assume measurements of μ in CT-images acquired at two different x-ray energies E_L and E_H (for simplification we assume monochromatic x-rays)

$$\mu_L = \frac{a_p}{E_L^3} \cdot \rho Z_{\text{eff}}^4 + a_c f_{KN}(E_L) \cdot \rho Z_{\text{eff}}$$

$$\mu_H = \frac{a_p}{E_H^3} \cdot \rho Z_{\text{eff}}^4 + a_c f_{KN}(E_H) \cdot \rho Z_{\text{eff}}$$

- We know a_p , a_c and $f_{KN}(E)$ and can solve these two equations for the two unknowns ρ and Z_{eff}

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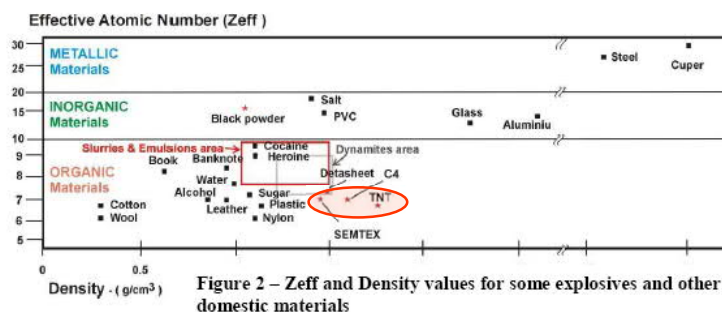
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Basic applications: imaging of electron density and effective atomic number

Computation of ρ - and Z_{eff} - images: detection of explosives with baggage scanners

- Explosives are characterized by high density $\rho \sim 1.5 \text{ g/cm}^3$
low atomic number $Z_{\text{eff}} \sim 7 - 8$



M. Iovea, High Accuracy X-Ray Dual-Energy Experiments ..for Explosives Detection Techniques in Luggage Control,
Page 14 DIR Symposium Digital Industrial Radiology and Computed Tomography, June 2007, Lyon, France

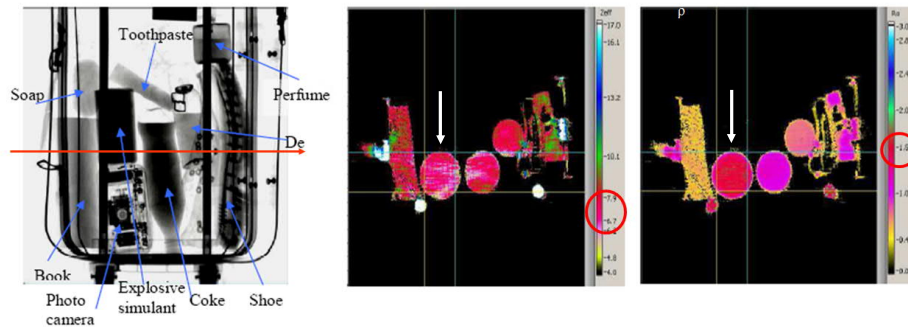
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Basic applications: computation of base material images

Two-material decomposition into two base materials A and B of unknown densities ρ_A and ρ_B

- The pure physics representations— photoelectric and Compton effects—do not provide a direct linkage to human anatomy, pathology, or physiology.
- Alternatively, $\left(\frac{\mu}{\rho}\right)$ of any material can be decomposed into a linear combination of $\left(\frac{\mu}{\rho}\right)_A$ and $\left(\frac{\mu}{\rho}\right)_B$ for two base materials A and B, which differ in their photoelectric and Compton characteristics.
- The attenuation coefficient is then

$$\mu(E) = \rho_A \left(\frac{\mu}{\rho}\right)_A(E) + \rho_B \left(\frac{\mu}{\rho}\right)_B(E)$$

- Suitable **base materials A and B** are **water and calcium**, or **water and iodine**

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Basic applications: computation of base material images

Two-material decomposition into two base materials A and B of unknown densities ρ_A and ρ_B

