Assessment of image quality for the new CT:
Statistical reconstruction methods

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...
Qualitative comparison

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The nonlinear reconstructions appear to have "better" image quality...
Focus on MBIR methods because image-domain methods are a black-box...

Primary challenges for IQ assessment

- Instrumentation (geometries)
- Reconstruction methods

Mathematical challenges
- Nonlinearity
- Nonstationariness (shift variance)

Practical challenges
- Relating mathematical characteristics to human observer performance
- ...
Sources of nonstationarity for FBP reconstruction

- Heteroscedastic data statistics
- Divergent ray (cone-beam, fan-beam) geometries
- Irregular sampling patterns
  - cone-beam scanners, particularly with larger cone angles
  - dual-source CT with two different detector sizes

Despite all these sources of nonlinearity and nonstationarity, traditional IQ measures like local MTF and local noise variance are useful for evaluating FBP reconstruction (provided the preprocessing steps adequately handle the nonlinearities).

MBIR reconstruction review

Penalized weighted least-squares (PWLS) cost function:

\[ \hat{x} = \arg\min_x \Psi(x), \quad \Psi(x) = \frac{1}{2} \| y - Ax \|_W^2 + \beta R(x) \]

- \( y \): sinogram data, fully preprocessed including logarithm
- \( A \): system matrix (forward projector)
- \( W = \text{diag}(w_i) \): diagonal data dependent statistical weighting matrix; ideally should account for all pre-correction steps and both photon and electronic noise.
- \( \beta \): regularization parameter, controls resolution/noise trade-off
- \( R(x) \): regularizer, often has the form \( R(x) = \sum_i \psi(C_i x_i) \)
  for some potential function \( \psi \)


New challenges for statistical image reconstruction

PWLS reconstruction:

\[ \hat{x} = \arg\min_x \Psi(x), \quad \Psi(x) = \frac{1}{2} \| y - Ax \|_W^2 + \beta R(x) \]

Q: Which of the following may cause nonlinear, shift-variant behavior?

1. Data-dependent weighting \( W \)
2. Non-quadratic regularizers \( R(x) \)
3. Nonnegativity constraints \( x \geq 0 \)
4. Incomplete algorithm convergence “arg min”
5. All of the above
Which of the following may cause nonlinear, shift-variant behavior?

- 30% 1. Data-dependent weighting \( W \)
- 23% 2. Non-quadratic regularizers \( R \)
- 20% 3. Nonnegativity constraints
- 13% 4. Incomplete algorithm convergence
- 13% 5. All of the above

Complications

Nonlinearity and nonstationarity complicate everything about IQ:

- resolution properties
  - local impulse response (point-spread function)
  - local modulation transfer function (MTF)
- noise properties
  - local variance
  - local autocorrelation function
  - local noise power spectrum
  - local distribution (e.g., kurtosis)
  - "texture" of noise
- contrast properties
- detection properties
  - analysis of model observers
  - empirical studies with human observers
The LIR of a statistical reconstruction methods depends on:

- 17% 1. Point location
- 20% 2. Point amplitude
- 13% 3. Surrounding object
- 30% 4. Data Statistics
- 20% 5. All of the above
Q: How does FWHM of LIR for PWLS method compare at center of 4 disks?

1. same
2. higher attenuation disks have bigger FWHM (worse LIR)
3. lower attenuation disks have smaller FWHM (better LIR)
4. no relationship between attenuation and LIR
5. none of the above
FHMW of LIR example

FWHM (angularly averaged) of LIR at center of each disk

- Standard quadratic regularizer: differences between 6 neighboring pixels
- Modified quadratic regularizer: attempts to give uniform spatial resolution
  (Feissler & Rogers, IEEE T-IP, Sep. 1998)
- Other regularizers would induce yet different results
- Unweighted least squares with standard quadratic regularizer
  would be similar to FBP

Towards understanding LIR

Any “black box” algorithm can be studied empirically
(e.g., previous disk example).

Analysis can help obtain insight
(e.g., to help understand what results are generalizable).

PWLS $\hat{x}$ is not only a nonlinear function of the (precorrected) data $y$:

$$\hat{x} = \arg \min_{x \in \mathbb{R}^N} \{ \Psi(x) \}, \quad \Psi(x) = \frac{1}{2} \| y - Ax \|^2_W + \beta R(x),$$

$\hat{x}$ is defined implicitly in terms of $y$, complicating analysis.

