Radiographic Tomosynthesis II: reconstruction algorithms

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FDA statement: discussion will include off-label uses and applications not yet approved

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Tomosynthesis: section imaging from multi-projection image reconstruction (limited angle tomography)

Geometry of tomosynthesis image acquisition

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Geometries of motion

Parallel path
Partial isocentric
Isocentric

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3D Chest Radiography (Tomosynthesis)

• Vertical tube motion
• Total tube angle: 20-35°
• Number of Projected Images: 60 - 71
• Exam length: 10-11 sec (single breath-hold)
• Slice thickness: 4-5 mm

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Tomosynthesis image formation

Tomosynthesis algorithms

- Shift-and-add
- ART (algebraic reconstruction techniques)
- Tuned aperture computed tomography (TACT)
- Iterative methods (MLEM)
- Matrix inversion tomosynthesis (MITS)
- Filtered backprojection (FBP)
- Feldkamp (limited angle CBCT)
Shift-and-add reconstruction
(simple backprojection)

Acquisition geometry

Shift-and-add image formation

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The importance of deblurring

Conventional tomo section

After deblurring

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Development of deblurring methods

Self-masking subtraction tomosynthesis
Chakraborty et al, 1984

Selective plane removal
Ghosh Roy et al, 1985

Matrix inversion tomosynthesis
Dobbins, 1986

TACT and image restoration
Webber, Ruttimann, 1990s

Filtered backprojection
2000s

Matrix Inversion Tomosynthesis (MITS)
Removing the blur with MITS

Direct solution using linear algebra and the known acquisition geometry

• Much faster computationally than iterative deblurring
• Better performance at narrow tube angles than filtered backprojection
• However…. susceptible to noise at the lowest spatial frequencies ( < ~ 0.1 cycles/mm)

Conventional tomosynthesis planes

\[
\begin{align*}
t_1 &= s_1 \otimes f_{11} + s_2 \otimes f_{12} + \cdots + s_n \otimes f_{1n} \\
t_2 &= s_1 \otimes f_{21} + s_2 \otimes f_{22} + \cdots + s_n \otimes f_{2n} \\
&\vdots \\
t_n &= s_1 \otimes f_{n1} + s_2 \otimes f_{n2} + \cdots + s_n \otimes f_{nn}
\end{align*}
\]
In the frequency domain...

\[
\begin{align*}
T_1 &= S_1 \times F_{11} + S_2 \times F_{12} + \cdots + S_n \times F_{1n} \\
T_2 &= S_1 \times F_{21} + S_2 \times F_{22} + \cdots + S_n \times F_{2n} \\
& \quad \vdots \\
T_n &= S_1 \times F_{n1} + S_2 \times F_{n2} + \cdots + S_n \times F_{nn}
\end{align*}
\]

Removing the blur with MITS

- Matrix form (freq space)
- Rewritten
- Solving for true structures

\[
T = M \times S
\]

\[
S = FT^{-1}(M^{-1} \times T)
\]

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Computing the MITS blurring functions

Shift for $k$th projection image to tomosynthesize $j$th plane: $-p_{jk}$

Blurring function for $\delta$-function in $i$th plane when $j$th plane is tomosynthesized:

$$f_{ij} = \sum_k \delta(x - p_{ik} + p_{jk})$$

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MITS requires a correction for low-frequency noise susceptibility

1. High pass MITS planes (Gaussian filter, $\sigma \sim 0.1$ cycles/mm)
2. Low pass conventional planes (Gaussian filter, $\sigma \sim 0.1$ cycles/mm)
3. Add 1.25% of the filtered conventional spectra to the high-passed MITS spectra, to restore lung opacity in chest images

$$MITS_{fb}(f) = \left[1 - \exp\left(-\frac{f^2}{2\sigma^2}\right)\right] \cdot MITS(f) + FW \cdot \exp\left(-\frac{f^2}{2\sigma^2}\right) \cdot CONV(f)$$

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Frequency blending with MITS

