Radiotherapy Physics Quick Reference
Ryan Flynn, 7/26/2011

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1. General
1.1. Important constants

<table>
<thead>
<tr>
<th>Charge/electron</th>
<th>$1.602 \times 10^{-19}$ C</th>
<th>$1$ mg-Ra eq</th>
<th>$8.25 \times 10^{-4}$ R/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W / e$</td>
<td>$33.97$ J/C or eV/ion pair</td>
<td>$1$ U (air kerma strength)</td>
<td>$7.227 \mu$Gy m$^2$/h</td>
</tr>
<tr>
<td>$eV$</td>
<td>$1.602$ J</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electron rest mass</td>
<td>$511$ keV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1$ amu</td>
<td>$931$ MeV $= 1.66 \times 10^{-27}$ kg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ci</td>
<td>$3.7 \times 10^{13}$ Bq</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ci (defn.)</td>
<td>Activity of $1$ gm of $^{226}$Ra</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-to-rad in air</td>
<td>$0.876$ rad/R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1$ R</td>
<td>$2.58 \times 10^{-4}$ C/kg</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1.2. Isotopes: $\gamma$-emitters

<table>
<thead>
<tr>
<th>Radioisotope</th>
<th>Half-life</th>
<th>Average $\gamma$-ray energy (keV)</th>
<th>$\Gamma_x$-Value (R-cm$^2$/mCi-hr)</th>
<th>Apparent activity</th>
<th>HVL (mm Pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{226}$Ra</td>
<td>$1,622$ y</td>
<td>$830$</td>
<td>$8.25$ (R-cm$^2$/mg-hr)</td>
<td></td>
<td>$12$</td>
</tr>
<tr>
<td>$^{222}$Rn</td>
<td>$3.83$ d</td>
<td>$830$</td>
<td>$10.15$</td>
<td></td>
<td>$12$</td>
</tr>
<tr>
<td>$^{60}$Co</td>
<td>$5.26$ y</td>
<td>$1,250$</td>
<td>$13$</td>
<td></td>
<td>$11$</td>
</tr>
<tr>
<td>$^{137}$Cs</td>
<td>$30.0$ y</td>
<td>$660$</td>
<td>$3.26$</td>
<td>$0.348$</td>
<td>$5.5$</td>
</tr>
<tr>
<td>$^{192}$Ir</td>
<td>$73.83$ d</td>
<td>$380$</td>
<td>$4.69$</td>
<td>$0.243$</td>
<td>$2.5$</td>
</tr>
<tr>
<td>$^{198}$Au</td>
<td>$2.7$ d</td>
<td>$412$</td>
<td>$2.38$</td>
<td>$0.486$</td>
<td>$2.5$</td>
</tr>
<tr>
<td>$^{103}$Pd</td>
<td>$16.97$ d</td>
<td>$21$</td>
<td>$1.48$</td>
<td>$0.773$</td>
<td>$0.008$</td>
</tr>
<tr>
<td>$^{125}$I</td>
<td>$59.4$ d</td>
<td>$28$</td>
<td>$1.46$</td>
<td>$0.787$</td>
<td>$0.025$</td>
</tr>
</tbody>
</table>

1.3. Isotopes: $\beta$-emitters

- $^{90}$Sr ($0.546$ MeV, $28.8$ y) $\rightarrow ^{90}$Y ($2.28$ MeV, $64$ hr) $\rightarrow ^{90}$Zr
- $^{89}$Sr ($1.46$ MeV, $50$ d) $\rightarrow ^{89}$Y
- Electron range in air: $4$ m for $2$ MeV
2. Radiation protection

2.1. Dose equivalent and effective dose equivalent

\[
H_E = \sum w_T H_T = \sum T \sum w_T D_T
\]

- \(H_T\) = dose equivalent delivered to tissue \(T\) [Sv]
- \(w_T\) = weighting factor for tissue, \(T\) [unitless]
- \(D_T\) = absorbed dose delivered to tissue \(T\) from radiation \(R\) [Gy]
- \(H_E\) = effective dose equivalent [Sv]