- Assume measurements of μ in CT-images acquired at two different x-ray energies E_L and E_H (for simplification we assume monochromatic x-rays)

$$\mu(E_L) = \rho_A \left(\frac{\mu}{\rho}\right)_A(E_L) + \rho_B \left(\frac{\mu}{\rho}\right)_B(E_L)$$

$$\mu(E_H) = \rho_A \left(\frac{\mu}{\rho}\right)_A(E_H) + \rho_B \left(\frac{\mu}{\rho}\right)_B(E_H)$$

- $\left(\frac{\mu}{\rho}\right)_A$ and $\left(\frac{\mu}{\rho}\right)_B$ are available from tabulated values for all chemical elements indexed by their atomic number.

The United States National Institute of Standards and Technology (NIST) hosts the constantly updated XCOM source at www.nist.gov

- These two equations can be solved for the two unknown densities ρ_A and ρ_B of the base materials A and B

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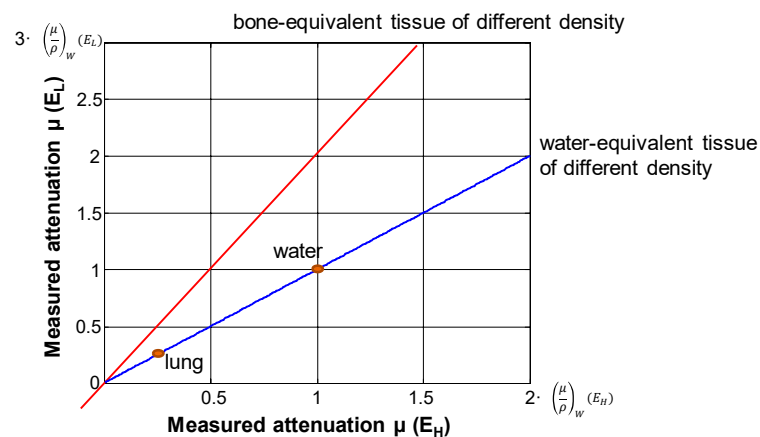
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Basic applications: computation of base material images

Two-material decomposition into two base materials A and B of unknown densities ρ_A and ρ_B

- Intuitive graphical solution for „image-based two material decomposition“, base materials water and bone



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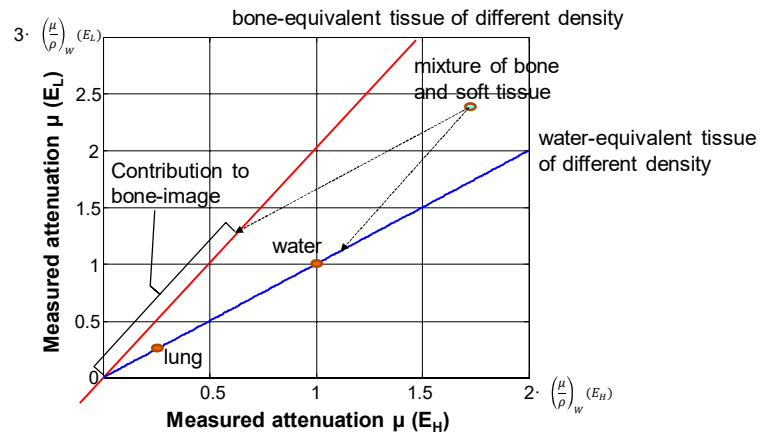
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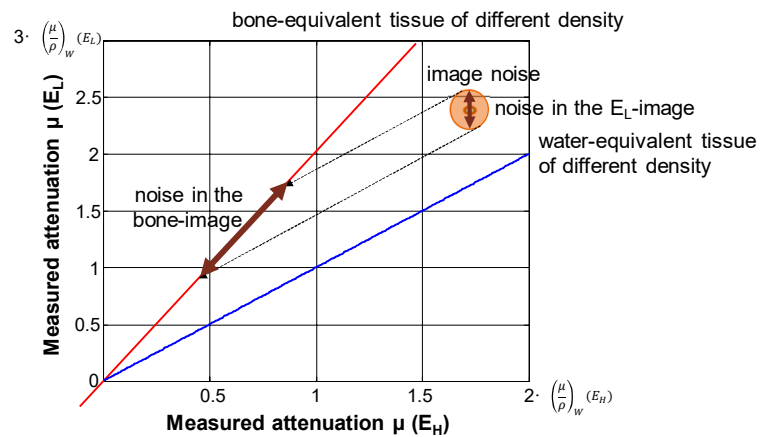
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Basic applications: computation of base material images

