Radiotherapy Physics Quick Reference

Ryan Flynn, 7/26/2011

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1. General

1.1. Important constants

Charge/electron	1.602×10^{-19} C		1 mg-Ra eq	8.25×10^{-4} R/h 7.227 μGy m^2/h
$\bar{W} / e$	33.97 J/C or eV/ion pair		1 U (air kerma strength)	cGy cm^2/hr = μGy m^2/hr
eV	1.602 J			
Electron rest mass	511 keV			
1 amu	931 MeV = 1.66×10^{-27} kg			
Ci	3.7×10^{13} Bq			
Ci (defn.)	Activity of 1 gm of ^{226}Ra			
R-to-rad in air	0.876 rad/R			
1 R	2.58×10^{-4} C/kg			

1.2. Isotopes: γ-emitters

Radioisotope	Half-life	Average γ-ray energy (keV)	Γ_x-Value (R-cm^2/mCi-hr)	Apparent activity	HVL (mm Pb)
^{226}Ra	1,622 y	830	8.25 (R-cm^2/mg-hr)		12
^{222}Rn	3.83 d	830	10.15		12
^{60}Co	5.26 y	1,250	13		11
^{137}Cs	30.0 y	660	3.26	0.348	5.5
^{192}Ir	73.83 d	380	4.69	0.243	2.5
^{198}Au	2.7 d	412	2.38	0.486	2.5
^{103}Pd	16.97 d	21	1.48	0.773	0.008
^{125}I	59.4 d	28	1.46	0.787	0.025

1.3. Isotopes: β-emitters

- ^{90}Sr (0.546 MeV, 28.8 y) -> ^{90}Y (2.28 MeV, 64 hr) -> ^{90}Zr
- ^{89}Sr (1.46 MeV, 50 d) -> ^{89}Y
- Electron range in air: 4 m for 2 MeV

2. Radiation protection

2.1. Dose equivalent and effective dose equivalent

$H_E = \sum_T w_T H_T$ $= \sum_T w_T \sum_R w_R D_T$	w_T = weighting factor for tissue, T [unitless] H_T = **dose equivalent** delivered to tissue T [Sv] w_R = relative weighting factor for radiation R [Sv/Gy] D_T = **absorbed dose** delivered to tissue T from radiation R [Gy] H_E = **effective dose equivalent** [Sv]

2.2. Permissible doses

Exposure limits			Shielding thicknesses (cm)						
Occupational	50 mSv/yr		**Energy (MV)**	4	6	10	15	18	20
lens	150 mSv/yr		Concrete	35	37	41	44	45	46
other organs	500 mSv/yr		Steel	10	10	11	11	11	11
cumulative	age (yrs) x 10 mSv/yr		Lead	5.7	5.7	5.7	5.7	5.7	5.7
Public			**Transporting radioactive isotopes**						
Frequent	1 mSv/yr					White I	Yellow I	Yellow III	
Infrequent	5 mSv/yr		Max surface (mrem/hr)			0.5	50	200	
lens, other	50 mSv/yr		Transport index			0	1.0	10.0	
Fetal	5 mSv total		**Cancer induction probability**						
	0.5 mSv/mo		5% per Sv (ICRP Report 60)						

2.3. Shielding

Primary: $P = \frac{\text{WUT}}{d^2} B_p$	**Secondary:** $P = \frac{\alpha \text{WT}}{d_{\text{sec}}^2 d_{\text{sca}}^2} \frac{F}{400} B_s$	**Leakage:** $P = \frac{0.001\,\text{WT}}{d_\ell^2} B_\ell$

P = permissible dose, W = workload (500 - 1000 Gy/wk),
U = use factor = 1 for controlled area, 1/4 for ceiling, 1/4 for walls),
T = occupancy factor = 1 controlled area, 1/4 partially occupied, 1/16 occasionally occupied,
α = the scatter-to-primary ratio off the scatterer at 1 m, F = field size at scatterer

Rule of thumb: If the secondary and leakage barriers differ by at least three HVLs, then the thicker of the two will suffice. Otherwise add one HVL onto the thicker of the two and use that.

Skyshine calculations

Photons: $D = 0.249 \times 10^6 \frac{B_{xs} D_{\text{io}} \Omega^{1.3}}{(d_i d_s)^2}$	**Neutrons:** $H = 0.84 \times 10^{-5} \frac{B_{\text{ns}} \Phi_0 \Omega}{d_i^2}$

D = dose equivalent rate at ground level [nSv/s],
D_{io} = x-ray dose rate at 1 m from target [cGy/s], Ω = solid angle of radiation beam [steradians],
B_{xs} = roof shielding transmission ratio, d_i = distance [m] from x-ray target to 2 m above the roof,
d_s = distance [m] from isocenter to the point where the dose equivalent rate is D,
H = nSv/s due to neutrons at ground level, B_{ns} = roof shielding transmission ratio for neutrons,
Φ_0 = neutron fluence rate ($cm^{-2}\,s^{-1}$) at 1 m from the target

2.4. Scatter and secondary particles

- Maximum energy of 90° Compton scattered photon = 511 keV

- Photoneutron production energy threshold for photons: 10 MeV

2.5. Radioactive seed disposal

Seeds can be discarded after having decayed away for 10 half-lives. For ^{125}I this is 594 days.

