Purpose: To derive a set of simple differential equations, which can represent temporal growth of typical tumors, by taking into account the effects of the tumor microenvironment on cell proliferation.

Methods: Cause-effect relationships between cell proliferation and nutrients contained in blood stream, which included oxygen and glucose, were modeled. By formulating a set of rate equations and simplifying those, we finally obtained two differential equations. One equation represents the change of the tumor mass and included time-varying growth rate, which was a solution of another equation relating the growth rate with the blood volume in the tumor. The set of differential equations were numerically solved by varying a parameter, kappa, which represented the growth rate of the blood volume relative to the tumor mass.

Results: Through numerical experiments we showed that the tumor volume increased exponentially if the blood volume increased at the same rate as the tumor. However, the blood volume did not increase rapidly enough as the tumor grew, the tumor showed the saturation in the growth when the tumor grew by 1000-fold. Such a retardation of tumor growth was experimentally observed and the function representing the growth curve is called as Gompertz equation. One of solutions also showed that the tumor stopped growing when the blood supply was shut off (or kappa = 0).

Conclusions: A solution of simple differential equations derived in this study could reproduce the tumor growth which can be represented by the well known Gompertz equation. Using these equations we were able to theoretically show that the saturation of tumor growth observed often for larger tumors could be explained by a relative lack of blood supply which leads to a decrease in the cell proliferation rate. In a future study the effects of radiation on the tumor growth will be included in the model.

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no conflict of interest