Two-material decomposition into two base materials A and B of unknown densities ρ_A and ρ_B

- Intuitive visualization of noise-amplification in the base material images



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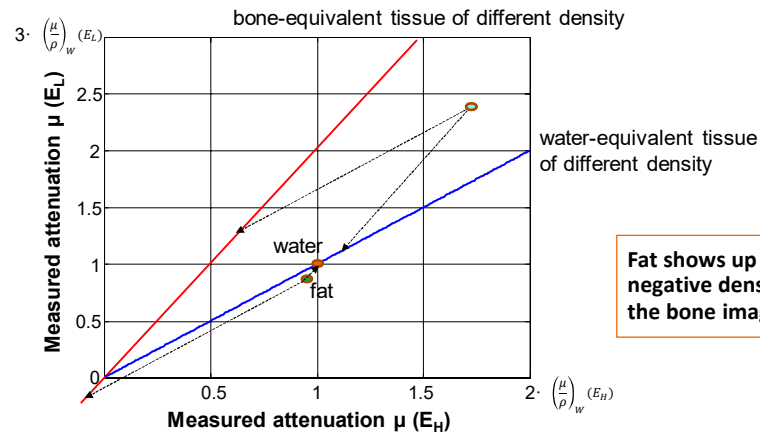
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Basic applications: computation of base material images

Two-material decomposition into two base materials A and B of unknown densities ρ_A and ρ_B

- What happens to materials outside the bone-water coordinate system?



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Basic applications: computation of base material images

Two-material decomposition into two base materials A and B of unknown densities ρ_A and ρ_B

- Material decomposition can be performed in the image space (as shown) and in the raw-data space
- Using base materials A and B, and assuming the object only consists of a mixture of the two base materials, the measured line integrals L can be expressed as a linear combination

$$L(E_L) = \rho_A d_A \cdot \left(\frac{\mu}{\rho}\right)_A(E_L) + \rho_B d_B \cdot \left(\frac{\mu}{\rho}\right)_B(E_L)$$

$$L(E_H) = \rho_A d_A \cdot \left(\frac{\mu}{\rho}\right)_A(E_H) + \rho_B d_B \cdot \left(\frac{\mu}{\rho}\right)_B(E_H)$$

d_A and d_B are the path lengths of the two base materials along the x-ray

- These two equations can be solved for $\rho_A d_A$ and $\rho_B d_B$ to generate material specific raw-data

$$L_A(E) = \rho_A d_A \cdot \left(\frac{\mu}{\rho}\right)_A(E) \quad \text{and} \quad L_B(E) = \rho_B d_B \cdot \left(\frac{\mu}{\rho}\right)_B(E) \quad \text{to reconstruct material-specific images}$$

