GPU Based Convolution/Superposition Dose Calculation

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1) Johns Hopkins University
2) Elekta – Research

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ICCR 2000 Monte-Carlo Photon Benchmark

Water-Aluminum-Lung-Water Phantom
18MV 1.5 cm x 1.5 cm

Dose (cm²/g) vs Depth (cm)
JHU GPU Superposition

Water-Aluminum-Lung-Water Phantom
18MV 1.5 cm x 1.5 cm

- Monte Carlo
- Density Scaled Superposition
- Heterogeneity Compensated Superposition
Superposition/Convolution

\[ D(r) = \iiint \sum_E \frac{\mu_E}{\rho(r')} \Psi_{E,0}(r') e^{\int_s^{r'} - \mu_E(t) dt} \left( \int_r^{r'} \rho(t) dt, \omega(r, r') \right) K_E \left( \int_r^{r'} \rho(t) dt, \omega(r, r') \right) \int_{r'}^{\infty} \frac{1}{\| r - r' \|^2} d\rho \]

Figure courtesy of Michael Sharpe
Incident Fluence Generation

Primary Source

Secondary Scatter Source
- **CPU**
  - # Rays = incident fluence pixels
  - Serial processing of each ray
  - Requires post processing to remove sampling noise

- **GPU**
  - # Rays = TERMA voxels
  - Each voxel is its own process
  - Avoids write on write conflicts
  - Cache $T/\Psi$ per energy bin per voxel
**CPU**

- Pre-compute lookup table for attenuation to avoid exponential
- Fixed step length
- Spectral changes in lookup table

**GPU**

- Use GPU exponential
- Voxel boundary stepping
- **Attenuate each energy independently!**
• Lookup table based attenuation
• Assumes material above is same as current position

• Exponential evaluation
• Enables correct beam hardening through heterogeneous media
TERMA Accuracy

Stair stepping from lookup table resolution in CPU method
TERMA Performance

- Rearranged attenuation equation improves back-projection performance

\[ \int_{r_1}^{r_2} -\mu_E(t)\, dt = -\sum_{\alpha} \frac{\mu_{E,M}}{\rho_M} \int_{r_1}^{r_2} \alpha_M(t)\rho(t)\, dt \]

- ‘Attenuation’ volume caching for Intensity Modulation:

\[ A(r') = \sum_{E} W_E(r') \frac{\mu_E}{\rho(r')} e^{\int_{s'}^{-\mu_E(t)dt}} \]

\[ T(r') = \Psi_0(r') A(r') \]
Superposition

**CPU**
- Each TERMA voxel is a process
- Read once from TERMA
- Write many to Dose
  - Write on Write errors

**GPU or CPU**
- Each dose voxel is a separate process
- Read many from TERMA
- Write once to Dose
  - No Write on Write errors
  - Compute dose to single point
Superposition

**CPU**
- Not Tilted
  - Shared ray-tracing
  - Process single ray direction for all voxels

**GPU**
- Tilted
  - Each voxel is thread
  - Read many write once
Multi-resolution Superposition

Ideal

Standard

50% Azimuth Phase

Multi-resolution
Heterogeneity Compensation

Density Scaled

Heterogeneity Compensated

\[ D(r) = \iiint_E T_E(r') K_E(r'-r) dE dr' \]
ρ-eff for different locations of dose deposition
Effective Distance for Kernel Lookup

- Effective Distance (cm)
- Distance (cm)

Key:
- $\rho D$ Left->Right
- $D_{eff}$ Left->Right
- $D_{eff}$ Right->Left
- $\rho D$ Right->Left

Graph shows the effective distance changes with distance for left to right and right to left scenarios.
Kernel Hardening

- For each mono-energetic bin there is no hardening
  - Too many calculations
- Solution is to break spectrum up into parts and compute each part separately in parallel as full computation
- QUAD = 4 parts
- DUAL = 2 parts
Effect of Spectral Binning

Water-Bone-Lung-Water Phantom
18MV 1.5 cm x 1.5 cm

- Monte Carlo
- Traditional HCS
- Poly-Energetic HCS
- Dual-Energetic HCS
- Quad-Energetic HCS
- Multi-Energetic HCS

