AAPM 2021

Conceptual Overview of Contemporary Photon Dose Computation Algorithms

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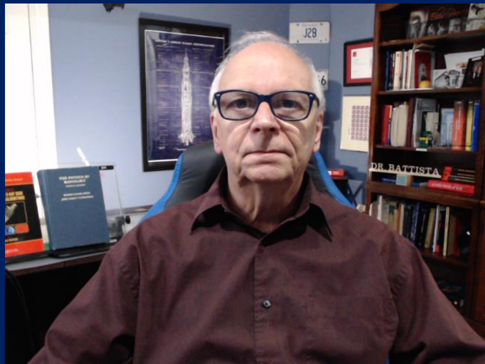
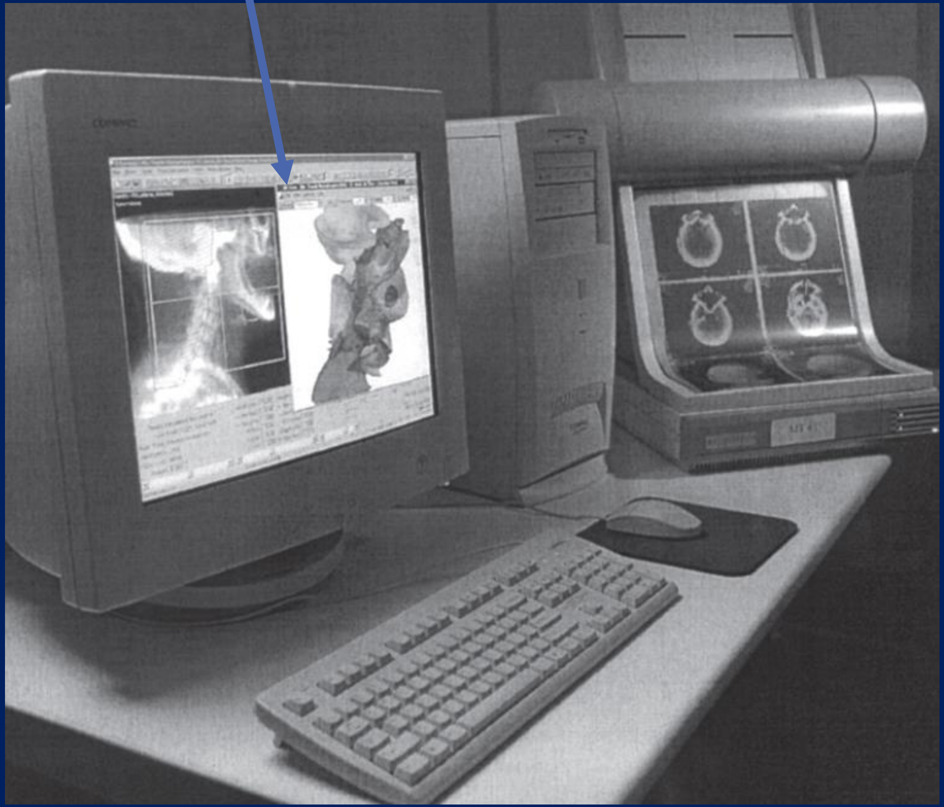
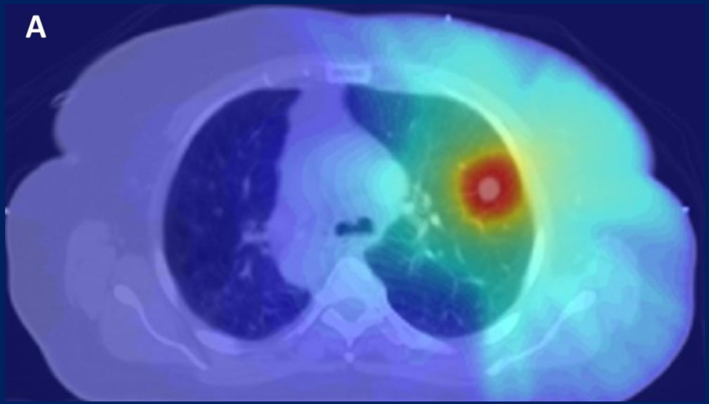


Dose Computation Epochs

BC – **Before Cunningham**

AD – **Anno Dosimetria**





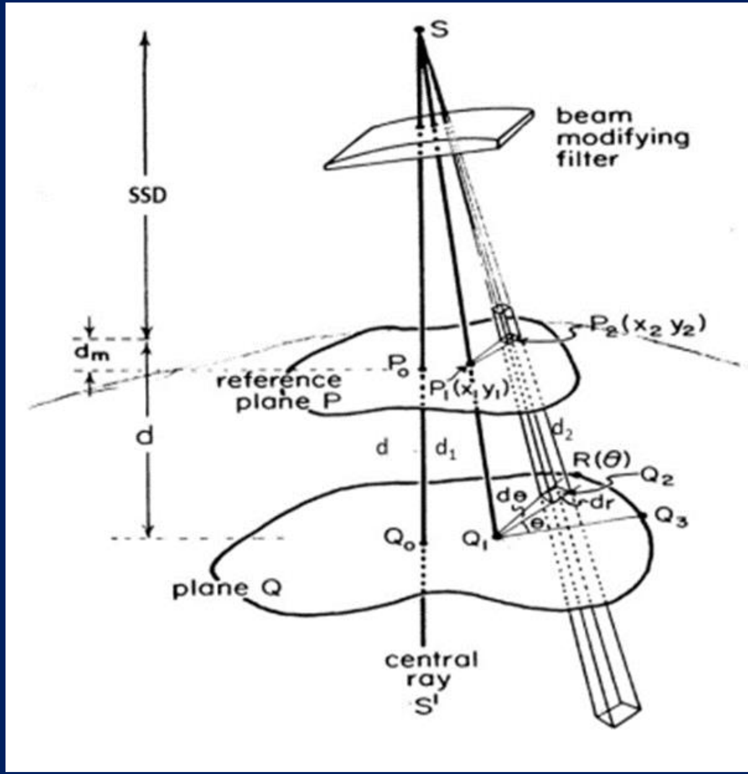
Topics *du Jour*

- Historical Evolution
- Monte Carlo Simulation
- Convolution–Superposition
- Boltzmann Transport
  - Phase Space
  - Divergence Theorem
- Summary

Disequilibrium Trouble

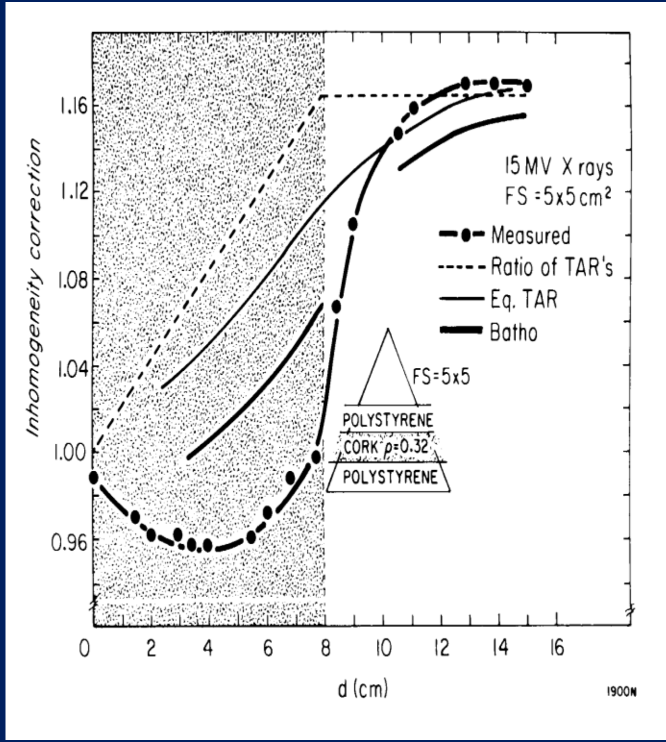
First Giant Leap: Splitting of Primary and Secondary Dose