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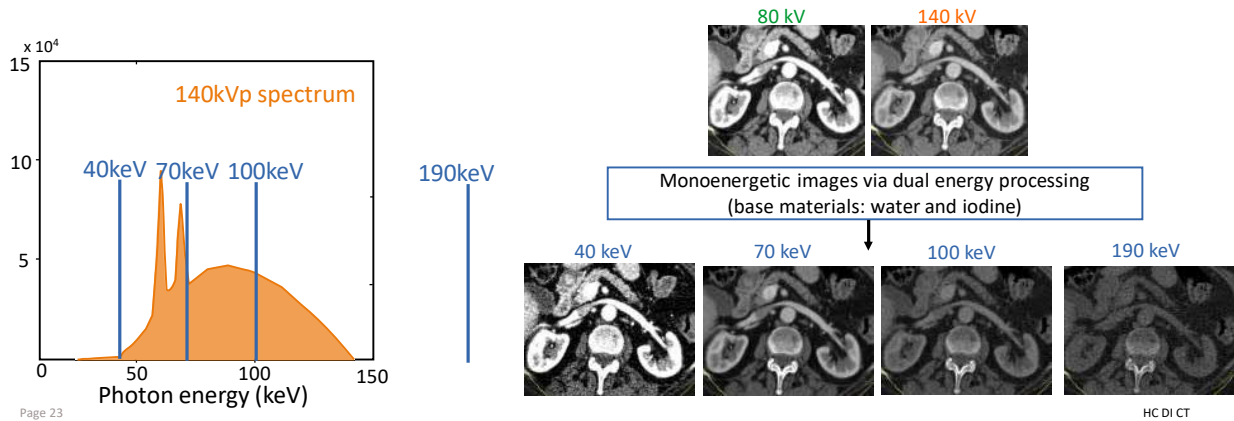
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Basic applications: computation of virtual monoenergetic images

Images at user-selectable “virtual” energies, e. g. in the range from 40 keV to 190 keV

- Derived from a **two-material decomposition into water and iodine images**
- Iodine and water images are **scaled to represent the attenuation of iodine and water at the selected keV** and added
- keV-dependency of other materials (e. g. Ca) may not be correct



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Computation of three-material images with a boundary condition

Three-material decomposition into three materials with known densities ρ , but unknown volume fractions

- Three materials can only be differentiated by assuming an additional boundary condition, e. g. volume conservation, to solve two equations with three unknowns
- With f_A , f_B , and f_C as the volume fractions of materials A, B, and C, volume conservation implies $f_A + f_B + f_C = 1$
- The attenuation coefficient of the mixture is then

$$\mu(E) = f_A \rho_A \left(\frac{\mu}{\rho} \right)_A (E) + f_B \rho_B \left(\frac{\mu}{\rho} \right)_B (E) + (1 - f_A - f_B) \rho_C \left(\frac{\mu}{\rho} \right)_C (E)$$

- Different from two-material composition into two base materials with unknown densities, we now assume **base materials with known densities and can solve for the volume fractions f_A , f_B , and $f_C = 1 - f_A - f_B$**

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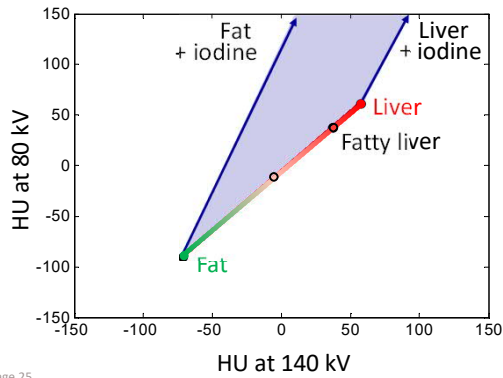
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Computation of three-material images with a boundary condition

Special, but relevant case: two materials with known density and unknown volume fractions f_A and $f_B = 1 - f_A$ plus one material of unknown density ρ with very high absorption coefficient and negligible volume fraction (e. g. iodine)