3. Monitor unit calculations

3.1. General equations

SAD setup (TPR/TMR/TAR calculation): $$D = \text{MU} \cdot \text{TMR}(r_d, d) \cdot S_c(r_c) \cdot S_c(r_d) \cdot \text{OF} \cdot \left(\frac{\text{SAD} + d_{\max}}{\text{SSD} + d}\right)^2$$	D = dose delivered MU = monitor units SSD = source-surface distance SAD = source-axis distance r = field size at surface r_d = field size at calculation point r_c = coll. setting at SAD d = depth of calculation point $d_{\max}$ = depth of maximum dose
SSD setup (PDD calculation): $$D = \text{MU} \cdot \text{OF} \cdot \text{PDD}(r, d, \text{SSD}) \cdot S_c(r_c) \cdot S_c(r_{d_{\max}})$$	

3.2. Converting PDD measurements to TMR and TAR data

$$\text{TMR}(r_d, d) = \frac{\text{PDD}(r,d)}{100}\left(\frac{\text{SSD} + d_{\max}}{\text{SSD} + d}\right)^2 \frac{S_p(r_{d_{\max}})}{S_p(r_d)}$$

$$\text{TAR}(r_d, d) = \frac{\text{PDD}(r,d)}{100}\left(\frac{\text{SSD} + d_{\max}}{\text{SSD} + d}\right)^2 \text{BSF}(r)$$

3.3. Converting PDDs at one SDD to PDDs at another SSD

Mayneord factor: $$\frac{\text{PDD}(\text{SSD}_2, r, d)}{\text{PDD}(\text{SSD}_1, r, d)} = \frac{\text{TAR}(r_{d,\text{SSD}_2}, d)}{\text{TAR}(r_{d,\text{SSD}_1}, d)}\left(\frac{\text{SSD}_2 + d_{\max}}{\text{SSD}_1 + d}\right)^2$$

3.4. Scatter-air-ratio calculations

$\text{SAR}(d, r_d) = \text{TAR}(d, r_d) - \text{TAR}(d, 0)$	SAR isolates the scatter component from TAR values. For irregular fields, $\overline{\text{TAR}}$ is obtained and used for MU calc.
$\overline{\text{TAR}} = \text{TAR}(d,0) + \overline{\text{SAR}}$, $\text{TAR}(d,0) = e^{-\bar{\mu}(d - d_{\max})}$	

3.5. Penumbra, gap, and collimator rotation calculations

$\text{gap} = \frac{d}{2}\left(\frac{L_1}{\text{SSD}_1} + \frac{L_1}{\text{SSD}_1}\right)$	$\text{penumbra} = \frac{s(\text{SSD} + d - \text{SCD})}{\text{SCD}}$	$\tan\theta_{\text{coll}} = \frac{L/2}{\text{SSD}}$

d = prescription depth, SCD = source-collimator distance, $L_{1,2}$ = size of field 1 or 2, s = source focal spot size (~3 mm for a linac), θ_{coll} = collimator angle for craniospinal field

3.6. Linac beam characteristics for photons and electrons

	$d_{\max}$	$d_{90\%}$	$d_{80\%}$	%DD(10)	%DD(20)	TPR(10)	TPR(20)	PDD(0)
6 MV	1.5	4.0	6.5	67.0	38.8	0.778	0.523	52.8
10 MV	2.3	5.5	8.0	73.6	46.6	0.846	0.627	37.9
18 MV	3.0	6.5	9.5	77.8	51.1	0.884	0.673	31.6
6 MeV	1.1	1.6	1.8	**Electron beam properties:**				77.3
9 MeV	1.9	2.6	2.9	$d_{90\%}$ [cm] ~ E [MeV]/4, $d_{80\%}$ ~ E/3, range ~ E/2				80.5

12 MeV	2.5	3.5	3.8	$\overline{E}_0 = 2.33R_0, E_p = \overline{E}(1 - d / R_p)$	84.7
15 MeV	3.0	4.4	4.9		89.6
18 MeV	2.4	5.5	6.2		93.4
21 MeV	2.1	6.0	6.9		94.8

3.7. Heterogeneity corrections

$\text{CF} = \dfrac{\text{TAR}(d_1 + \rho_2 d_2 + d_3)}{\text{TAR}(d_1 + d_2 + d_3)}$	CF corrects the dose calculation for heterogeneities. The MU value will be increased by a factor of 1/CF.