Second Giant Leap: CT Tissue Densitometry



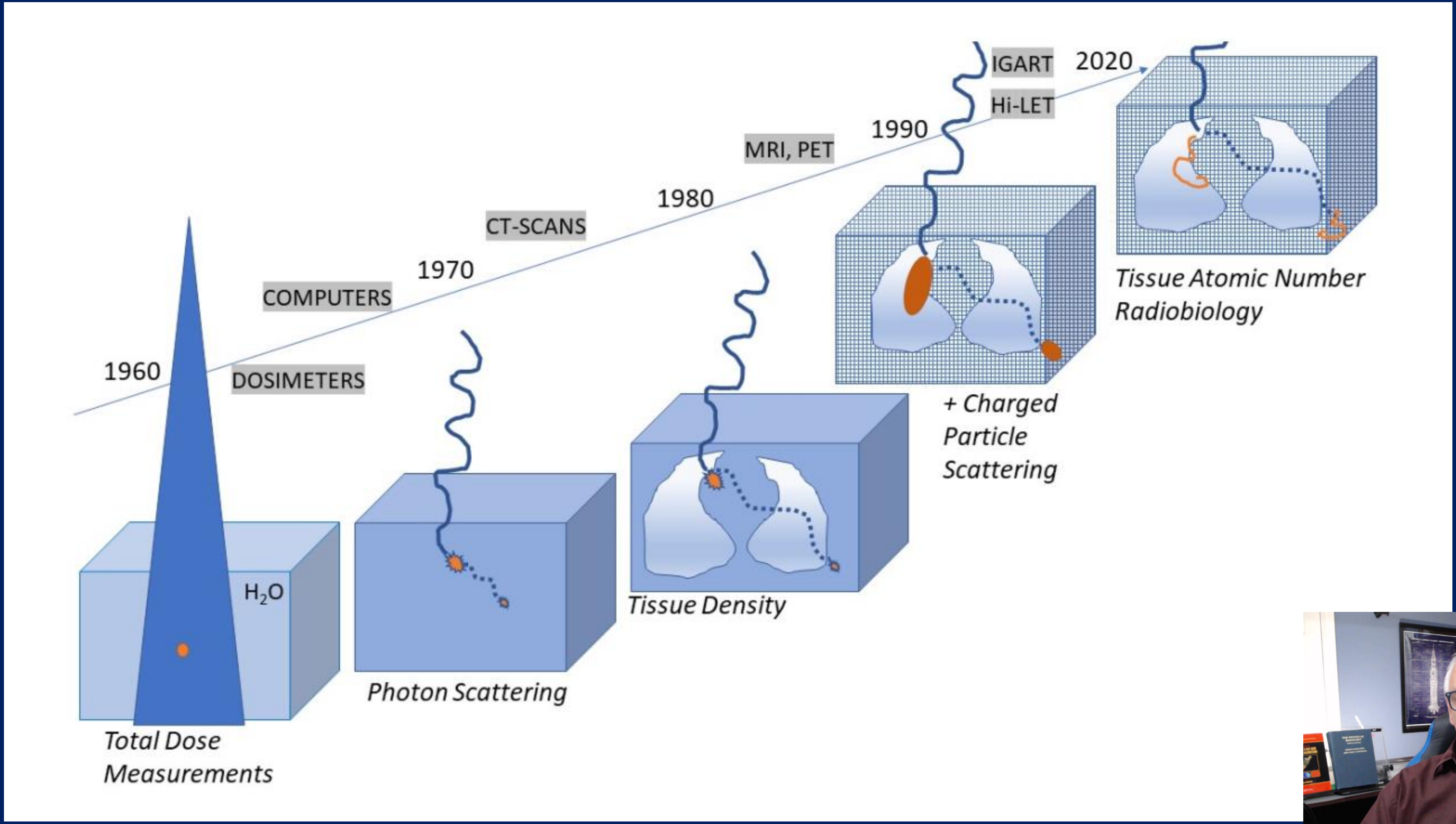
1972 Scatter-Air Ratios

1971 Brain CT Scanner



1985 Mackie et al.

Algorithm Evolution



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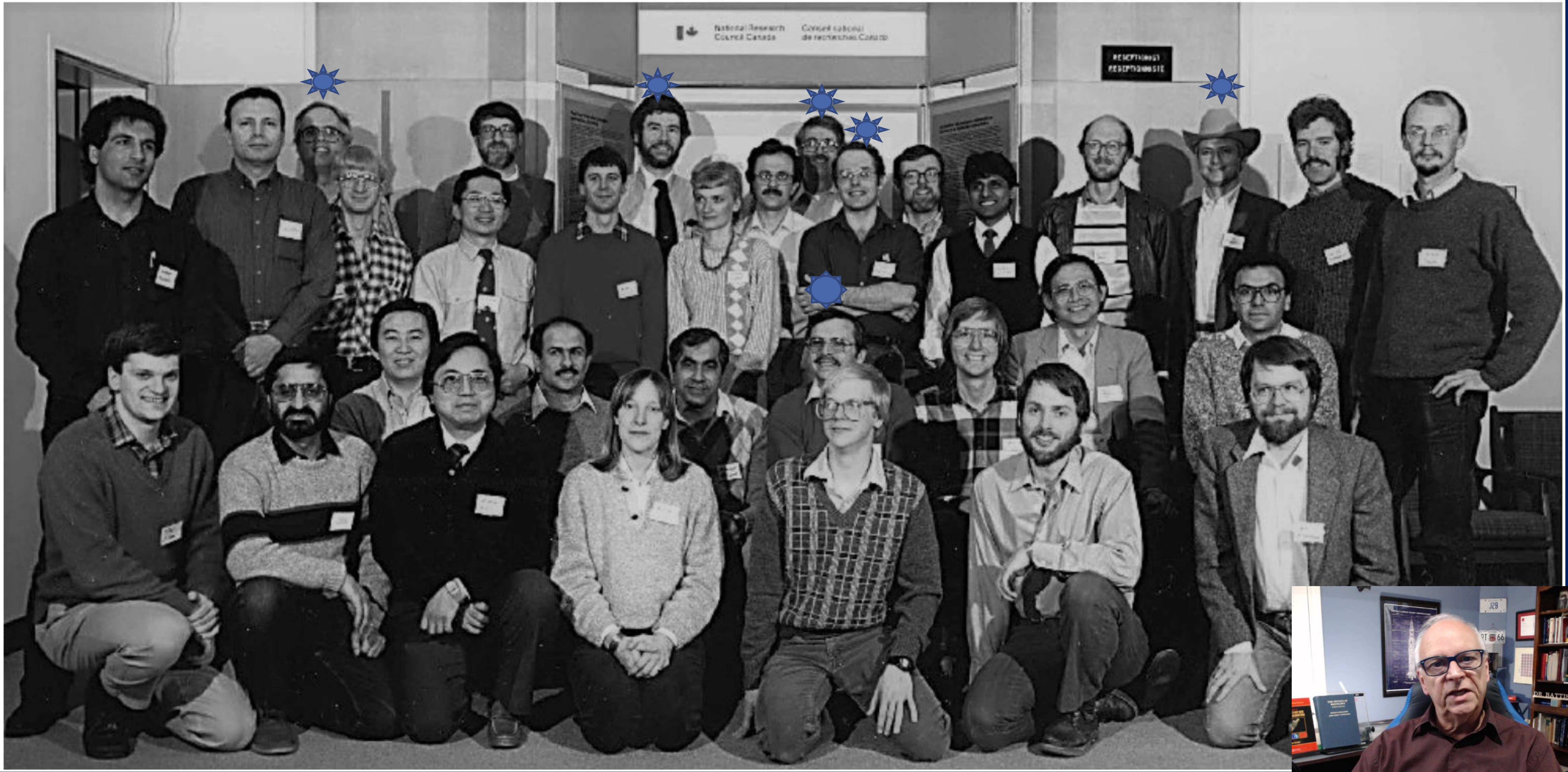
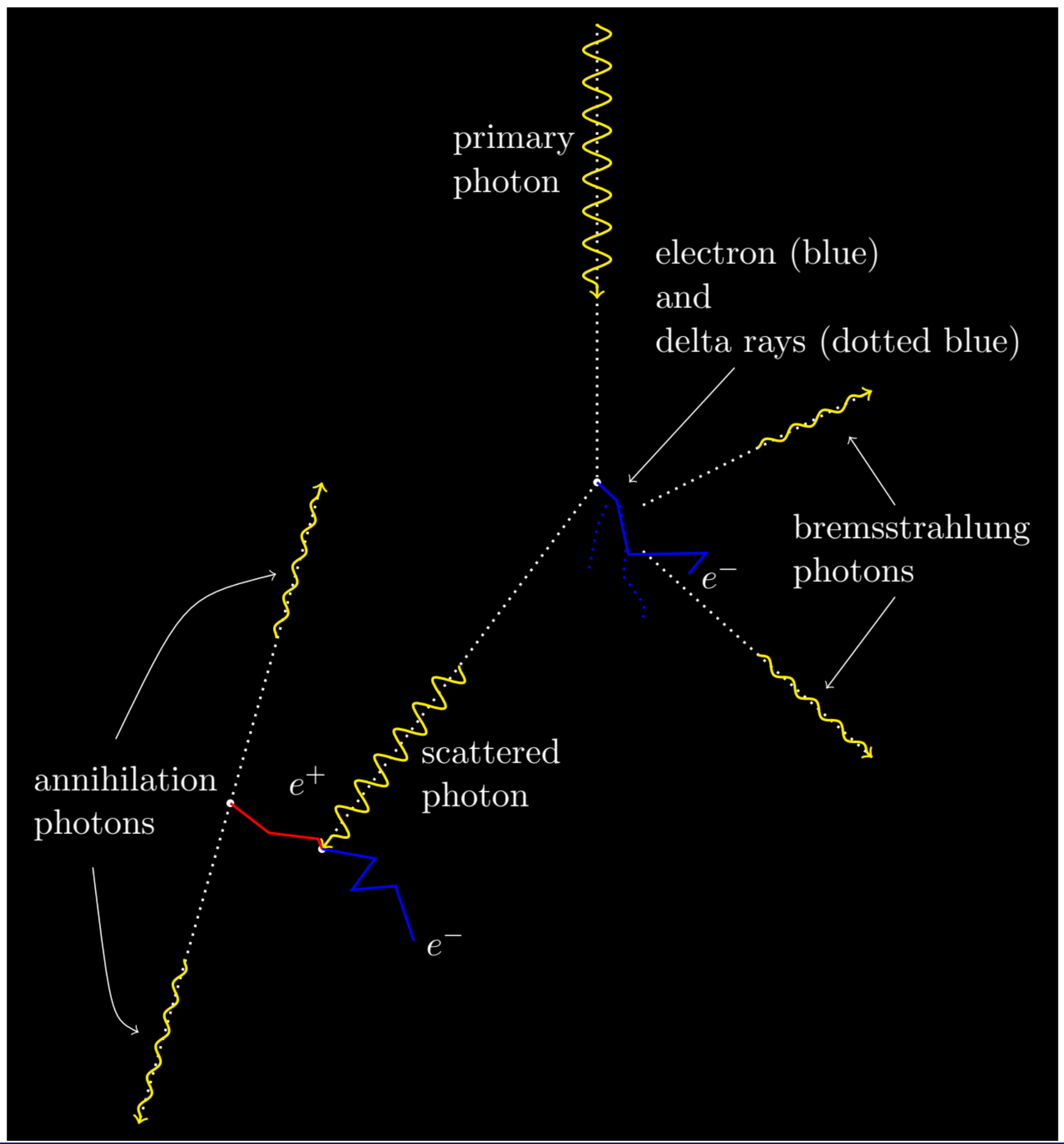
First EGS (Electron Gamma Shower) Course (1986)



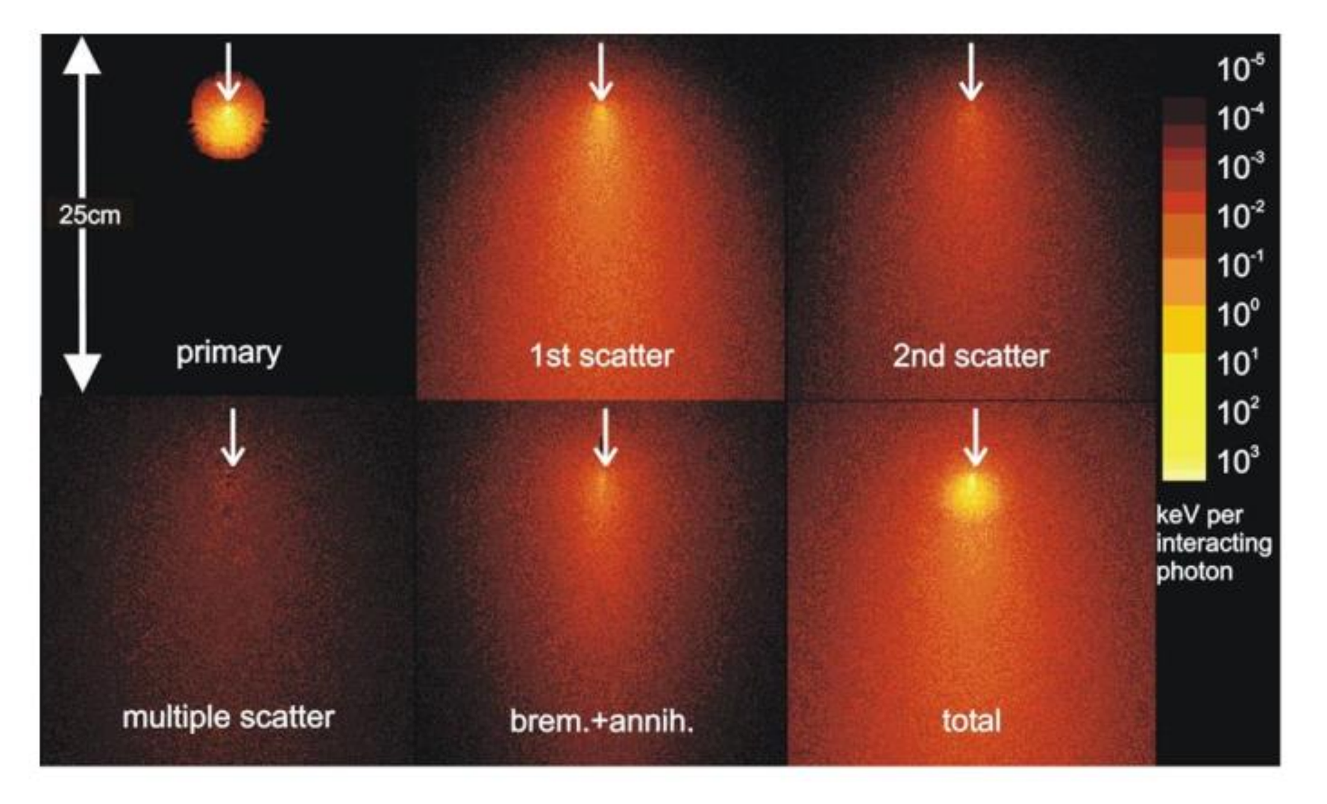
Biased Random Walks of Individual Particles



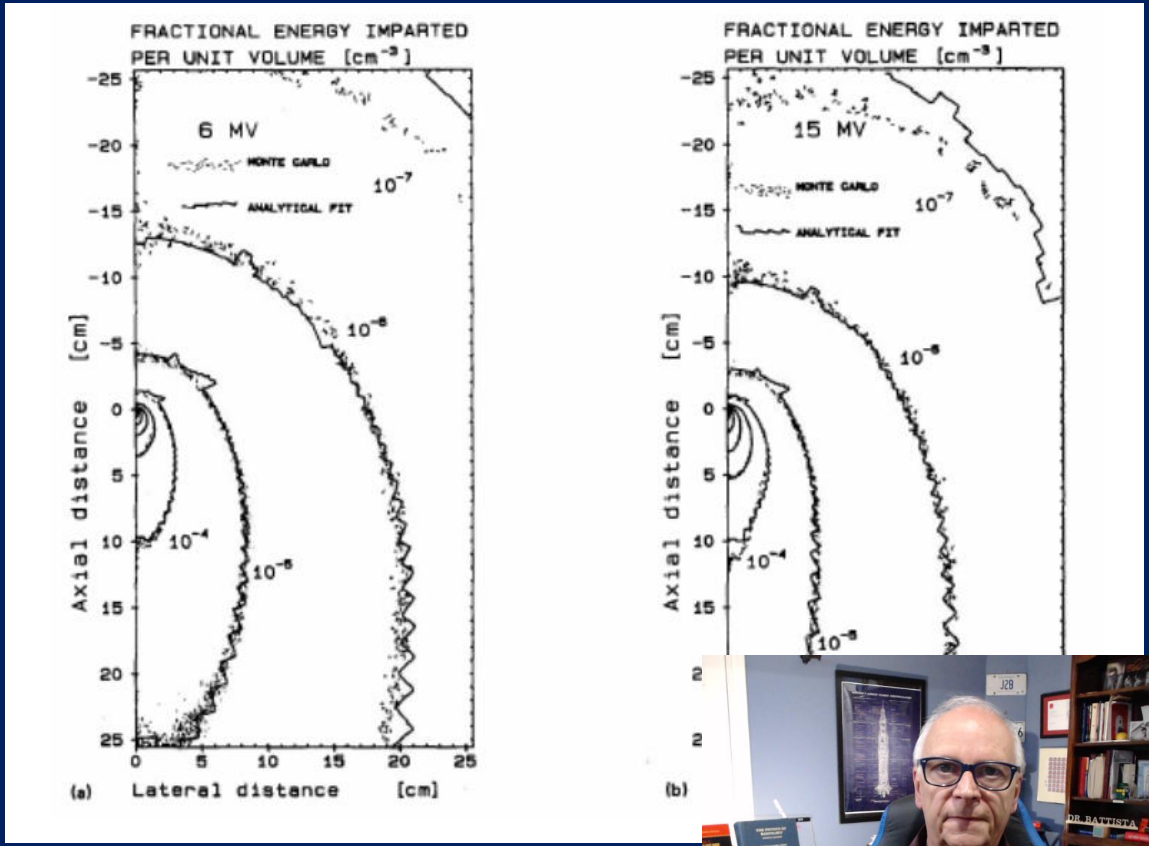
- Geometry of beams and patient (CT)
- Media (Z,ρ) and interaction x-sections
- Trusted Random Numbers
- Many events (> $10^{6 \text{ to } 9}$ "histories")
  - Fluences, Dose can be scored
- Particle Tagging (family tree)
- Average ± Statistical Variance (Noise)