- Differentiation and quantification of **fat**, **liver** and **iodine**



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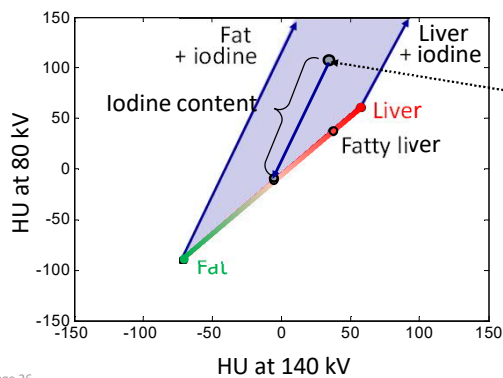
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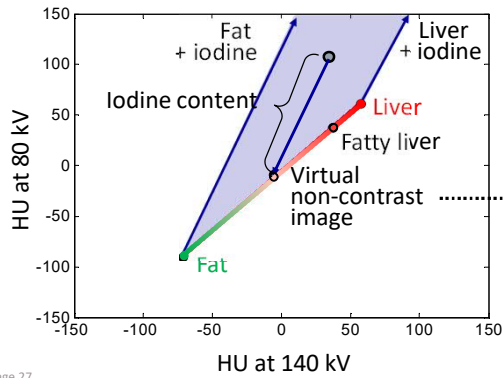
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Computation of three-material images with a boundary condition

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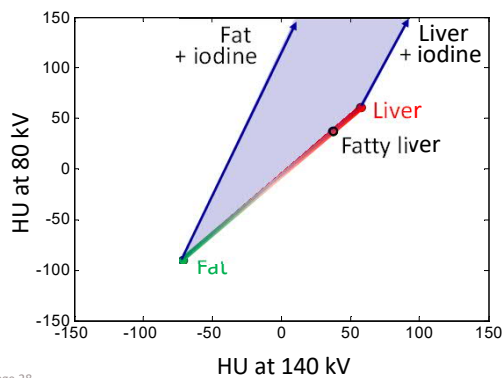
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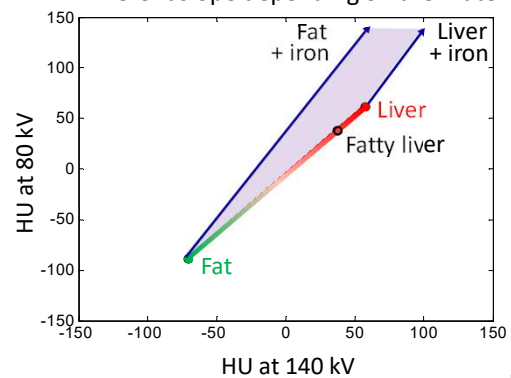
Computation of three-material images with a boundary condition

The modified three-material decomposition can be extended to other materials as well:

- Differentiation and quantification of **fat**, **liver** and **iodine-iron**



Different slope depending on the material!



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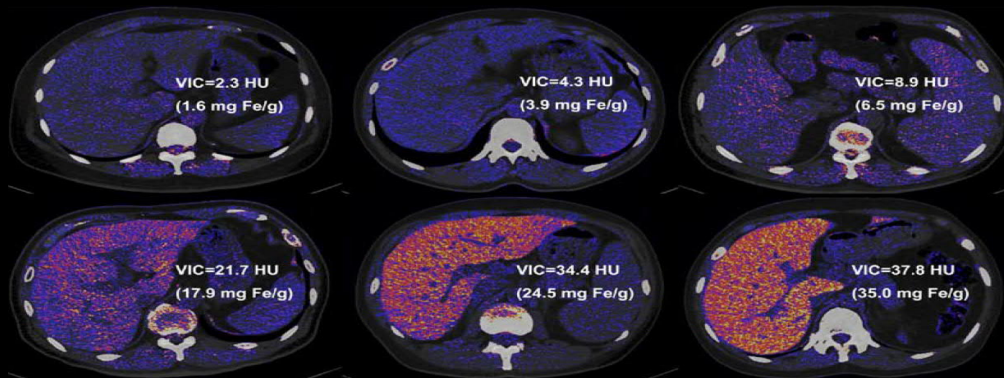
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Computation of three-material images with a boundary condition

Assessment of iron concentration in the liver; quantification in mg/cm^3 by DE DSCT