3.8. Wedges

3.8.1. Hinge angles

$\theta_{\text{wedge}} = 90° - \varphi_{\text{hinge}} / 2$	θ_{wedge} = wedge angle φ = hinge angle = angle between central axes of wedged fields

3.8.2. Generating wedge angles with a universal wedge

$D_{\text{tot}} = D_0 + D_W = \text{MU}_0 \cdot \text{OF} + \text{MU}_W \cdot \text{OF} \cdot \text{WF},$ $A = D_0/D_{\text{tot}} = \text{MU}_0 \cdot \text{OF}/D_{\text{tot}}$ $B = D_0/D_{\text{tot}} = \text{MU}_W \cdot \text{OF} \cdot \text{WF}/D_{\text{tot}}.$	$A = 1 - B$ $B = \dfrac{\tan\theta_E}{\tan\theta_W} \cong \dfrac{\theta_E}{\theta_W}$	$\text{MU}_0 = (1 - B) \cdot D_{\text{tot}} / \text{OF}$ $\text{MU}_W = \dfrac{B \cdot \text{MU}_0}{(1 - B)\text{WF}} = \dfrac{B \cdot D_{\text{tot}}}{\text{OF} \cdot \text{WF}}$
D_{tot} = total dose prescribed, D_0 = dose from open field, D_W = dose from universal wedged field θ_W = universal wedge angle, θ_E = desired effective wedge angle, OF = open field output factor, WF = wedge factor, MU_0 = MU from open field, MU_W = MU from wedged field		

3.9. Timer error for superficial x-rays or ^{60}Co

$M_1 = n\dot{M}(t_{\text{short}} + \Delta t) = \dot{M}(t_{\text{tot}} + n\Delta t)$	$M_2 = \dot{M}(t_{\text{tot}} + \Delta t)$	$\Delta t = \dfrac{t_{\text{tot}}(M_2 - M_1)}{M_1 - nM_2}$
$\dot{M}$ = charge collection rate, n = number of short measurements (10 at least), t_{short} = short measurement time, Δt = timer error, $t_{\text{tot}} = nt_{\text{short}}$ = total measurement time, M_1 = total charge collected over n short measurements, M_2 = total charge collected over one measurement session of length t_{tot}.		

3.10. Patient thickness vs. dose uniformity

Energy	Max/midline dose for 30 cm thickness	
^{60}Co	1.4	See Khan 3rd Ed., p.211-212
4 MV	1.25	Dose ratios start at 1.0 for 10 cm thickness
10 MV	1.15	
24 MV	1.05	Dose ratios increase approximately as thickness squared

3.11. Compensators/bolus/spoilers

Compensators: account for missing tissue while maintaining buildup region
Spoilers: increase angular spread of phase space of electrons, degrade e^- beam energy
Bolus: shift PDD buildup region toward skin surface

3.12. Photon compensator and lead shield thicknesses determination

$t_c = \text{TD}\,\dfrac{\tau}{\rho_c}$	t_c = **photon compensator thickness** TD = tissue deficit (cm) τ = thickness ratio, function of distance between compensator and absorber ρ_c = density of compensator material (Al is 2.7 g/cm^3)
Lead shield thickness for e^- beams: 0.5 mm / MeV. Cerrobend must be 20% thicker than Pb.	