Energy Deposition Point Kernels

Mackie 1988



(Ahnesjö 1989)



Dose Accuracy
- Fewer assumptions
- Less statistical noise
- More particle histories

Big Field Size

Large Patient

High Spatial Resolution

Slow Computer Hardware

Long Computation Times

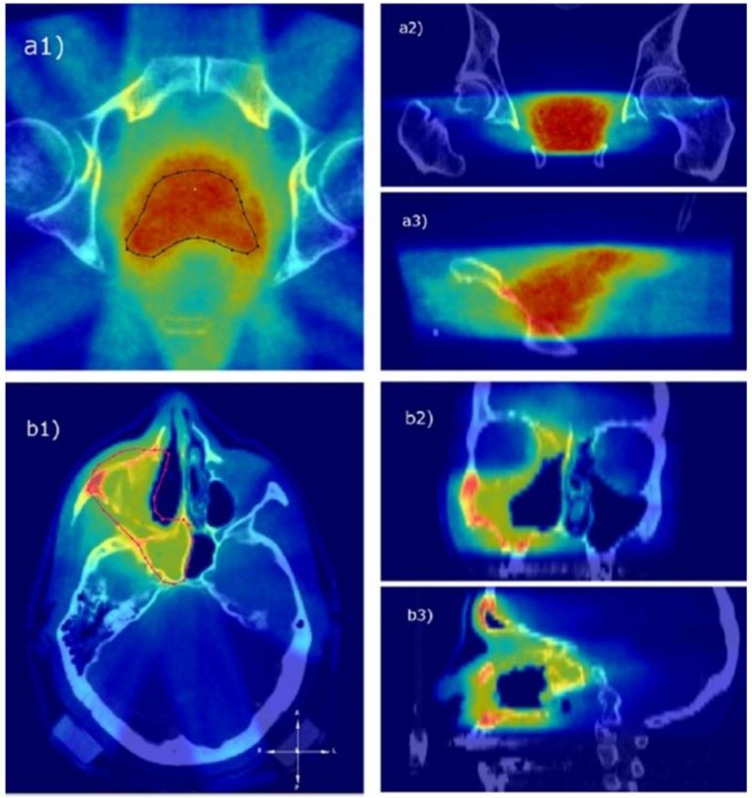


Figure 7.16: IMRT dose distributions for a prostate case (top panels) and head and neck case (bottom panels). These were calculated in about 10 seconds for 1% precision using PhiMC Monte Carlo code on a multi-core CPU (XeonV3). Reproduced under a creative commons license (https://creativecommons.org/licenses/by/3.0/) (Ziegenhein et al. 2015).

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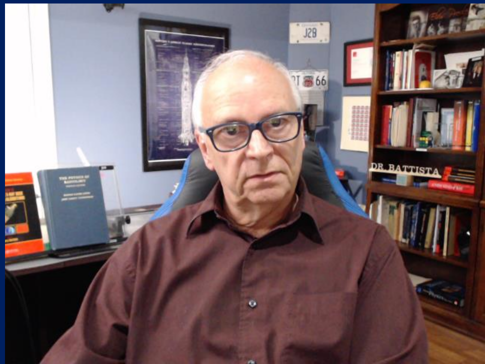
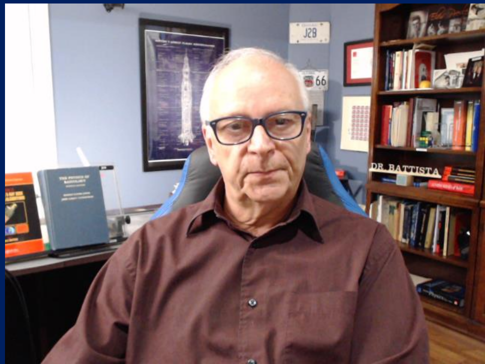



Mackie The Mover
WORLD WIDE MOVING
'Miss Conduct'
1950
MACKIE
The
MOVER
50 KW


An Elocution on the Evolution of the Convolution Revolution



Impulse = primary photon interactions
Response = spray of secondary energy

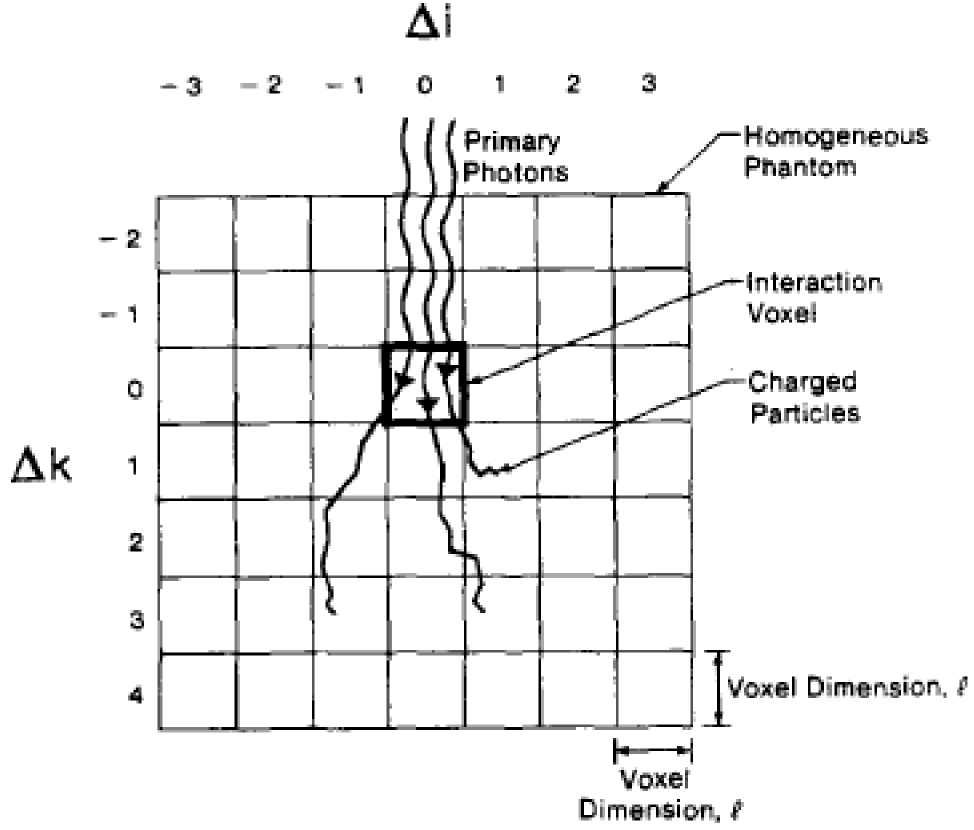
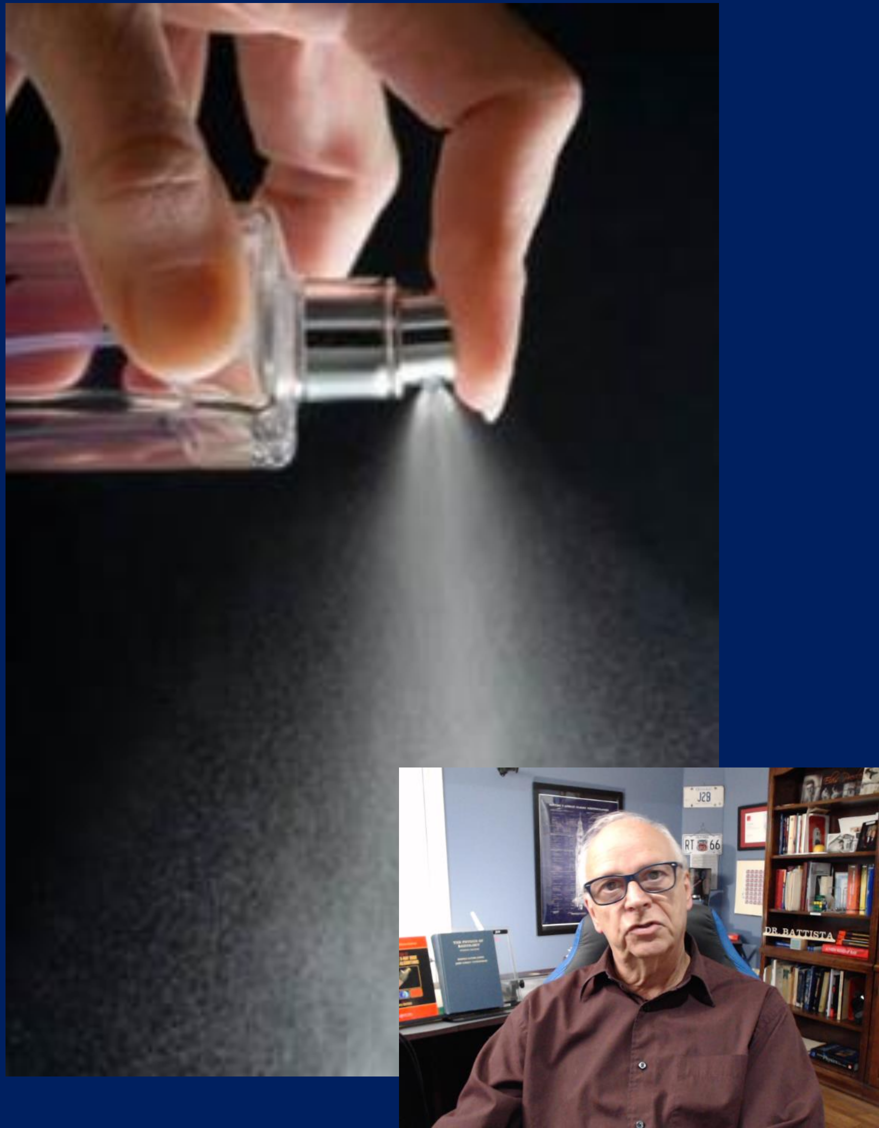


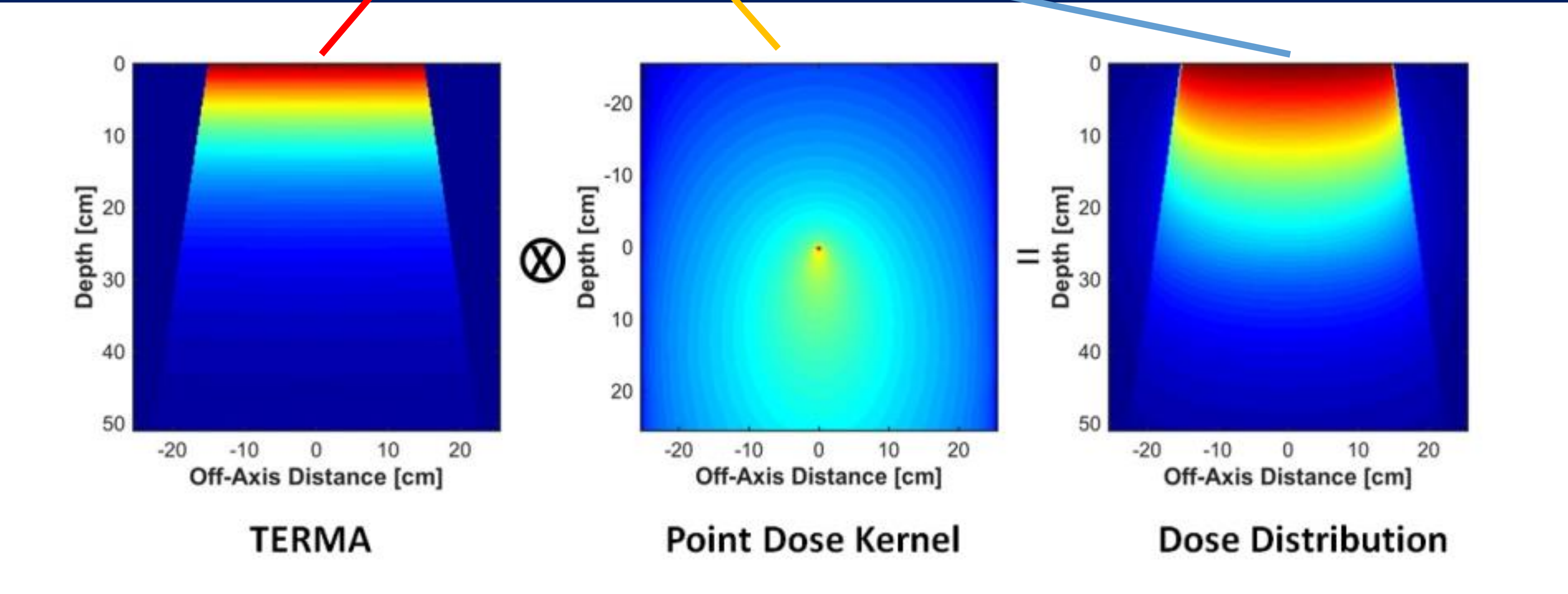
FIG. 1. Schematic representation of the generation of a primary dose spread array.

