Purpose: Introducing an extended, phenomenological, generalized equivalent uniform dose (eEUD) that incorporates multiple volume-effect parameters for different dose-ranges.

Methods: The generalized EUD (gEUD) was introduced as an estimate of the EUD that incorporates a single, tissue-specific parameter - the volume-effect-parameter (VEP) 'a'. As a purely phenomenological concept, its radio-biological equivalency to a given inhomogeneous dose distribution is not a priori clear and mechanistic models based on radio-biological parameters are assumed to better resemble the underlying biology. However, for normal organs mechanistic models are hard to derive, since the structural organization of the tissue plays a significant role. Consequently, phenomenological approaches might be especially useful in order to describe dose-response for normal tissues.

However, the single parameter used to estimate the gEUD may not suffice in accurately representing more complex biological effects that have been discussed in the literature.

For instance, radio-biological parameters and hence the effects of fractionation are known to be dose-range dependent. Therefore, we propose an extended phenomenological eEUD formula that incorporates multiple VEPs accounting for dose-range dependency.

Results: The eEUD introduced is a piecewise polynomial expansion of the gEUD formula. In general, it allows for an arbitrary number of VEPs, each valid for a certain dose-range. We proved that the formula fulfills required mathematical and physical criteria such as invertibility of the underlying dose-effect and continuity in dose. Furthermore, it contains the gEUD as a special case, if all VEPs are equal to 'a' from the gEUD model.

Conclusion: The eEUD is a concept that expands the gEUD such that it can theoretically represent dose-range dependent effects. Its practicality, however, remains to be shown. As a next step, this will be done by estimating the eEUD from patient data using maximum-likelihood based NTCP modelling in the same way it is commonly done for the gEUD.