Recurrence Quantification Analysis of lung cancer patients' breathing pattern

Suppose x_i are scalar samples acquired at times t_i separated by a fixed time interval t_s, yielding the scalar time series $S = \{x_i\}_{i \in T}; T = \{1, 2, \ldots, N\}$. Using time delay embedding, the data can be represented in an m-dimensional state space where τ is the embedding time delay and m is the embedding dimension.

$$\tilde{\boldsymbol{x}}_i = \left(\boldsymbol{x}_{i-(m-1)\tau}, \boldsymbol{x}_{i-(m-2)\tau}, \ldots, \boldsymbol{x}_{i-\tau}, \boldsymbol{x}_i\right) \qquad (1)$$

Recurrence plots represent a 2 dimensional sub-space that enables one to investigate an m-dimensional state space trajectory through its time recurrences.
A thresholded RP is mathematically expressed as follows

$$R_{i,j} = \Theta\left(\varepsilon - \left\| x_i - x_j \right\|\right), x_i \in \Re^m, i, j = 1, \ldots, N \qquad (2)$$

Where N is the number of $\mathbf{x}_i$ considered, ε is the threshold distance, $\|\bullet\|$ denotes the Euclidean norm and Θ is the Heaviside function.

$$\Theta(\mathrm{x}) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

The purpose of RPs is the visual inspection of higher dimensional state space trajectories in a 2D time recurrence representation. Some of the important RQA parameters are:

Recurrence Rate (RR)*: A measure of the density of recurrence points*
%Determinism*: Stochastic processes have very few short diagonals where as deterministic processes have longer diagonals. Thus the percentage of recurrent points forming line segments parallel to the main diagonal is an indirect measure of determinism or predictability of the system.*
L_{mean}:*. The average diagonal line length is the average time that two segments of the trajectory that is close to each other and can be interpreted as the mean prediction time.*
DIV*: L_{max} , the length of the longest diagonal or its inverse , the divergence (DIV) is related to the largest positive Lyapunov exponent. The higher the divergence, more chaotic the orbit.*
Entropy*: The measure entropy refers to the Shannon entropy of the frequency distribution of the diagonal lengths. It reflects the complexity of the deterministic structure in the system.*
The above parameters, were computed for each of the breathing series obtained using RPM system (Varian, Palo Alto) for seven healthy volunteers. Nonlinear prediction based on state-space prediction methods were implemented on these volunteer breathing data. Initially, 300 seconds of data was used for non-linear prediction and the size of the input data set increased moving forward in time. The NRMSE calculated over 300 sec of predicted signal for all volunteers was plotted against the RQA parameters calculated.

A strong variation was found among the seven healthy volunteers, which is presented here. The 1-D breathing pattern, reconstructed state-space in 3 dimensions, and Recurrence plot thresholded with ε such that RR is equal to 1% of two volunteers (4 and 7) is shown in figures 1(a) through 1(c), and 1(d) through 1(f) respectively. Fig 2(a) and 2(b) shows a strong linear correlation between NRMSE and Entropy & NRMSE and Determinism with correlation coefficients of 82% and 95% respectively.

For Time resolved RQA, the data was windowed into smaller segments (Wsmall = 2000 points), 40 smaller and overlapping RP squares over the entire main RP window. Time resolved RR for Volunteer 4 is shown in figure 3. The peak recurrence detects the location of strong UPOs i.e. patterns of uninterrupted equally spaced diagonal lines, which can be used to coach patients.. RQA analysis on the UPOs shows long diagonals with larger Determinism values. This shows that if patients can be coached to comfortably re-enter their UPOs, larger prediction horizon can be achieved.