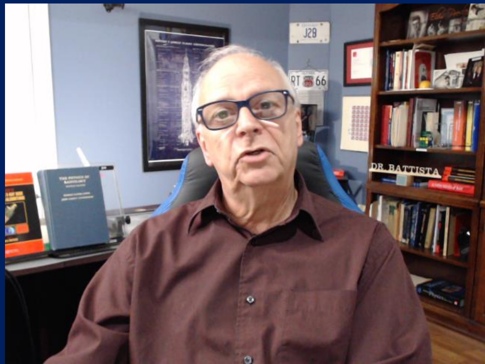


Convolution in 2D

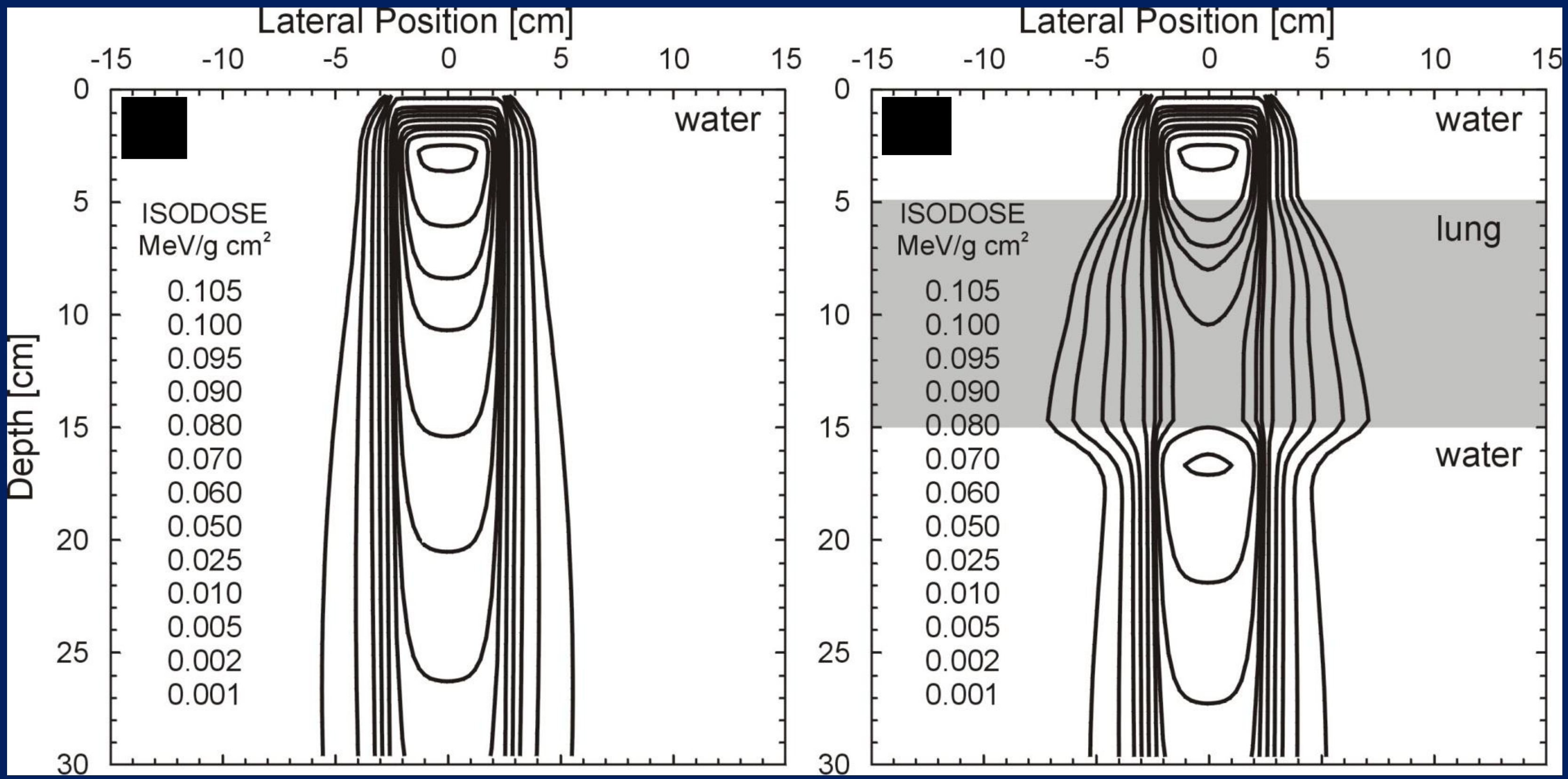
$$D(\mathbf{r}) = \iiint T(\mathbf{s})k(\mathbf{r}-\mathbf{s})\mathrm{d}^3 s$$

