Hybrid-Input-Output Algorithm for IMRT Optimization with Dose-Volume Histogram Constraints

Introduction Dose-volume histogram (DVH) is a clinically relevant criterion to evaluate a treatment plan quality. It is hence desirable to incorporate the DVH constraints into the treatment plan optimization for intensity modulated radiation therapy (IMRT). Yet, this usually lead to computational difficulties due to the non-convex nature of these constraints. Recently, hybrid-input-output (HIO) algorithm, a projection-based algorithm, has been widely utilized to solve non-convex optimization problems. It is found to be very effective, in terms of finding a feasible solution, despite the non-convex nature. The purpose of this project is to solve the IMRT optimization problem with DVH constraints using the HIO algorithm.

Methods and Materials Let us denote a fluence map by a vector $x$ and the corresponding dose distribution by $y = fx$, where $F$ is the dose deposition matrix. The IMRT optimization problem finds a dose distribution $y$ under two constraints. First, there exist a fluence map $x$ such that $Fx = y$ and $x > 0$. This constraint defines a set $A = \{x: Fx = y, x > 0\}$. Second, the dose distribution $y$ should satisfy the DVH constraints, which defines another feasible set $B$. The solution to the IMRT problem lies in the intersection of these two sets $A \cap B$.

Let us define the projection operators $P_A$ and $P_B$ corresponding to the projection operation to these two sets $A$ and $B$, respectively, and the reflection operators $R_i = 2P_i - I$ for $i = A, B$. Our method finds the solution via the HIO algorithm that iteratively updates the dose distribution $y$ according to its projections to the two sets until convergence as $y^{(k+1)} = \frac{1}{2}[(1 + \beta)P_B - I + (1 - \beta)P_A]y^{(k)}$, where $k$ is the iteration step index and $\beta$ is a relaxation factor. The projection to $A$ is handled by solving a least square problem and the projection to $B$ is achieved by gradually adjusting voxel doses that validate the DVH criteria to meet the constraints.

Results We test our algorithm on prostate cancer cases. The prescription dose to planning target volume (PTV) is 75 Gy. We used a beamlet size of $5 \times 5$ mm$^2$ and voxel size of $2.5 \times 2.5 \times 2.5$ mm$^3$. A beam configuration with 7 equiangular beams is used. The DVH constraints in the treatment planning are $V_{75 Gy} > 95\%$, $V_{82.5 Gy} < 5\%$ for PTV, $V_{65 Gy} < 17\%$, $V_{40 Gy} < 35\%$ for rectum, and $V_{65 Gy} < 25\%$, $V_{40 Gy} < 50\%$ for bladder. After optimization with our algorithm, the DVH curves in four patient cases are shown in Fig. 1, where the circles indicate the locations of those DVH constraints. After optimization, all the DVH curves satisfy the constraints. In particular, in Fig. (a)~(c), the DVH curves tightly bend around some constraint points, indicating the effectiveness of our algorithm.

Conclusion We have developed an IMRT optimization algorithm with DVH constraints based on the HIO approach. Tests conducted in prostate cancer cases have demonstrated the effectiveness of our algorithm.