2.2. Permissible doses

<table>
<thead>
<tr>
<th>Exposure limits</th>
<th>Shielding thicknesses (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Occupational</td>
<td></td>
</tr>
<tr>
<td>50 mSv/yr</td>
<td>Energy (MV)</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>lens</td>
<td>150 mSv/yr</td>
</tr>
<tr>
<td>other organs</td>
<td>500 mSv/yr</td>
</tr>
<tr>
<td>cumulative</td>
<td>age (yrs) x 10 mSv/yr</td>
</tr>
<tr>
<td>Public</td>
<td>Transporting radioactive isotopes</td>
</tr>
<tr>
<td>Frequent</td>
<td>1 mSv/yr</td>
</tr>
<tr>
<td>Infrequent</td>
<td>5 mSv/yr</td>
</tr>
<tr>
<td>lens, other</td>
<td>50 mSv/yr</td>
</tr>
<tr>
<td>Fetal</td>
<td>5 mSv total</td>
</tr>
<tr>
<td>0.5 mSv/yr</td>
<td>Cancer induction probability</td>
</tr>
<tr>
<td></td>
<td>5% per Sv (ICRP Report 60)</td>
</tr>
</tbody>
</table>

2.3. Shielding

Primary: \(P = \frac{WUT}{d^2} B_p\)

Secondary: \(P = \frac{\alpha WT}{d_{ss}^2 d_{sa}^2} F B_s\)

Leakage: \(P = \frac{0.001 WT}{d_i^2} B_i\)

- \(P\) = permissible dose, \(W\) = workload (500 - 1000 Gy/wk), \(U\) = use factor = 1 for controlled area, 1/4 for ceiling, 1/4 for walls), \(T\) = occupancy factor = 1 controlled area, 1/4 partially occupied, 1/16 occasionally occupied, \(\alpha\) = the scatter-to-primary ratio off the scatterer at 1 m, \(F\) = field size at scatterer

Rule of thumb: If the secondary and leakage barriers differ by at least three HVLs, then the thicker of the two will suffice. Otherwise add one HVL onto the thicker of the two and use that.

Skyshine calculations

Photons: \(D = 0.249 \times 10^6 \frac{B_{ss} D_{xo} \Omega^{1/3}}{(d_i d_s)^2}\)

Neutrons: \(H = 0.84 \times 10^{-5} \frac{B_{ns} \Phi_0 \Omega}{d_i^2}\)

\(D\) = dose equivalent rate at ground level [nSv/s], \(D_{xo}\) = x-ray dose rate at 1 m from target [cGy/s], \(\Omega\) = solid angle of radiation beam [steradians], \(B_{ss}\) = roof shielding transmission ratio, \(d_i\) = distance [m] from x-ray target to 2 m above the roof, \(d_s\) = distance [m] from isocenter to the point where the dose equivalent rate is \(D\), \(H\) = nSv/s due to neutrons at ground level, \(B_{ns}\) = roof shielding transmission ratio for neutrons, \(\Phi_0\) = neutron fluence rate \((\text{cm}^{-2} \text{s}^{-1})\) at 1 m from the target

2.4. Scatter and secondary particles

- Maximum energy of 90° Compton scattered photon = 511 keV
2.5. Radioactive seed disposal

Seeds can be discarded after having decayed away for 10 half-lives. For $^{125}$I this is 594 days.

3. Monitor unit calculations

3.1. General equations

<table>
<thead>
<tr>
<th>SAD setup (TPR/TMR/TAR calculation):</th>
<th>SSD setup (PDD calculation):</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D = MU \cdot TMR(r_d, d) \cdot S_c(r_c) \cdot S_s(r_d) \cdot OF \cdot \left(\frac{SAD + d_{max}}{SSD + d}\right)^2$</td>
<td>$D = MU \cdot OF \cdot PDD(r, d, SSD) \cdot S_c(r_c) \cdot S_s(r_d)$</td>
</tr>
</tbody>
</table>

3.2. Converting PDD measurements to TMR and TAR data

| TMR($r_d, d$) = $\frac{PDD(r, d)}{100} \left(\frac{SSD + d_{max}}{SSD + d}\right)^2 \frac{S_p(r_{d_{max}})}{S_p(r_d)}$ | TAR($r_d, d$) = $\frac{PDD(r, d)}{100} \left(\frac{SSD + d_{max}}{SSD + d}\right)^2 BSF(r)$ |

3.3. Converting PDDs at one SSD to PDDs at another SSD

Mayneord factor: $\frac{PDD(SSD_2, r_d, d)}{PDD(SSD_1, r_d, d)} = \frac{TAR(r_d, SSD_2, d)}{TAR(r_d, SSD_1, d)} \left(\frac{SSD_2 + d_{max}}{SSD_1 + d}\right)^2$

3.4. Scatter-air-ratio calculations

| SAR($d, r_d$) = TAR($d, r_d$) − TAR($d, 0$) |
| SAR isolates the scatter component from TAR values. For irregular fields, TAR is obtained and used for MU calc. |