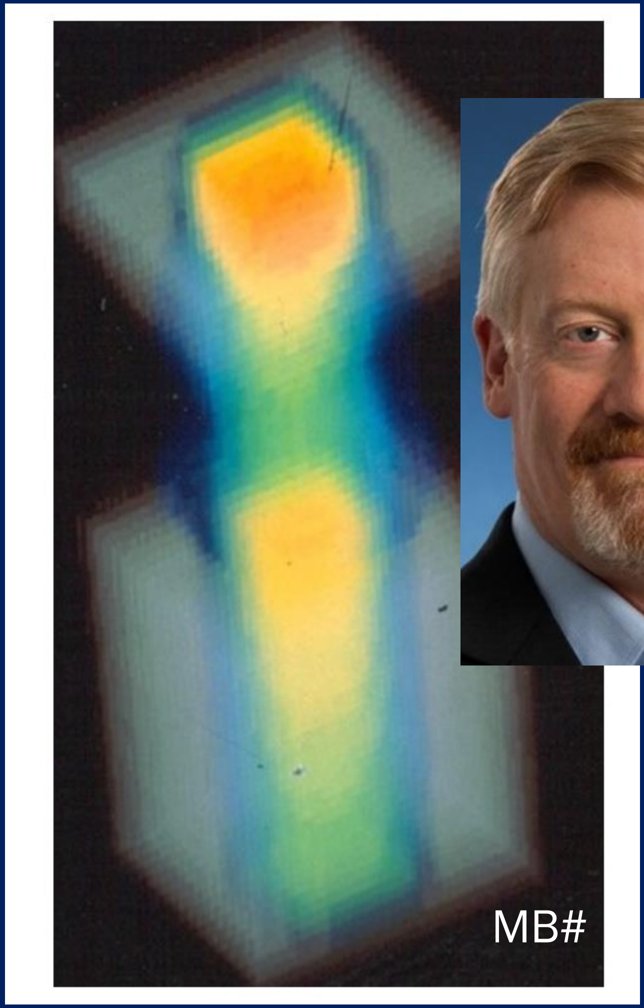
Monoenergetic Beam
All-Water Medium

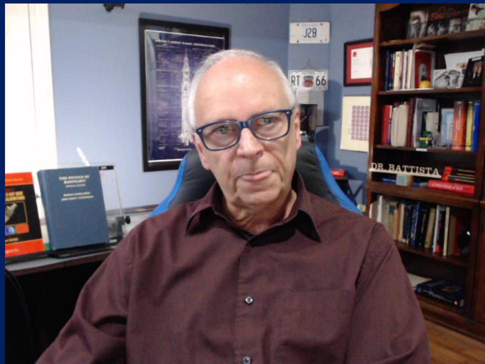




3D Superposition in Lung
Spatially-Variable Kernels



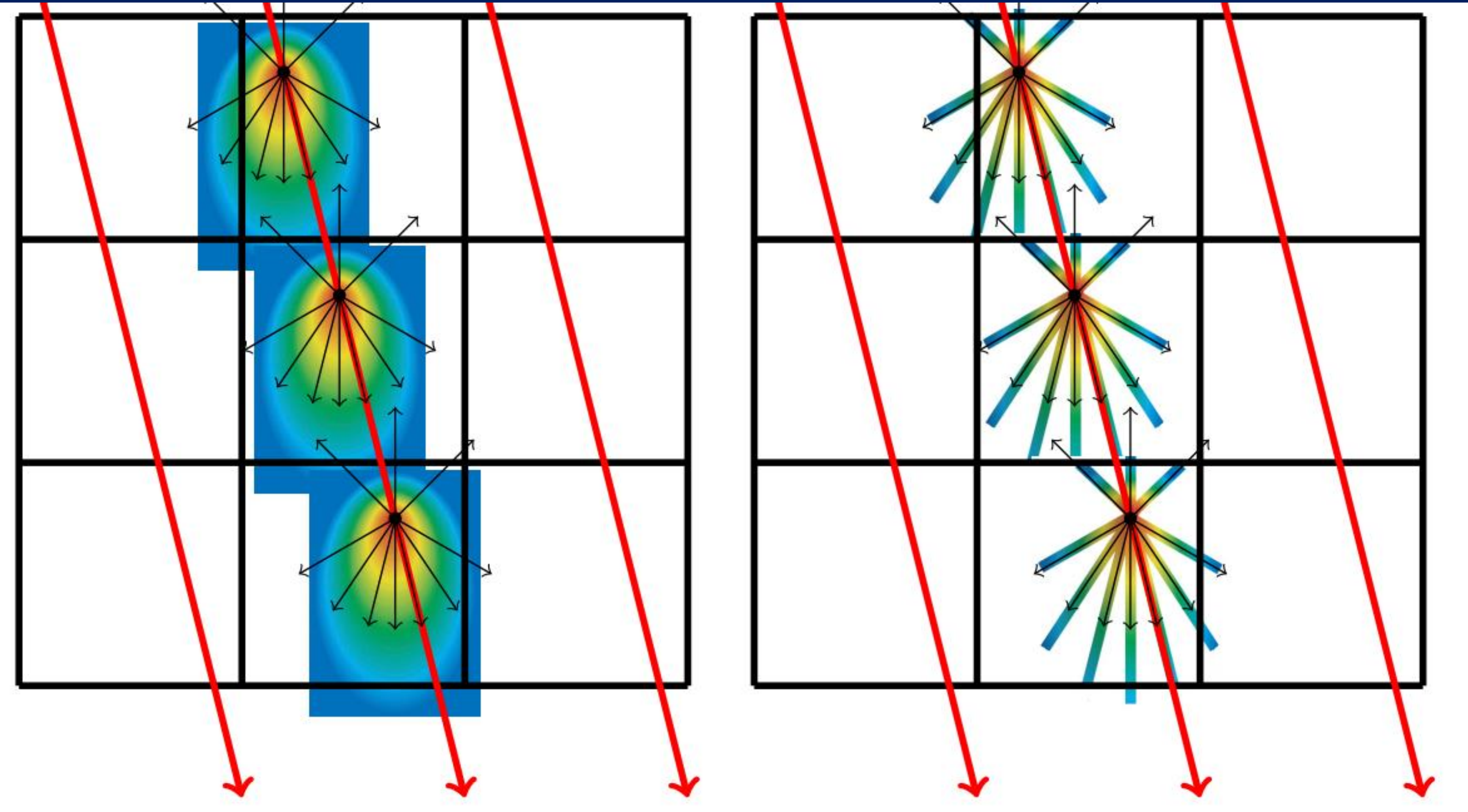


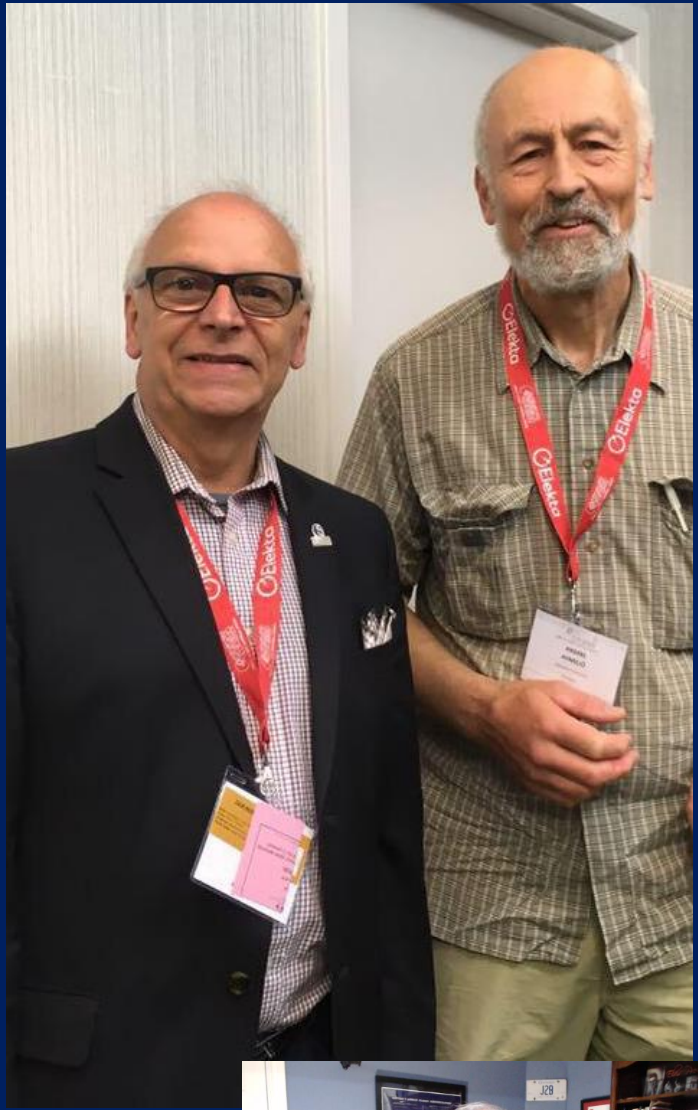


Collapsed Cone Convolution (CCC)

Smart Assumption and Recursive Relations

[M cones x N^3] vs N^7 Operations





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First Boltzmann Course (1887)

Ludwig Boltzmann (centre) and his colleagues in 1887. Boltzmann was an Austrian physicist who established the foundation of statistical mechanics with emphasis on the kinetic theory of gas molecules. In Chapter 6, the same principles are applied to the transport of megavoltage x-rays and secondary electrons. Photo courtesy of Universitat Graz.

Introduction to Phase Space

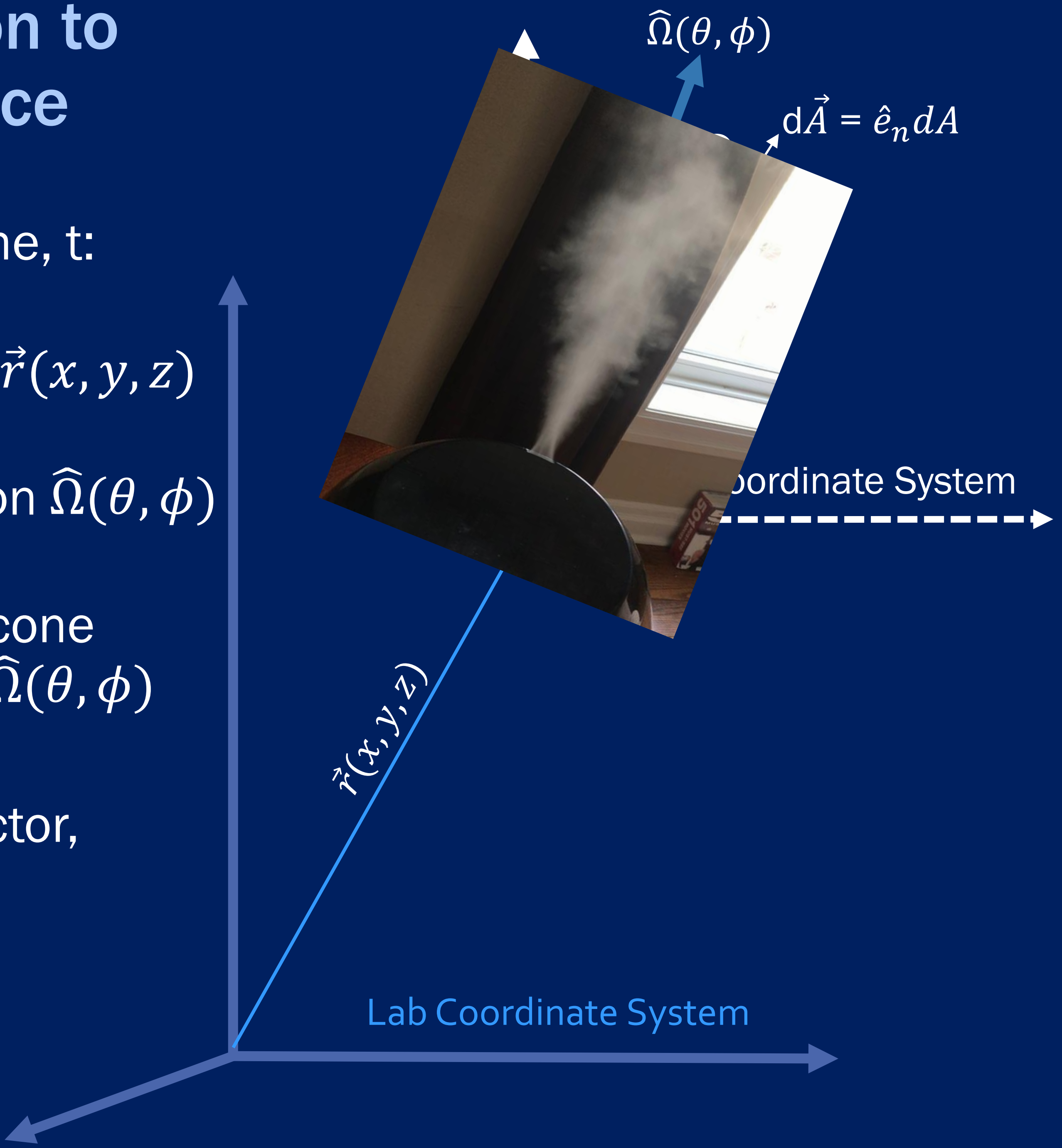
At observation time, t:

Particle Location $\vec{r}(x, y, z)$

Trajectory Direction $\widehat{\Omega}(\theta, \phi)$

dΩ is solid angle cone wrapped around $\widehat{\Omega}(\theta, \phi)$

d$\vec{A}$ is the area vector, normal to surface



Radiation particles are triaged into "phase space" bins with 6D tags:

$\vec{r}(x, y, z), \widehat{\Omega}(\theta, \phi)$, E

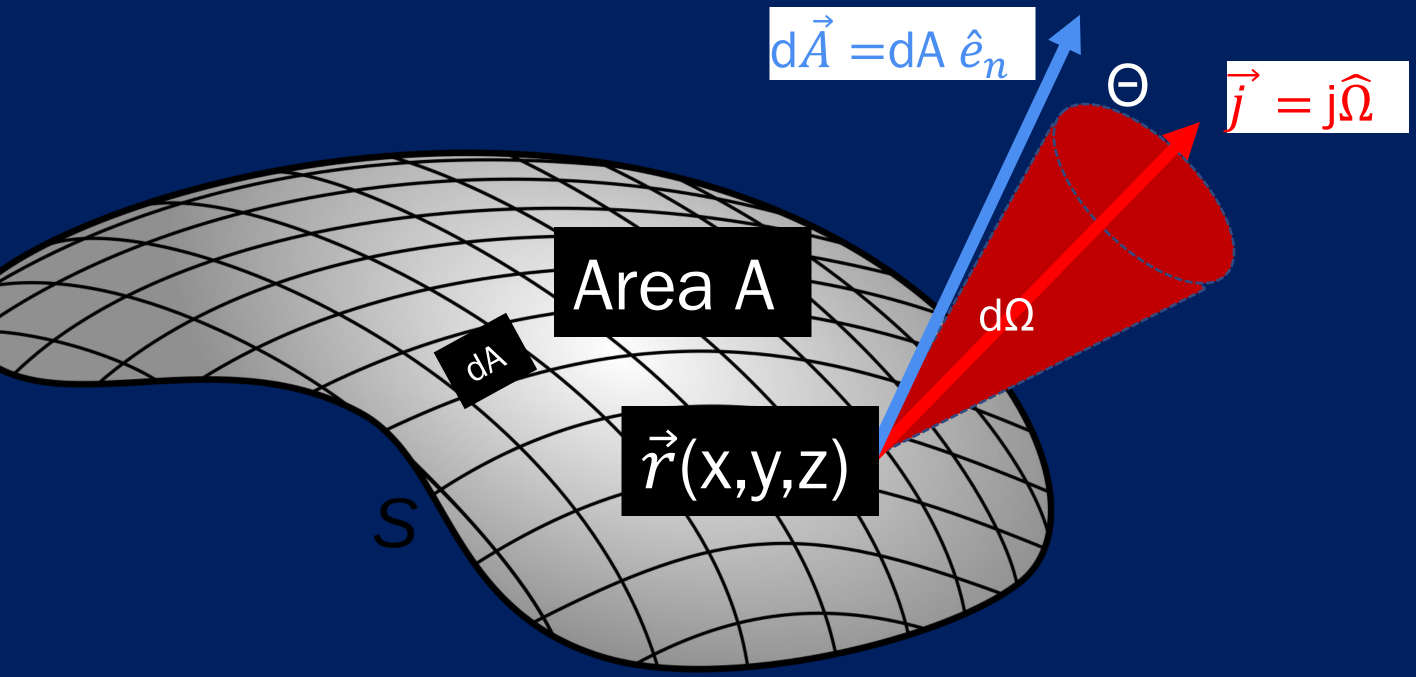
"Ensemble" of cohort particles emerging from dV at $\vec{r}$ with similar phase tags

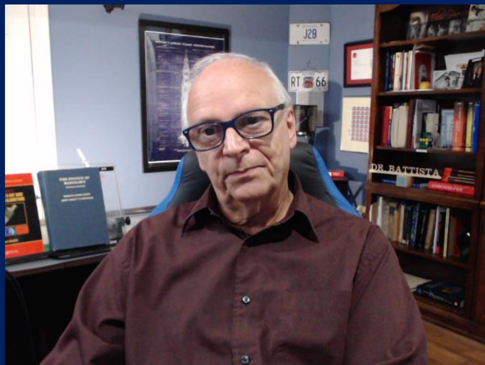
Net Flow of Radiation Particles

$\vec{j}\,(\vec{r},\widehat{\Omega},\mathrm{E},t)$ - current density distribution at $\vec{r}$ as a function of direction, and energy, at time, t.