To simplify analysis:

- Focus on case where algorithm is iterated “to convergence.”
  Eliminates the iterative algorithm from consideration. Only $\Psi$ matters.
- Ignore the nonnegativity constraint (which is quite nonlinear).
  (It mainly affects background air regions for well regularized cases.)
- Look at the limit of a low-contrast point source (low-contrast case)

LIR expression for PWLS

The LIR at the $j$th voxel is (Feissler & Rogers, IEEE T-IP, Sep. 1998):

$$\text{LIR}_j = [\hat{A} W A + \beta \nabla^2 R(x)]^{-1} \hat{A} W A x_j + \epsilon_j$$

LIR depends on:

- point location $j$
- type of regularizer through its Hessian $\nabla^2 R$
- surrounding object $x$ (for non-quadratic regularizers)
- data statistics $W$
- true system response $A_{data}$ and system model $A$

Using this analysis, we can design regularizer $R(x)$ to guide spatial resolution
properties, e.g., make resolution approximately uniform and isotropic,
and largely independent of the object and statistics, at least for quadratic
regularizers.


However, uniform spatial resolution usually means nonuniform noise in CT.
(probably always)
For PWLS reconstruction, compared to the noise variance of the heart region, the noise variance in the lung region is:

1. Much lower
2. Somewhat lower
3. Comparable
4. Higher
5. None of the above

Noise properties depend on reconstruction method (A, W, R, ...).
Empirical noise maps for PWLS image reconstruction

CT simulation with XCAT phantom:

standard regularizer:  

"uniform resolution" regularizer:  

These are both results from simple quadratic regularizers. Edge-preserving regularizers produce more variable noise maps.

(Zhang-O'Connor & Fessler, IEEE TML, 2007)

Predicting noise properties

For PWLS with quadratic regularization:  

\[
\text{Cov}(\hat{x}) \approx \left(A^TWA + \beta V^2 R^{-1}\right)^{-1} A^T\text{Cov}(y)A \left(A^TWA + \beta V^2 R^{-1}\right)^{-1}
\]

Useful for predicting:
- local reconstructed image variance
- local image autocorrelation
- local noise power spectrum

Empirical:  

Predicted:  

In principle can use such noise predictions to inform regularization design and selection of regularization parameter \(\beta\).

Unfortunately, analysis for non-quadratic regularization is very difficult. For TV and \(l_1\)-based sparsity regularizers even harder.

Empirical noise properties: Kurtosis

![Diagram showing empirical noise properties: Kurtosis](image)
Kurtosis continued

For non-Gaussian images, second moments (NPS) are an incomplete story.

Contrast-dependent edge resolution: 1D

Challenge: Shape of edge response depends on contrast when “preserving edges.”

Contrast-dependent edge resolution: 2D CT

Challenge: Shape of edge response depends on contrast for edge-preserving regularization.
How much does (quadratic) regularization improve SNR of CHO for SKE detection task?

1. A lot, if we select proper beta
2. At best only a little
3. Makes no difference
4. Quadratic regularization degrades SNR due to blur

Optimizing regularizers for signal detection

SNR of channelized Hotelling observer (CHO) for signal-known-exactly (SKE) task, applied to PWLS reconstruction with quadratic regularizer.

Q: How much does regularization ($\beta > 0$) improve SNR over $\beta = 0$?
1. A lot, if we select proper $\beta$
2. At best only a little
3. Makes no difference
4. Quadratic regularization degrades SNR due to blur

Choosing $\beta$: Unknown location

AUC for signal detection with unknown location task.

Benefits of $\beta$ depend on ability of observer to perceive gradient.

Other complications

- 3D regularization (vs FBP)
- Temporal/dynamic CT reconstruction (inherently missing data)
- Dual-energy, dual-source, ...
- ...

(Vendelli & Fessler, IEEE TMI, Jan. 2006)

Summary

- For quadratic regularization we have good understanding of local resolution and noise properties.
- Nonquadratic case is less well understood, though progress has been made for smooth-edge-preserving regularizers:

Take aways

- Resolution/noise properties depend on the reconstruction method including all of its specific models and components (e.g., regularizer)
- Report local L/P or PSF and local MTF and local NPR.
- Focus on low-contrast signals for comparing FBP vs "R".
- Include unknown location tasks in IQ assessment.
- Be wary of general claims about statistical image reconstruction methods vs FBP.
- When publishing (or reviewing) comparisons, provide (or require) a description of the statistical image reconstruction method.

Bibliography