20-degree tube motion, 61 proj images, 59 planes recon, 5 mm plane spacing

\[ MITS_\sigma(f) = \left[ 1 - \exp\left( -\frac{f^2}{2\sigma^2} \right) \right] \cdot MITS(f) + \text{FW} \cdot \exp\left( -\frac{f^2}{2\sigma^2} \right) \cdot \text{CONV}(f) \]

\( \sigma = 0.01 \text{ mm}^{-1} \)

\( \sigma = 0.1 \text{ mm}^{-1} \)

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Slice sensitivity profile

Single-slice MITS

7-slice sliding average MITS

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MITS provides excellent reconstruction even at very narrow tube angles

~ 12° tube movement  ~ 6° tube movement

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Filtered backprojection

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Filtered backprojection methodology

- Ramp filter corrects for the 1/f inherent point response in frequency space
- Apodization (roll-off filter) suppresses high-frequency noise enhancement following ramp filter

Acquire projection images; take Fourier transform
Multiply by ramp filter
Multiply by roll-off filter
Reconstruct by shift-and-add

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Projection/Slice Theorem

1D FT
2D FT
Radial slice

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Simple Backprojection

<table>
<thead>
<tr>
<th>Back Projection</th>
<th>Reconstructed Fourier</th>
<th>Original Fourier</th>
</tr>
</thead>
</table>

- Very blurry.
+ Noise tolerant

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Principles of FBP for cone-beam imaging

Filtered Backprojection

Intuitive Interpretation
- Backprojection causes blur
- Correct the blur with an “inverse filter”

MTF($k$)

Inverse filter = $1 / \text{MTF}(k)$

Ramp Filter
Reconstruction Filter

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Filtered Backprojection

- Ramp filter provides exact reconstruction when:
  - Noise free
  - Sufficient samples (no missing data)

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Tomosynthesis
(Limited View / Limited Angle)

Effect of Thick Slices
Slice Thickness Correction Filters

- View-dependent (angle) apodization

Apodizer

Ramp * Apodizer

T Mertelmeier, Optimizing filtered backprojection reconstruction for a breast tomosynthesis prototype device, Proc SPIE 6142, 2006

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Back Projection  Reconstructed Fourier  Original Fourier

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Comparison of MITS and FBP

- MITS uses direct solution using linear algebra and the known acquisition geometry (perfect rendition of in-plane structures)
- FBP uses well-known algorithm from CT
- MITS performs better at narrow tube angles
- Both are much faster computationally than iterative methods
- MITS is susceptible to noise at the lowest spatial frequencies (< ~ 0.1 cycles/mm)
- FBP must use roll-off filter to avoid noise at high-frequencies

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Iterative reconstruction strategies

- Breast volume is sampled using a three-dimensional matrix of elements (voxels)
- Typical voxel size: 0.1 mm × 0.1 mm × 1 mm
- The value of a voxel is the linear x-ray attenuation coefficient μ of that element

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Maximum Likelihood Expectation Maximization (ML-EM)

Initial 3D Model

\[ \mu^{(n)} \]

Forward projection

\[ \mu^{(n+1)} \]

Update

Optimized Likelihood Function

\[ \mu^{(\text{end})} \]

Measured Projections:
\[ Y \]

Calculated Projections:
\[ Y^{(n)} \]

\[ \Delta \mu^{(n+1)} \]

ML-EM Reconstruction: Likelihood Function

Likelihood Function

\[ L = P(Y | \mu) \]: probability of getting the measured projections \( Y \), given a 3D model

\[ \mu^{(n)} \] is updated iteratively so that \( L^{(n+1)} > L^{(n)} \)

The reconstruction solution is the 3D attenuation distribution model that maximizes \( L \)
Advantages/disadvantages of iterative methods relative to FBP/MITS

- Better modeling of system, including truncation effects
- Potentially better noise properties
- Potentially better with fewer projections
- Much slower computationally than FBP or MITS

Translational issues remaining

Reconstruction algorithms:

- Low-freq contrast in FBP (less high-pass filtered look)
- Noise improvement: MITS+FBP
- SART
- MLEM - multiple processors
- Clinical evaluation/comparison of various algorithms
Review articles:


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