The Dot Product of $\vec{j}\bullet \mathbf{d}\vec{\boldsymbol{A}}$ = j dA cos(Θ) discriminates direction of flow across surface 'patches' dA. Net Outflow is:

$$\oiint \vec{j}(\vec{r},\widehat{\Omega},\mathrm{E},t)\cdot d\vec{A} = \iiint \nabla\cdot\vec{j}(\vec{r},\widehat{\Omega},\mathrm{E},t)\,dV$$





Boltzmann Transport Equation

Steady State (Time–Independent); no internal sources

Net Streaming

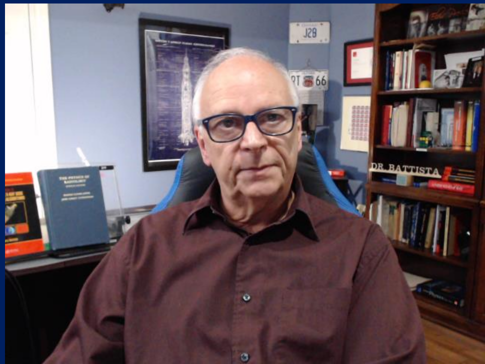
$$\hat{\Omega}\cdot\vec{\nabla}\varphi(\vec{r},\hat{\Omega},E) = \int \mathrm{d}E' \int \mathrm{d}\hat{\Omega}'\,\Sigma_s(\vec{r},\hat{\Omega}'\to\hat{\Omega},E'\to E)\,\varphi(\vec{r},\hat{\Omega},E') - \Sigma_t(\vec{r},E)\,\varphi(\vec{r},\hat{\Omega},E)$$

Gains
-
Losses

NET
Gain

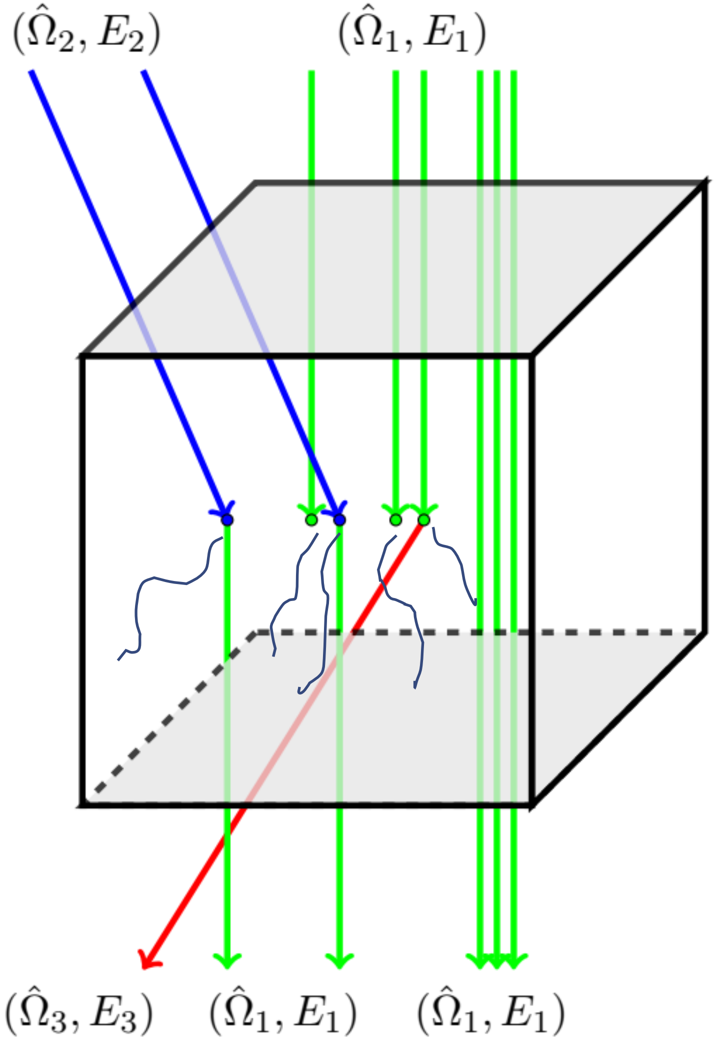
Fluence density distribution (location, direction, E)
(integrated over an exposure time)

GOAL:
Solve a set of equations for Fluence Distributions (γ ➡ e).
Then Convert to Dose



Particle Bin Accounting

Each ray represents 10 photons



Phase Space Bin	Incoming	Outgoing	Bin Change $(\hat{\Omega}_1, E_1)$
$(\hat{\Omega}_2, E_2)$	20	0	+20 In Scatter
$(\hat{\Omega}_3, E_3)$	0	10	−10 Out Scatter
Absorbed	-	-	−20
$(\hat{\Omega}_1, E_1)$	60	50	**-10**

Divergence (Ω1,E1)

ε1 = 50 - 60 = -10

Net Gain Inside

ε1 = 20 - 30 = -10

Dose Contributions

$$\epsilon = (R_{in} - R_{out})_u + (R_{in} - R_{out})_c + \Sigma Q$$

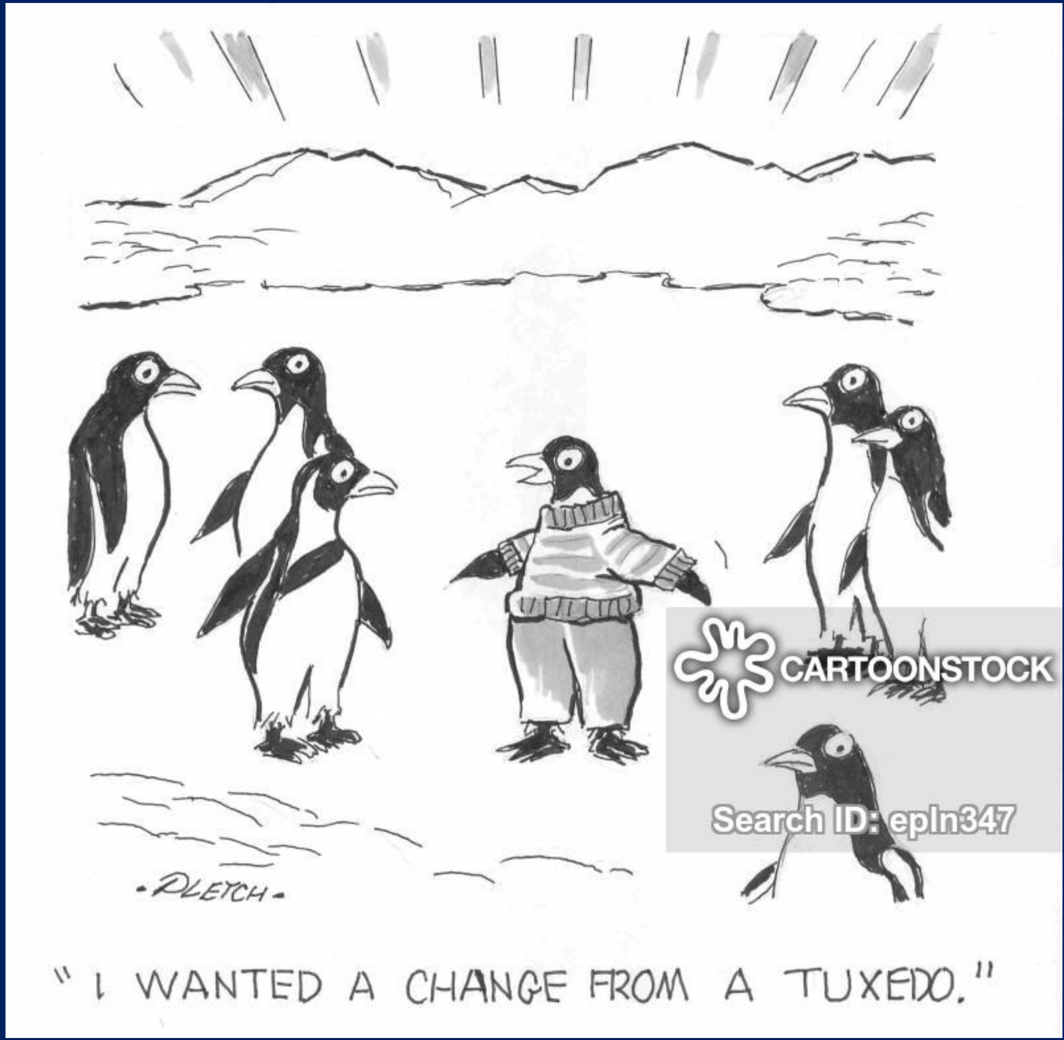
Dose = (20E2 + 60E1) – (50 E1 + 10E3)
= 10E1 + 20E2 - 10E3

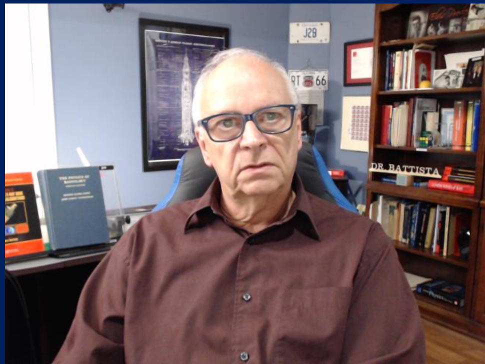
OR Electron Energy Deposits

= 20(E1) + 10 (E1-E3) + 20 (E2-E1)
= 10E1 + 20E2 – 10E3

An Overview of Deterministic Radiation Transport Approaches

Todd Wareing





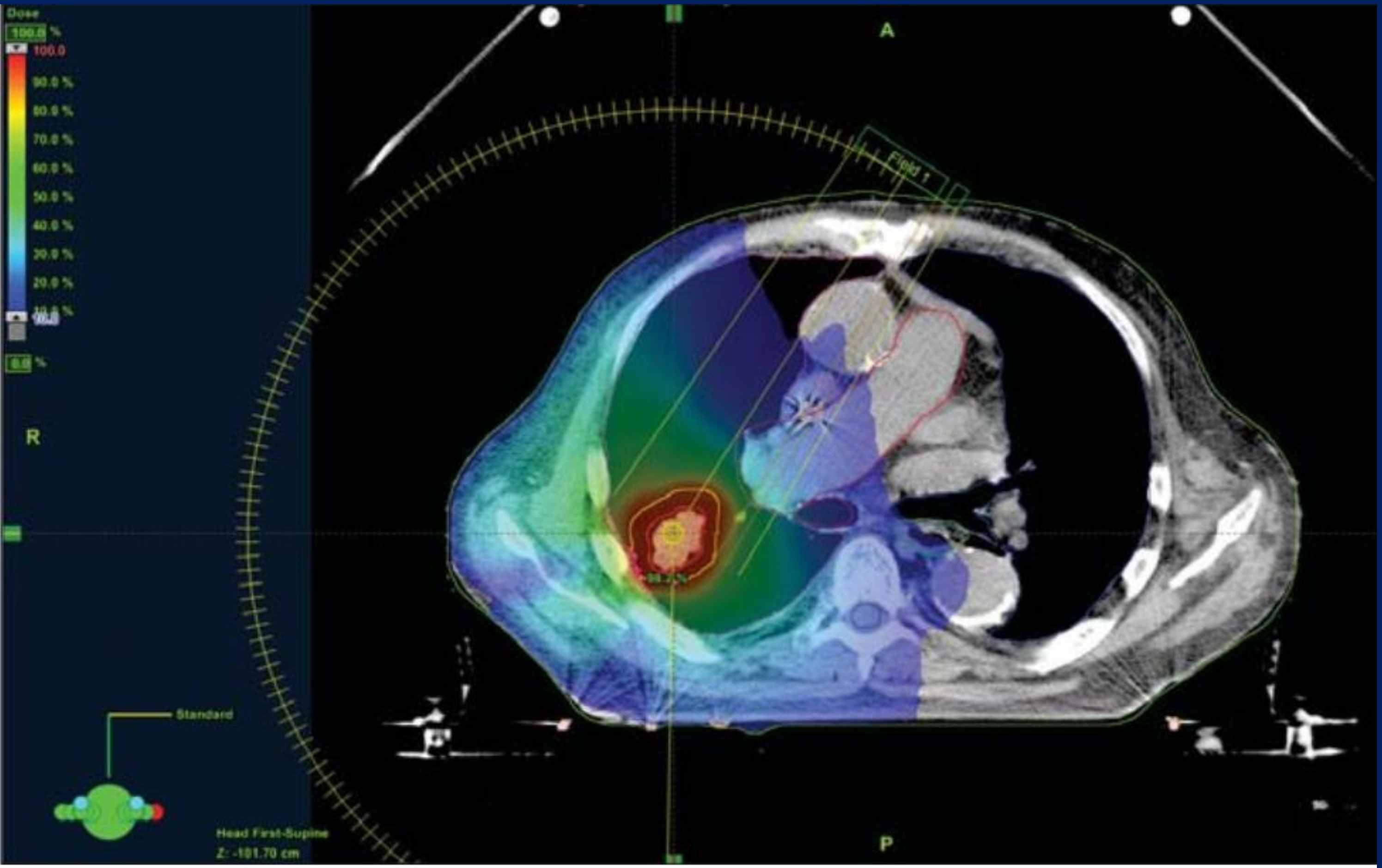


Figure 14. Acuros XB dose field (dose-to-medium) for a 6 MV RapidArc lung case. Total dose calculation time, including source model and patient transport, on a 2.5 mm voxel grid: 86 seconds (4 degree separation - 57 control points).