3.5. Penumbra, gap, and collimator rotation calculations

$\text{gap} = \frac{d}{2} \left(\frac{L_1}{SSD_1} + \frac{L_2}{SSD_2}\right)$

$\text{penumbra} = \frac{s(SSD + d - SCD)}{SCD}$

$\tan \theta_{\text{coll}} = \frac{L}{2SSD}$

$d = \text{prescription depth, SCD = source-collimator distance, } L_{1,2} = \text{size of field 1 or 2, } s = \text{source focal spot size (~3 mm for a linac), } \theta_{\text{coll}} = \text{collimator angle for craniospinal field}$

3.6. Linac beam characteristics for photons and electrons

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>$d_{max}$</th>
<th>$d_{90%}$</th>
<th>$d_{80%}$</th>
<th>%DD(10)</th>
<th>%DD(20)</th>
<th>TPR(10)</th>
<th>TPR(20)</th>
<th>PDD(0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 MV</td>
<td>1.5</td>
<td>4.0</td>
<td>6.5</td>
<td>67.0</td>
<td>38.8</td>
<td>0.778</td>
<td>0.523</td>
<td>52.8</td>
</tr>
<tr>
<td>10 MV</td>
<td>2.3</td>
<td>5.5</td>
<td>8.0</td>
<td>73.6</td>
<td>46.6</td>
<td>0.846</td>
<td>0.627</td>
<td>37.9</td>
</tr>
<tr>
<td>18 MV</td>
<td>3.0</td>
<td>6.5</td>
<td>9.5</td>
<td>77.8</td>
<td>51.1</td>
<td>0.884</td>
<td>0.673</td>
<td>31.6</td>
</tr>
<tr>
<td>6 MeV</td>
<td>1.1</td>
<td>1.6</td>
<td>1.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 MeV</td>
<td>1.9</td>
<td>2.6</td>
<td>2.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>77.3</td>
</tr>
</tbody>
</table>

**Electron beam properties:**

$d_{90\%}[\text{cm}] \sim E[\text{MeV}]/4, d_{80\%} \sim E/3, \text{range} \sim E/2$
3.7. Heterogeneity corrections

\[ \text{CF} = \frac{\text{TAR}(d_1 + \rho_2 d_2 + d_3)}{\text{TAR}(d_1 + d_2 + d_3)} \]

CF corrects the dose calculation for heterogeneities. The MU value will be increased by a factor of 1/CF.

3.8. Wedges

3.8.1. Hinge angles

\[ \theta_{\text{wedge}} = 90^\circ - \phi_{\text{hinge}} / 2 \]

\[ \phi = \text{hinge angle} = \text{angle between central axes of wedged fields} \]

3.8.2. Generating wedge angles with a universal wedge

\[ D_{\text{tot}} = D_0 + D_W = \text{MU}_0 \cdot \text{OF} + \text{MU}_W \cdot \text{OF} \cdot \text{WF}, \]

\[ A = \frac{D_0}{D_{\text{tot}}} = \frac{\text{MU}_0 \cdot \text{OF}}{D_{\text{tot}}} \]

\[ B = \frac{D_0}{D_{\text{tot}}} = \frac{\text{MU}_W \cdot \text{OF}}{D_{\text{tot}}} \]

\[ A = 1 - B \]

\[ B = \frac{\tan \theta_E}{\tan \theta_W} = \frac{\theta_E}{\theta_W} \]

\[ \text{MU}_0 = \left(1 - B\right) \cdot D_{\text{tot}} / \text{OF} \]

\[ \text{MU}_W = \frac{B \cdot \text{MU}_0}{(1 - B) \cdot \text{WF} / \text{OF} \cdot \text{WF}} \]

\[ D_{\text{tot}} = \text{total dose prescribed}, \ D_0 = \text{dose from open field}, \ D_W = \text{dose from universal wedged field} \]

\[ \theta_W = \text{universal wedge angle}, \ \theta_E = \text{desired effective wedge angle}, \ \text{OF} = \text{open field output factor}, \]

\[ \text{WF} = \text{wedge factor}, \ \text{MU}_0 = \text{MU from open field}, \ \text{MU}_W = \text{MU from wedged field} \]