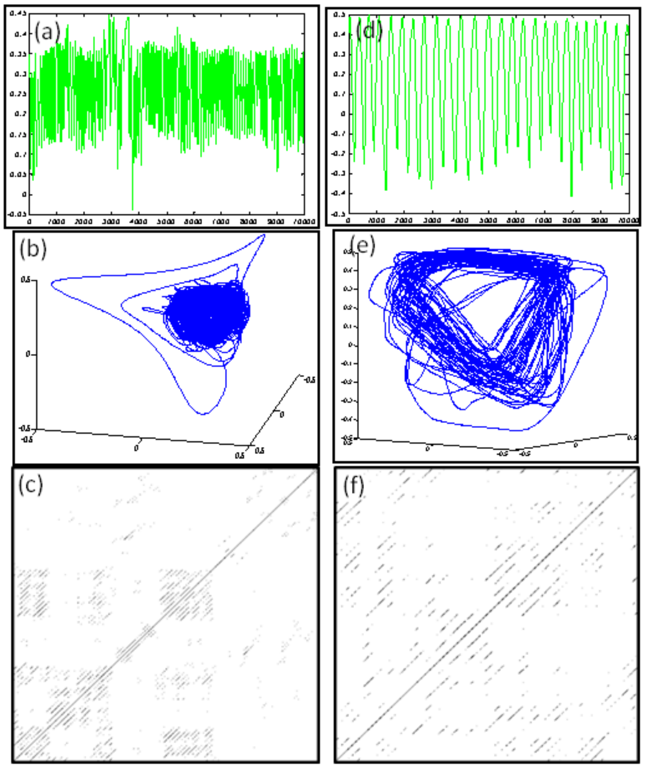

Fig 1(a) through (d) shows the RPM signal and 3D state space for V4 and V7 respectively. The recurrence plots for V4 and V7 are shown in figure (e) and (f) respectively

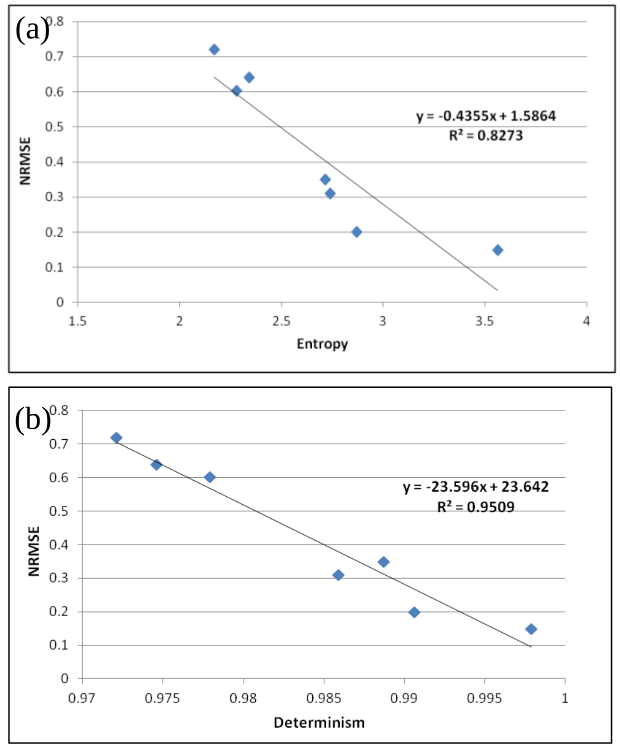

Fig 2(a) and 2(b) shows the correlation between NRMSE and Entropy & NRMSE and Determinism with correlation coefficients of 82% and 95% respectively.

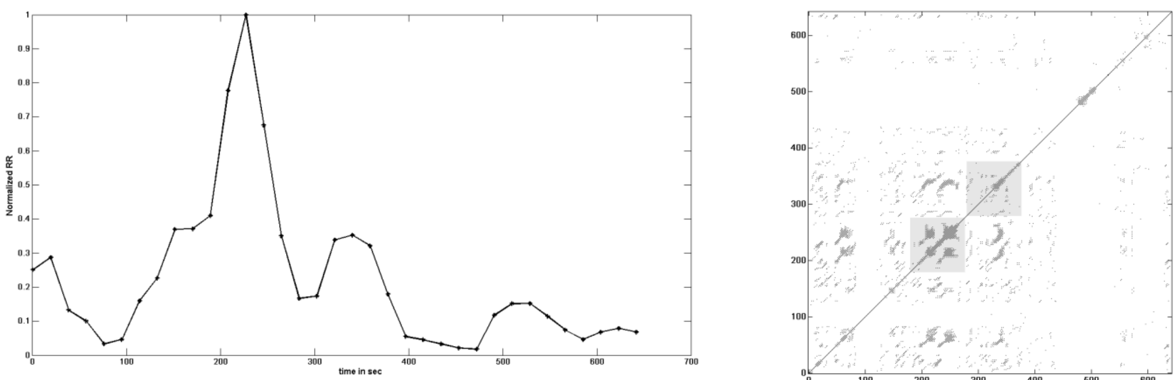

Fig 3(a) shows the time resolved RR for volunteer 4. The respective location for the 2 largest RR is shaded in the RP, which indicates presence of strong UPOs.

References:

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4. *Eckmann J, Kamphorst S and Ruelle D (1987) Recurrence plots of dynamical systems EPL (Europhysics Letters)* **4** *973*