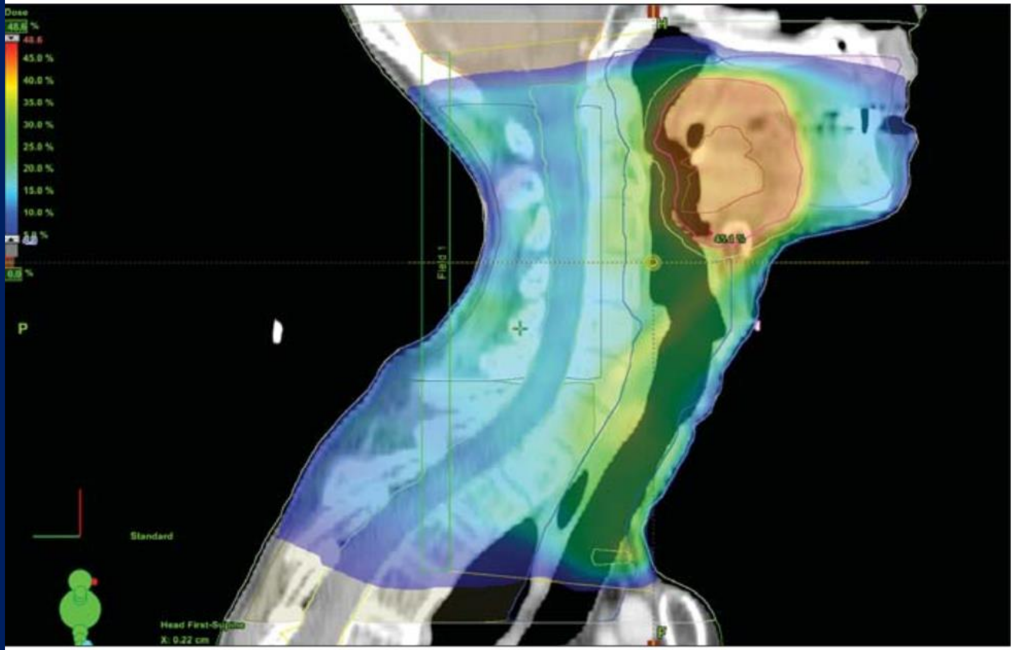
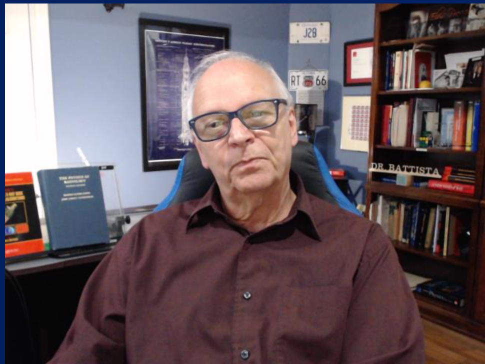


Figure 15. Acuros XB dose field (dose-to-medium) from a 6 MV RapidArc head and neck case. Total dose calculation time, including source model and patient transport, on a 2.5 mm voxel grid: 163 seconds (4 degree separation - 89 control points).



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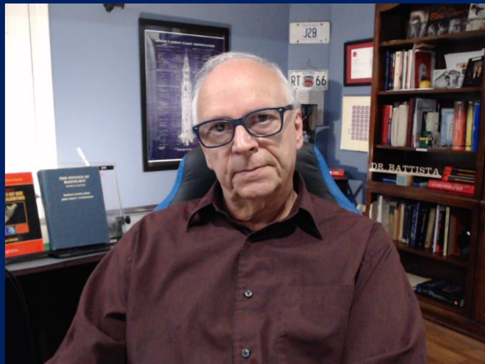
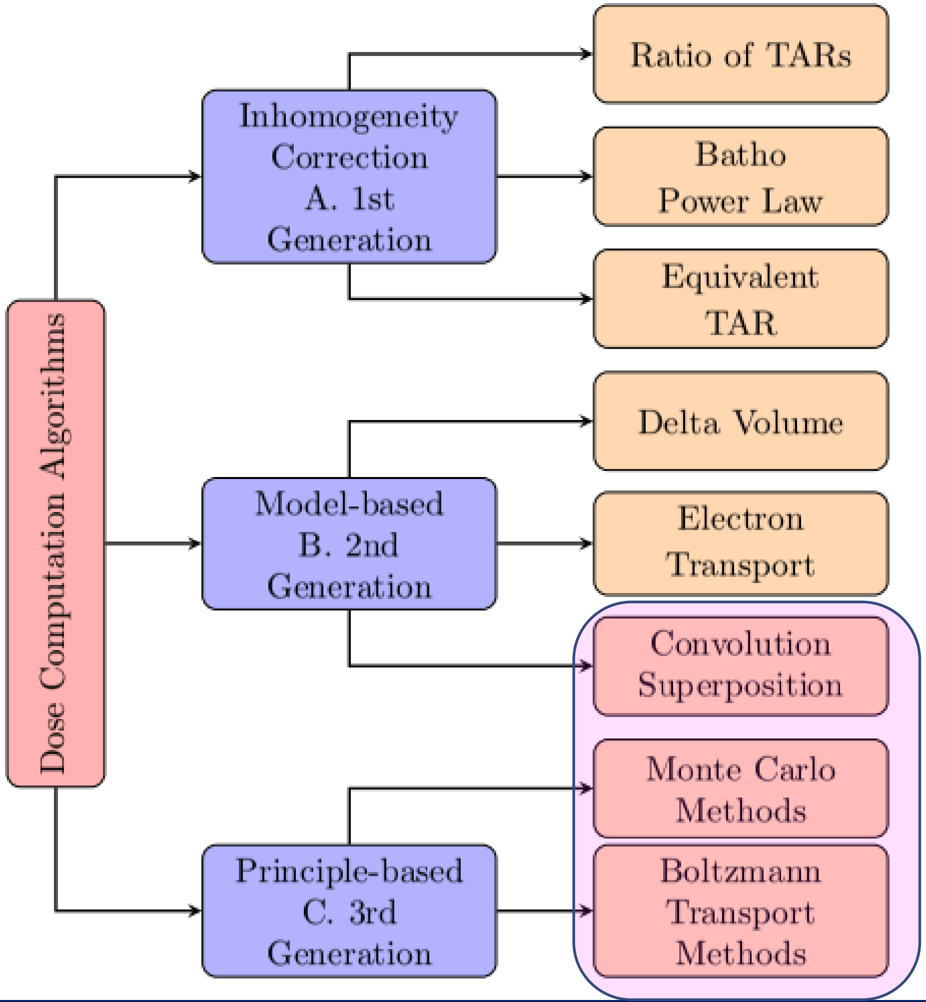
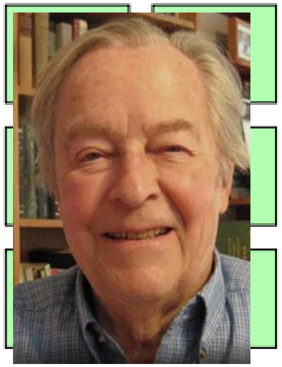
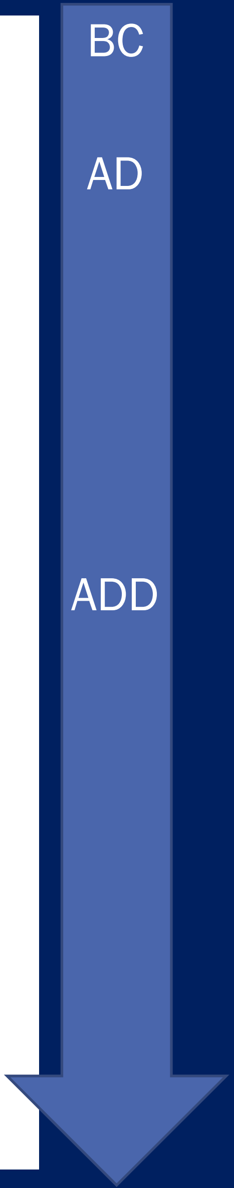


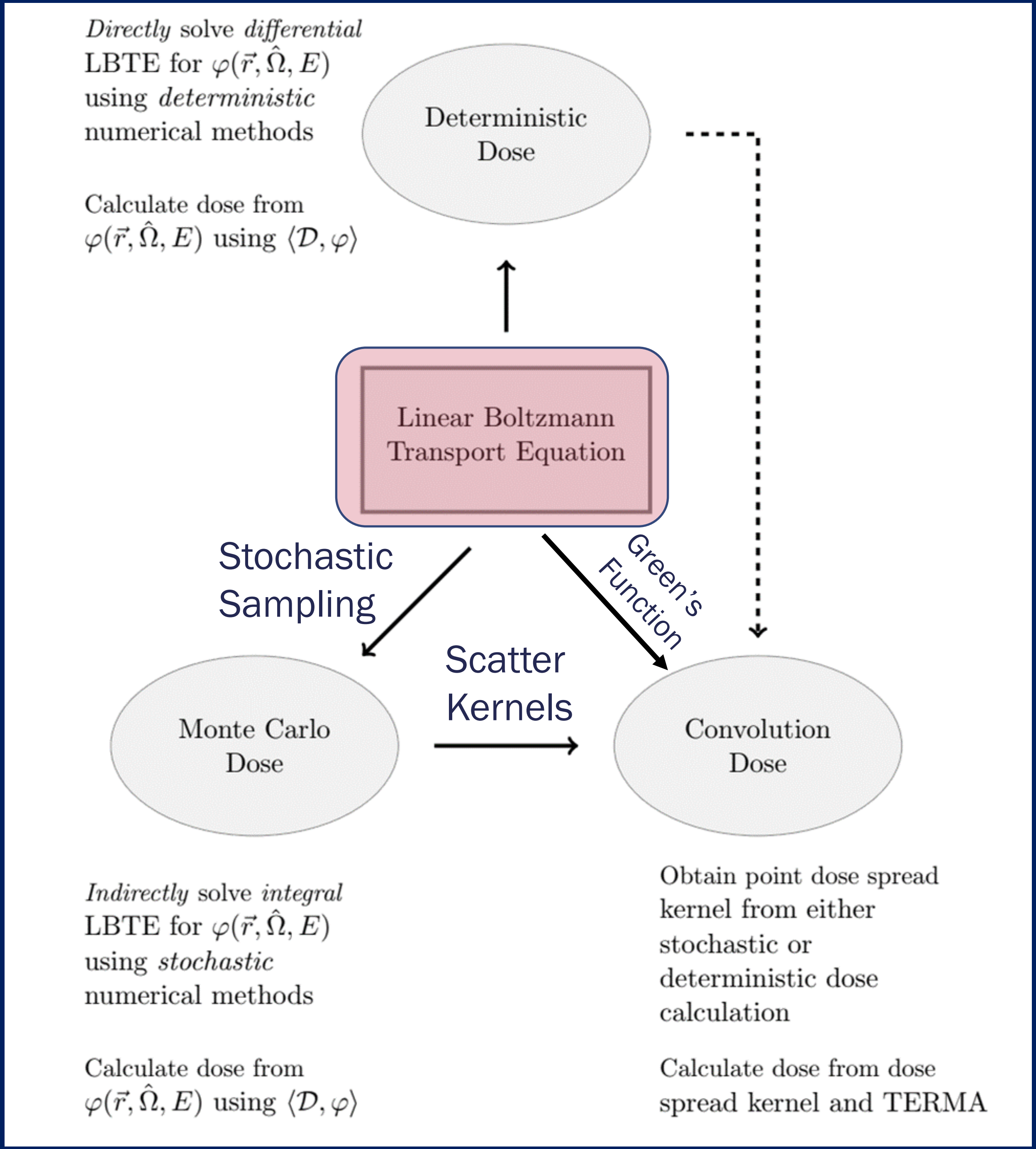
Three Generations of Dose Algorithms



	Primary Photons	Scattered Photons	Radiative Photons	non CPE Condition	Tissue Type
Ratio of TARs	✓	✓	[illegible]	[illegible]	ρ_e
Batho Power Law	✓	✓	[illegible]	[illegible]	ρ_e
Equivalent TAR	✓	✓	[illegible]	[illegible]	ρ_e
Delta Volume	✓	✓			ρ_e
Electron Transport	✓	✓		✓	$\rho_e,(Z)$
Convolution Superposition	✓	✓	✓	✓	$\rho_e,(Z)$
Monte Carlo Methods	✓	✓	✓	✓	ρ_e, Z
Boltzmann Transport Methods	✓	✓	✓	✓	ρ_e, Z