3.9. Timer error for superficial x-rays or $^{60}$Co

\[ M_1 = n \dot{M}(t_{\text{short}} + \Delta t) = \dot{M}(t_{\text{tot}} + n \Delta t) \]

\[ M_2 = \dot{M}(t_{\text{tot}} + \Delta t) \]

\[ \Delta t = \frac{t_{\text{tot}} (M_2 - M_1)}{M_1 - n M_2} \]

\[ \dot{M} = \text{charge collection rate}, \ n = \text{number of short measurements (10 at least)}, \]

\[ t_{\text{short}} = \text{short measurement time}, \ \Delta t = \text{timer error}, \ t_{\text{tot}} = nt_{\text{short}} = \text{total measurement time}, \]

\[ M_1 = \text{total charge collected over} \ n \text{ short measurements}, \]

\[ M_2 = \text{total charge collected over one measurement session of length} \ t_{\text{tot}}. \]

3.10. Patient thickness vs. dose uniformity

<table>
<thead>
<tr>
<th>Energy</th>
<th>Max/midline dose for 30 cm thickness</th>
<th>See Khan 3rd Ed., p.211-212</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{60}$Co</td>
<td>1.4</td>
<td>Dose ratios start at 1.0 for 10 cm thickness</td>
</tr>
<tr>
<td>4 MV</td>
<td>1.25</td>
<td>Dose ratios increase approximately as thickness squared</td>
</tr>
<tr>
<td>10 MV</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
<td>24 MV</td>
<td>1.05</td>
<td></td>
</tr>
</tbody>
</table>

3.11. Compensators/bolus/spoilers

Compensators: account for missing tissue while maintaining buildup region

Spoilers: increase angular spread of phase space of electrons, degrade e$^-$ beam energy

Bolus: shift PDD buildup region toward skin surface
### 3.12. Photon compensator and lead shield thicknesses determination

\[ t_c = \frac{TD \cdot \tau}{\rho_c} \]

- **\( t_c \)** = photon compensator thickness
- **TD** = tissue deficit (cm)
- **\( \tau \)** = thickness ratio, function of distance between compensator and absorber
- **\( \rho_c \)** = density of compensator material (Al is 2.7 g/cm\(^3\))

**Lead shield thickness** for \( e^+ \) beams: 0.5 mm / MeV. Cerrobend must be 20% thicker than Pb.

### 4. Dosimetry

#### 4.1. TG-51

**General:** The TG-51 protocol yields absorbed dose in water, in Gy, at the point of measurement of the absent ion chamber. Reference dosimetry must be performed in a water phantom with dimensions of at least 30 cm x 30 cm x 30 cm.

**For photon beams** with energies between \( ^{60}\text{Co} \) and 50 MeV

\[ D_w^q = M_{\text{raw}} N_{D_w} \]

\[ M = M_{\text{raw}} P_{\text{ion}} P_{\text{pol}} P_{\text{elec}} P_{\text{TP}} \]

\[ P_{\text{ion}} = \frac{1 - V_H / V_L}{M_{\text{raw}} / M_{\text{raw}} - V_H / V_L} \]

\[ P_{\text{pol}} = \frac{M_{\text{raw}} - M_{\text{raw}}^*}{2M_{\text{raw}}^*} \]

\[ P_{\text{TP}} = 273.15 + T \cdot 760 \]

\[ P_{\text{TP}} = 273.15 + 22 \cdot P \]

\[ D_w^q = \text{dose to water for beam of quality, } Q, \text{ determined by } %\text{dd}(10), \]

\[ M_{\text{raw}} = \text{raw charge measurement}, \]

**Correction factors:** \( P_{\text{ion}} = \) recombination, \( P_{\text{pol}} = \) polarity, \( P_{\text{elec}} = \) electrometer, \( P_{\text{TP}} = \) temp/press.

**For electron beams** with energies between 4 and 50 MeV

\[ D_w^q = M_{\text{raw}} k_{\text{ecal}} k_{R_{50}'} P_{\text{gr}} N_{D_w} \]

\[ R_{50} = \begin{cases} 1.029 & I_{50} = 0.06 \text{ cm for } 2 \leq I_{50} \leq 10 \text{ cm} \\ 1.059 & I_{50} = 0.37 \text{ cm for } 10 < I_{50} \end{cases} \]

\[ d_{\text{ref}} = 0.6R_{50} - 0.1 \text{ cm} \]

\[ P_{\text{gr}} = \frac{M(d + 0.5r_{\text{cav}})}{M(d)} \]

\[ E_{p,0} = 0.22 + 1.98R_p + 0.0025R_p^2 \text{ [MeV]} \]

\[ E_0 = 2.33R_{50} \text{ [MeV]} \]

\[ \overline{E_d} = E_0 \left(1 - d / R_p\right) \]