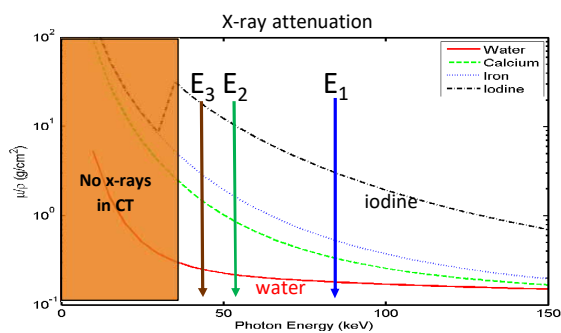
- Iron map and virtual non-iron image by decomposition into iron, fat and liver



Courtesy of Xian Fu Luo, Ruijin Hospital, China

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Is there a need for data acquisition with more than energies?

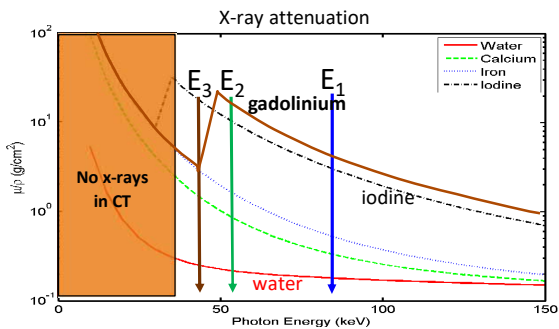


N energies = N materials?

- No! Only two basic interaction mechanisms in the relevant x-ray energy range of CT (40 – 150 keV)!

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Is there a need for data acquisition with more than energies?



N energies = N materials?

- No! Only two basic interaction mechanisms in the relevant x-ray energy range of CT (40 – 150 keV)!

How to differentiate 3 (or more) materials?

- K-edge needed!**
- Measurement at 3 (or more) energies
→ Base materials e. g. water and iodine + **material with a k-edge** in the relevant CT energy range, e. g. gadolinium (K-edge 50.2 keV)

$$\mu(x, E) = a_{photo}(x) \frac{1}{E^3} + a_{compton}(x) f_{KN}(E) + a_{K-edge}(x) f_{K-edge}(E)$$

Photon-counting detectors can provide CT data at more than 2 energies!

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Is there a need for data acquisition with more than energies?

Essential elements on the periodic table [edit]

Periodic table highlighting dietary elements

H																	He
Li	Be											B	C	N	O	F	Ne
Na	Mg											Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	* Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	** Lr	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Uut	Fl	Uup	Lv	Uus	Uuo

* La Ce Pr Nd Pm Sm Eu Gd Tb Dy Ho Er Tm Yb

** Ac Th Pa U Np Pu Am Cm Bk Cf Es Fm Md No

The four organic basic elements Quantity elements Essential trace elements
Possible structural or functional role in mammals

Increasing k-edge energy

- Elements naturally found in the human body have k-edges below the relevant x-ray energies in CT (40 – 150 keV)
- This includes iodine in contrast enhanced scans (k-edge 33.2 keV)
→ **Only two energy levels are needed for spectral CT in clinical routine**

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Is there a need for data acquisition with more than energies?

Essential elements on the periodic table [\[edit \]](#)

Periodic table highlighting dietary elements

H																	He
Li	Be											B	C	N	O	F	Ne
Na	Mg											Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	* Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	** Lr	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Uut	Fl	Uup	Lv	Uus	Uuo
		* La Ce Pr Nd Pm Sm Eu Gd Tb Dy Ho Er Tm Yb															
		** Ac Th Pa U Np Pu Am Cm Bk Cf Es Fm Md No															
				The four organic basic elements				Quantity elements				Essential trace elements					
Possible structural or functional role in mammals																	

Increasing k-edge energy ↓

- Relevant elements with higher k-edge: Gd 50.2 keV, Au 80.7 keV
- Only used in experimental applications, e. g. Au-nanoparticles
- Measurements at three or more energies may identify additional element with a k-edge → photon counting CT

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https://en.wikipedia.org/wiki/Composition_of_the_human_body

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