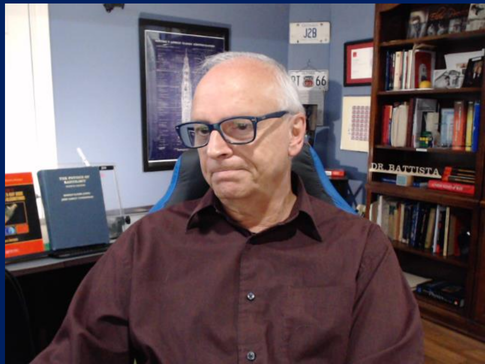




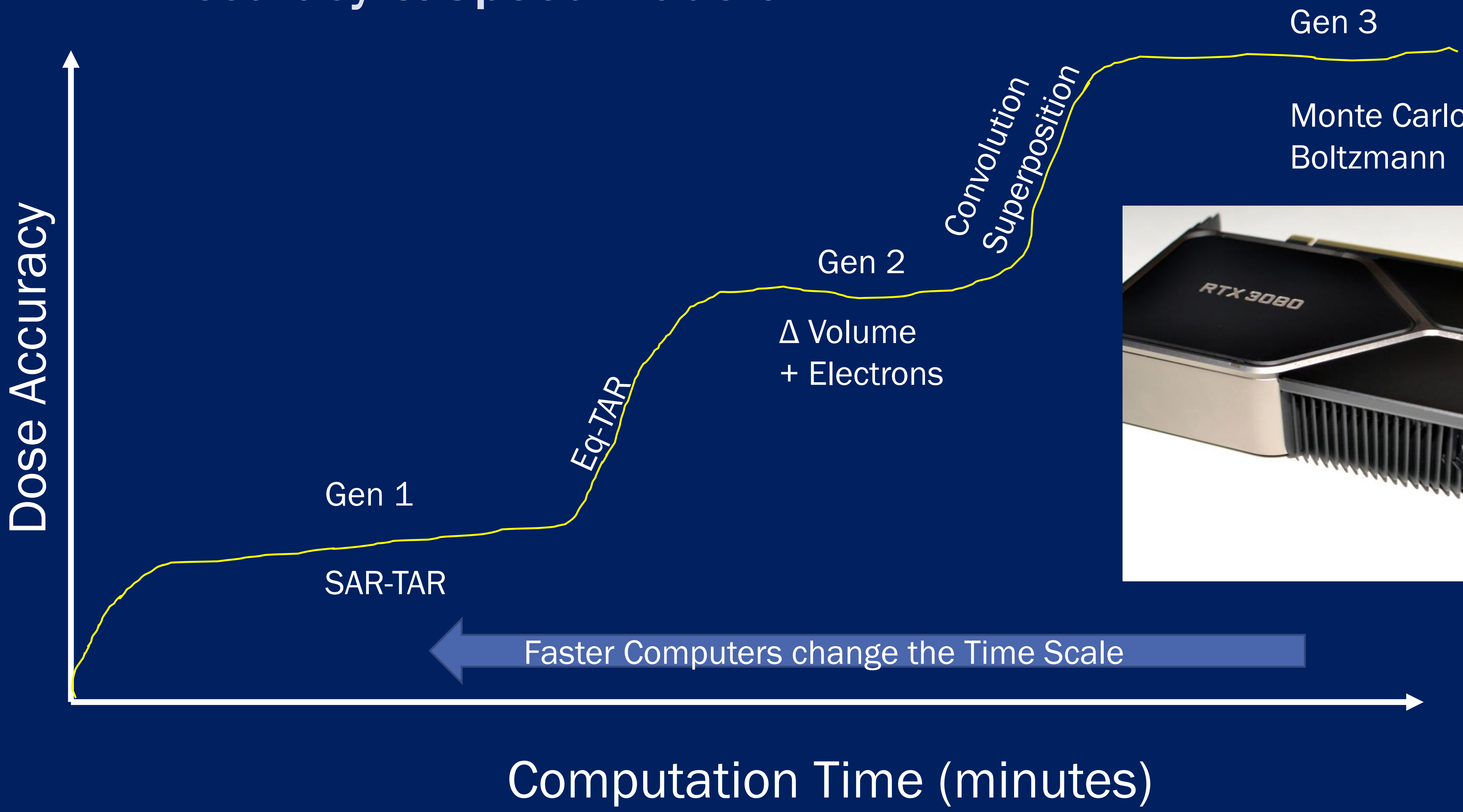


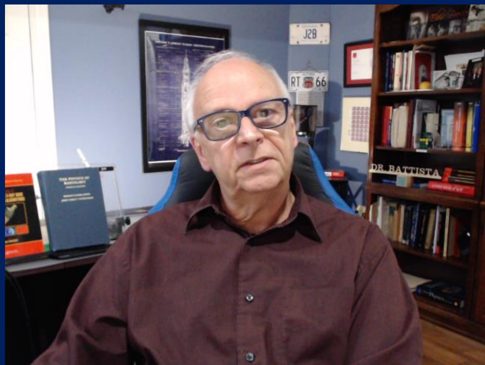
Equation Solver

Green's Method



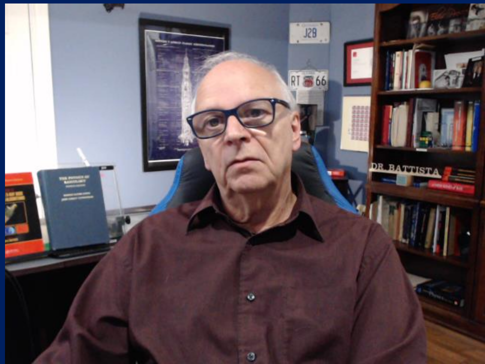
Accuracy & Speed Trade-off





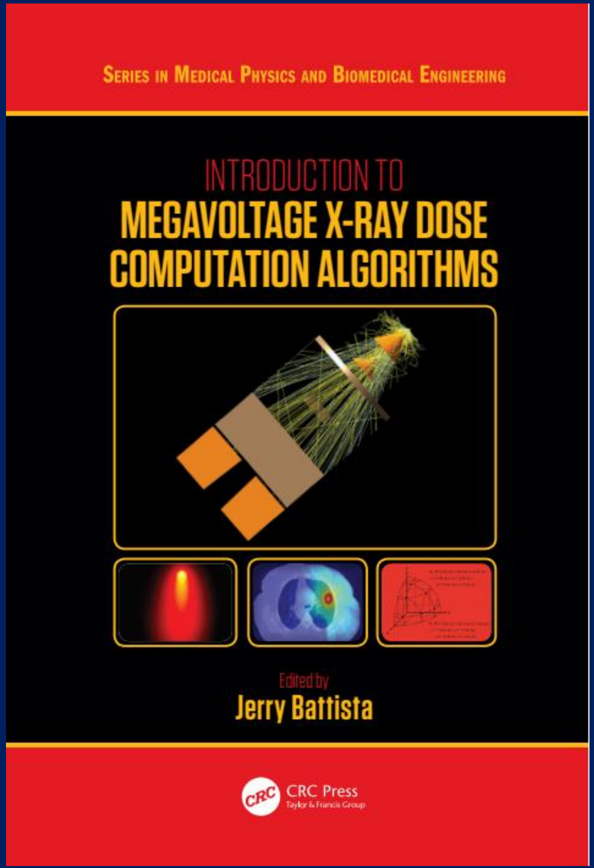
Monte Carloist
- Slow Starter

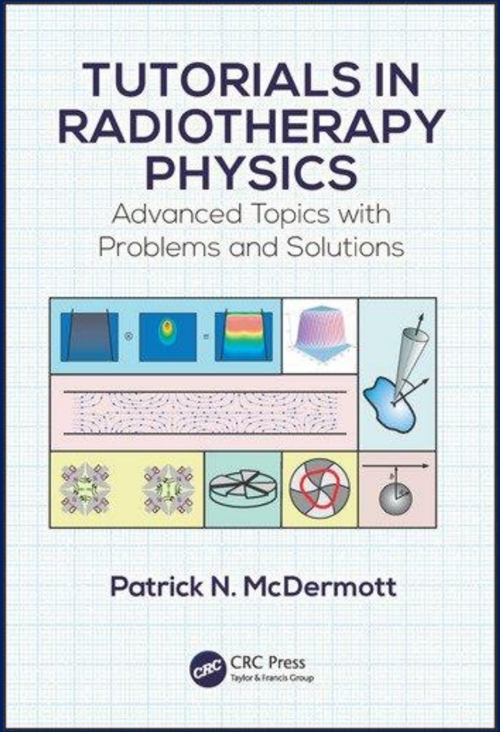
Ludwig Boltz
- Strong Closer

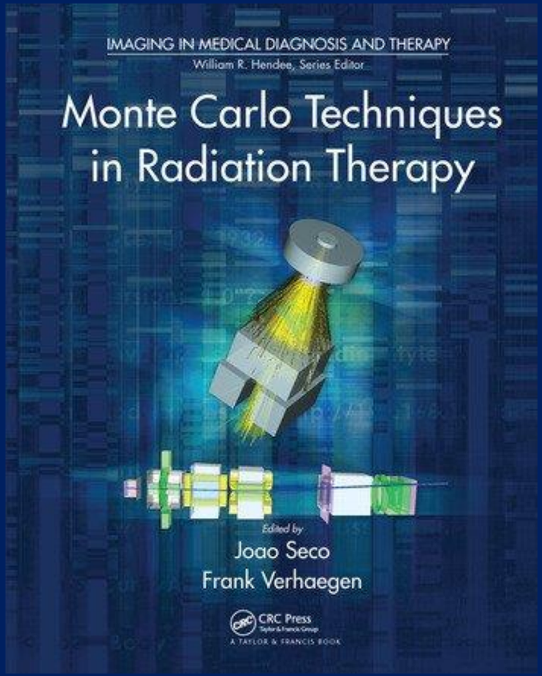


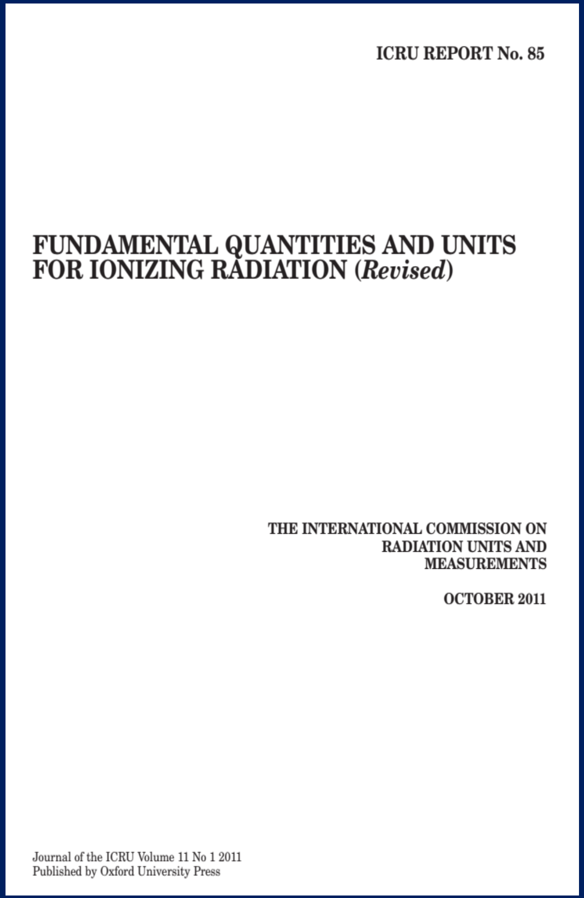
Conclusions

- Algorithms embed assumptions, some are tacit
  - Coding of algorithms can vary – unphysical short-cuts
  - Assumptions can compensate and hide errors
  - Assumptions can fail under different clinical situations
- Beware of "fast & accurate" commercials
  - Smart assumptions win the day
- Monte Carlo and Boltzmann yield "Gold Standard" results
- *Validation by measurement or computer modeling ?*
  - *Do you trust your dosimeter readings ?*





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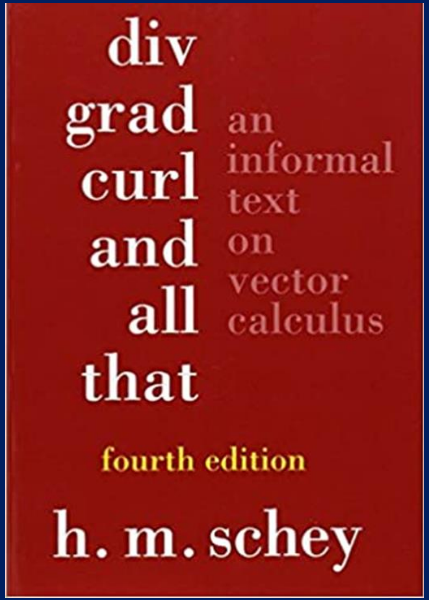

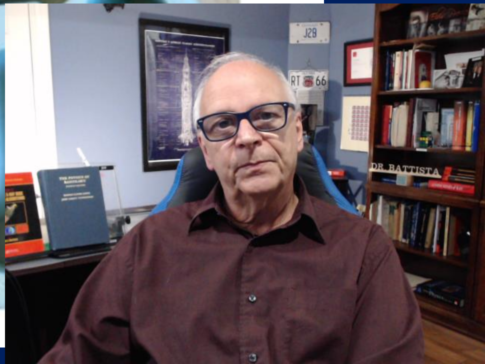

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