**\( k_{\text{ecal}} \)** = photon-electron quality conversion factor (chamber-dependent),

**\( k_{R_{50}'} \)** = electron beam quality conversion factor (chamber- and beam energy-dependent),

**\( R_{50} \)** = depth of 50% dose wrt maximum, **\( r_{\text{cav}} \)** = cylindrical ion chamber radius

**\( I_{50} \)** = depth of 50% ionization wrt maximum, after upstream shift of 0.5**\( r_{\text{cav}} \),**

**\( d_{\text{ref}} \)** = reference depth at which parallel plate to cylindrical chamber conversion factor valid

**\( P_{\text{gr}} \)** = electron gradient correction factor, **\( R_p \)** = practical electron range, **\( E_{p,0} \)** = most probable energy at depth 0 cm, **\( d \)** = depth in phantom, **\( \overline{E_d} \)** = mean electron energy at depth **\( d \)**

### 4.2. Cross-calibration of a parallel plate chamber for electron dosimetry

- Determine depth of **\( d_{\text{ref}} \)** (cm)
- Measure charge collected with both the cylindrical and parallel plate chambers with point of measurement at **\( d_{\text{ref}} \)**
- Calculate \( k_{\text{ecal}} N_{D_w}^{\text{\( ^{60}\text{Co} \)}} = \left(M_{\text{ecal}} k_{R_{50}'} P_{\text{gr}} N_{D_w}^{\text{\( ^{60}\text{Co} \)}}\right)^{\text{pp}} / \left(M_{R_{50}'}\right)^{\text{pp}} \)
4.3. Measuring electron PDD curves

- Measure ionization curve
- Shift curve upstream if necessary
- Correct for inverse square
- Determine average electron energy with depth
- Scale curve by stopping power ratios evaluated at each depth

5. Radiation Biology

5.1. Biological effective dose

\[
BED = N \left( 1 + \frac{d}{\alpha/\beta} \right) \quad \text{where} \quad N = \text{number of fractions} \\
\alpha/\beta = \text{linear/quadratic factors} = 3 \text{ for typical tumors, 10 normal tissue} \\
d = \text{dose per fraction (Gy)}
\]

\[
d_2 = \frac{1}{2} \left[ -\frac{\alpha}{\beta} + \sqrt{\left(\frac{\alpha}{\beta}\right)^2 + 4 \cdot \text{BED}_1 \cdot \frac{\alpha/\beta}{N_2}} \right] \quad \text{where} \quad \text{BED}_1 = \text{original BED} \\
N_2 = \text{new number of fractions}
\]

6. Film

6.1. Optical density measurement

\[
\text{OD} = \log_{10} \frac{I_0}{I} \
\text{OD} = \text{optical density} \\
I_0 = \text{light without film} \\
I = \text{light transmitted through film}
\]

6.2. Film dosimetry for electrons

- Kodak XV-2 film is linear up to 50 cGy
- When irradiating a film parallel to the central axis of the beam, it is very important to ensure that as much air as possible is removed from between the phantom slabs and that the film is flush with the surface of the phantom that the beam will be aimed at.

7. Brachytherapy

7.1. TG-43 dose calculation formalism

\[
D = S_\Lambda \frac{G(r, \theta)}{G(r_0, \theta_0)} \cdot g(r) \cdot F(r, \theta) \quad \text{where} \quad r = \text{radius from center of mass of source [cm]} \\
\theta = \text{angle between source axis and calculation point [degrees]} \\
r_0 = 1 \text{ cm, } \theta_0 = 90^\circ, S_\Lambda = \text{air kerma strength [U]} = [\text{cGy cm}^2/\text{hr}] \\
\Lambda = \text{dose rate constant at } (r_0, \theta_0) [\text{cGy/U}], \\
G(r, \theta) = \text{geometry factor [-], } g(r) = \text{radial dose function [-],} \\
F(r, \theta) = \text{anisotropy function [-]}
\]

\[
G(r, \theta) = \begin{cases} 
1/r^2 & \text{point source approximation} \\
\beta/L \sin \theta & \text{line source approximation} 
\end{cases} \\
\beta = \text{angle subtended by source [radians]} \\
L = \text{length of source}
\]

\[
g(r) = \frac{D(r, \theta_0)/G(r, \theta_0)}{D(r_0, \theta_0)/G(r_0, \theta_0)} \\
F(r, \theta) = \frac{D(r, \theta)/G(r, \theta)}{D(r_0, \theta_0)/G(r_0, \theta_0)}
\]