Compress sensing with L1-norm based total-variation (TV) regularization has been successfully applied in IMRT inverse planning by generating piecewise constant fluence maps. L1-norm was primarily chosen due to its computational efficiency; for sparse signal recovery, an ideal approach is to use L0-norm (the number of non-zero elements) or Lp-norm (p<1). It is, however, difficult to implement in practice as it is a highly nonlinear and non-convex problem. Recently, the reweighted L1-norm was proposed for closer approximation to L0-norm basis. This work proposes to use the iteratively reweighted L1-norm combined with the TV solver, called template for first-order conic solver (TFOCS), in order to further reduce the complexity of the fluence-map, hence improving the delivery efficiency in IMRT inverse planning, while not sacrificing the conformal dose distribution.

Methods The TV form for the fluence-map optimization is expressed as Eq.(1), while the L0-norm basis written as in Eq.(2), where x is the fluence-map and \( A, d_1, \) and \( \lambda \) are the dose matrix, the dose distribution and the importance factor of each structure. The primary feature of L0-norm is to find the number of non-zero elements, namely suppressing the effect of the amplitudes of elements unlike L1-norm. The reweighted L1-norm was proposed to resemble L0-norm combined with the weighted L1-norm computation, where the weights, denoted by \( w_{\mu,n,f} \), for the next iterate are inversely proportional to the value of the current solution, which is identical to oppressing the effect of the magnitudes by degrees, as presented in Eq.(3). The additional term by \( \delta \) is recommended to be lower than the amplitude of the element, defined as \( \delta = 0.95 \max(|Dx|_{u,v,f}) \) in this work. The proposed method with the iteratively reweighted L1-norm for the fluence-map optimization is summarized in Table.1.

To validate the proposed method, the prostate data with 7 beams (30°, 80°, 130°, 180°, 230°, 280°, 330°) was applied with 16x20 beamlets whose resolution is 5mm. It composed the strong dose conformity to the critical structures such as in rectum, bladder and seminal vesicle. The proposed algorithm was performed with large scale L1-solver (TFOCS) to solve TV form at each iterate. The conformal dose distribution is confirmed by measuring the conformation number (CN) and by looking at the DVHs and iso-dose distribution, while the delivery efficiency is quantified by the modulation index (MI) and the number of segments.

Results and Discussions Table.2 and Fig.1 demonstrate that the reweighted L1-norm outperforms the TV minimization in delivery efficiency and conformation number at 60 segments. The reweighted L1-norm turns out to reduce the complexity of fluence-map as seen in Fig.1 (a) and (b), lowering the MI by 20% (4.74 \( \rightarrow \) 3.86), while not impairing the conformal dose distribution at the PTV (by CN), and the critical structures (by DVHs and iso-dose distribution). By the variation of CN against the number of segments and MI, Fig.2 proves that the reweighted L1-norm enables the plan to reach the higher CN at lower complexity or fewer segments. The improvement will be surely more effective as the dose conformity to the critical structures gets stronger.

Conclusion: The proposed method with the iteratively reweighted L1-norm achieves more conformal dose distribution and improves the delivery efficiency by reducing the fluence-map complexity compared with the conventional TV minimization for IMRT inverse planning.

Reference:

Fig.1 (a),(b) Fluence-map at 6th field (c),(d) DVHs of all structures and (e),(f) iso-dose distribution (30,65,100% of the prescribed dose) of the plans without and with reweighting, respectively.

Table.2 The comparisons in delivery efficiency (MI) and conformal dose distribution (CN) at 60 segments

<table>
<thead>
<tr>
<th></th>
<th>Without reweight</th>
<th>With reweighted L1-norm</th>
</tr>
</thead>
<tbody>
<tr>
<td>MI</td>
<td>4.74</td>
<td>3.86</td>
</tr>
<tr>
<td>CN</td>
<td>0.8111</td>
<td>0.8326</td>
</tr>
</tbody>
</table>

Fig.2 The variation of CN depending on (a) the different beam segments and (b) MI at the fluence-maps