4. Dosimetry

4.1. TG-51

General: The TG-51 protocol yields absorbed dose in water, in Gy, at the point of measurement of the absent ion chamber. Reference dosimetry must be performed in a water phantom with dimensions of at least 30 cm x 30 cm x 30 cm.		
For **photon beams** with energies between ^{60}Co and 50 MeV		
$D_w^Q = M k_Q N_{D,w}^{^{60}Co}$	$M = M_{\text{raw}} P_{\text{ion}} P_{\text{pol}} P_{\text{elec}} P_{\text{TP}}$	$P_{\text{ion}} = \dfrac{1 - V_H / V_L}{M_{\text{raw}}^H / M_{\text{raw}}^L - V_H / V_L}$
$P_{\text{pol}} = \dfrac{\lvert M_{raw}^- - M_{raw}^+ \rvert}{2 M_{raw}^{+/-}}$	$P_{\text{TP}} = \dfrac{273.15 + T}{273.15 + 22}\,\dfrac{760}{P}$	
D_w^Q = dose to water for beam of quality, Q, determined by %dd(10)$_x$, M_{raw} = raw charge measurement, k_Q = quality factor, Correction factors: P_{ion} = recombination, P_{pol} = polarity, P_{elec} = electrometer, P_{TP} = temp/press.		
For **electron beams** with energies between 4 and 50 MeV		
$D_w^Q = M k_{ecal} k_{R_{50}}' P_{gr}^Q N_{D,w}^{^{60}Co}$	$R_{50} = \begin{cases} 1.029 I_{50} - 0.06 \text{ cm for } 2 \le I_{50} \le 10 \text{ cm} \\ 1.059 I_{50} - 0.37 \text{ cm for } 10 < I_{50} \end{cases}$	$d_{ref} = 0.6 R_{50} - 0.1\,\text{cm}$
$P_{gr}^Q = \dfrac{M(d + 0.5 r_{cav})}{M(d)}$	$E_{p,0} = 0.22 + 1.98 R_p + 0.0025 R_p^2$ [MeV] $\overline{E}_0 = 2.33 R_{50}$ [MeV]	$\overline{E}_d = \overline{E}_0 (1 - d / R_p)$
k_{ecal} = photon-electron quality conversion factor (chamber-dependent), $k_{R_{50}}'$ = electron beam quality conversion factor (chamber- and beam energy-dependent), R_{50} = depth of 50% dose wrt maximum, r_{cav} = cylindrical ion chamber radius I_{50} = depth of 50% ionization wrt maximum, after upstream shift of $0.5 r_{\text{cav}}$, d_{ref} = reference depth at which parallel plate to cylindrical chamber conversion factor valid P_{gr}^Q = electron gradient correction factor, R_p = practical electron range, $E_{p,0}$ = most probable energy at depth 0 cm, d = depth in phantom, $\overline{E}_d$ = mean electron energy at depth d		

4.2. Cross-calibration of a parallel plate chamber for electron dosimetry

- Determine depth of d_{ref} (cm)
- Measure charge collected with both the cylindrical and parallel plate chambers with point of measurement at d_{ref}.
- Calculate $\left(k_{\text{ecal}} N_{D,w}^{^{60}\text{Co}}\right) = \left(M k_{ecal} k_{R_{50}}' P_{gr}^Q N_{D,w}^{^{60}Co}\right)^{\text{cyl}} / \left(M k_{R_{50}}'\right)^{\text{pp}}$

4.3. Measuring electron PDD curves

- Measure ionization curve
- Shift curve upstream if necessary
- Correct for inverse square
- Determine average electron energy with depth
- Scale curve by stopping power ratios evaluated at each depth

5. Radiation Biology

5.1. Biological effective dose

$\text{BED} = Nd\left(1 + \frac{d}{\alpha/\beta}\right)$	N = number of fractions d = dose per fraction (Gy) α/β = linear/quadratic factors = 3 for typical tumors, 10 normal tissue
$d_2 = \frac{1}{2}\left[-\frac{\alpha}{\beta} + \sqrt{\left(\frac{\alpha}{\beta}\right)^2 + 4 \cdot \text{BED}_1 \frac{\alpha/\beta}{N_2}}\right]$	d_2 = dose/fraction for new number of fractions BED_1 = original BED N_2 = new number of fractions

6. Film

6.1. Optical density measurement

$\text{OD} = \log_{10} \frac{I_0}{I}$	OD = optical density I_0 = light without film I = light transmitted through film

6.2. Film dosimetry for electrons

- Kodak XV-2 film is linear up to 50 cGy
- When irradiating a film parallel to the central axis of the beam, it is very important to ensure that as much air as possible is removed from between the phantom slabs and that the film is flush with the surface of the phantom that the beam will be aimed at.

7. Brachytherapy

7.1. TG-43 dose calculation formalism

$D = S_k \Lambda \frac{G(r,\theta)}{G(r_0,\theta_0)} g(r) F(r,\theta)$	r = radius from center of mass of source [cm] θ = angle between source axis and calculation point [degrees] r_0 = 1 cm, θ_0 = 90°, S_k = air kerma strength [U] = [cGy cm^2 / hr] Λ = dose rate constant at (r_0, θ_0) [cGy/U], $G(r,\theta)$ = geometry factor [-], $g(r)$ = radial dose function [-], $F(r, \theta)$ = anisotropy function [-]
$G(r,\theta) = \begin{cases} 1/r^2 & \text{point source approximation} \\ \beta / Lr\sin\theta & \text{line source approximation} \end{cases}$	β = angle subtended by source [radians] L = length of source
$g(r) = \frac{\dot{D}(r,\theta_0)/G(r,\theta_0)}{\dot{D}(r_0,\theta_0)/G(r_0,\theta_0)}$	$F(r,\theta) = \frac{\dot{D}(r,\theta)/G(r,\theta)}{\dot{D}(r,\theta_0)/G(r,\theta_0)}$