Dose vs Depth (cm)
Water-Bone-Lung-Water Phantom
18MV 3 cm x 3 cm
Water-Bone-Lung-Water Phantom
4MV 1.5 cm x 1.5 cm

Dose

Depth (cm)
Water-Half Bone-Water Phantom
4MV 1.5 cm x 1.5 cm
Water-Half Lung-Water Phantom
4MV 1.5 cm x 1.5 cm
Water-Bone-Lung-Water Phantom
24MV 1.5 cm x 1.5 cm
Water-Half Bone-Water Phantom
24MV 1.5 cm x 1.5 cm
Heterogeneity Compensation

Dose

C/S

HCS
Heterogeneity Compensation

Dose

C/S

HCS
## Performance

<table>
<thead>
<tr>
<th>Method</th>
<th>Grid^3</th>
<th># rays</th>
<th>spectra</th>
<th>tilt</th>
<th>kernel</th>
<th>t (ms)</th>
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</thead>
<tbody>
<tr>
<td>HCS</td>
<td>64</td>
<td>72</td>
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<td>tilted</td>
<td>CCK</td>
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</table>

10x10 field using the ICCR 18 MV spectrum
NVIDIA Tesla GPU C2050 with ECC enabled

- Can achieve interactive rates with $64^3$ grid
- $128^3$ grid under 4 seconds
### Modeling Tools

#### Kerem Parameters

- **Number of Azimuth Angles**: 8
- **Number of Zenith Angles**: 10
- **Maximum Zenith Angle (rad)**: 0.000000
- **Phase Ratio**: 0.25000
- **Phase Offset Ratio**: 0.50000
- **Hardening Depth (cm)**: 10.00000
- **Hardening Density (g/ccm)**: 1.00000
- **Number of MeV Bins**: 16
- **Use Energetic Weighting**: [ ]
- **Use Forward Axis Ray**: [ ]
- **Boost First Ring**: [ ]
- **Use Bin Look Ahead**: [ ]
- **Add Backward Axis Ray**: [ ]

#### Superposition Parameters

- **Angles Per CUDA Call**: 1
- **ms Between CUDA Calls**: 50
- **Energetic Type**: Dual
- **Use Tilted Kerems**: [ ]
- **Use CCK**: [ ]
- **Use MS Attenuation**: [ ]
- **Use Projected Fluence**: [ ]
- **Compute Profiles Only**: [ ]
- **Point Dose Mode**: [ ]

#### MeV Divisions

<table>
<thead>
<tr>
<th>MeV Divisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
</tr>
</tbody>
</table>

[Calculate button]
Summary - Algorithm

- Reworked the C/S method from first principles for the GPU
- Significant accuracy gains by avoiding performance approximations used in CPU methods
- Able to obtain Monte Carlo accuracy with improvements in the heterogeneity compensation and kernel hardening methods
- Obtain performance in the 100-300 ms range for a typical beam calculation
Robert Allan Jacques
(July 25, 1982 - June 18, 2013)

Thank you! Robert
Arc Therapy: Arc Superposition
## Arc Therapy

<table>
<thead>
<tr>
<th>Rays</th>
<th>Arc Superposition</th>
<th>Standard Superposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>6x12</td>
<td>4x8</td>
<td>10x8 6x12 4x8 4x8</td>
</tr>
<tr>
<td>Tilt</td>
<td>✓</td>
<td>✓ x ✓ ✓ ✓</td>
</tr>
<tr>
<td>Multi-resolution</td>
<td>x ✓ ✓ ✓</td>
<td>x x x ✓ ✓</td>
</tr>
</tbody>
</table>

### High Dose Region

<table>
<thead>
<tr>
<th># of (\angle)'s</th>
<th>(\Delta\angle)</th>
<th>(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∞ 0°</td>
<td>0.12% 0.28% 0.99% 1.12%</td>
<td>0.27% 0.12% 0.28% 0.99% 1.12%</td>
</tr>
<tr>
<td>360 0.5°</td>
<td>0.12% 0.28% 0.99% 1.12%</td>
<td>0.32% 0.18% 0.32% 1.01% 1.14%</td>
</tr>
<tr>
<td>180 1°</td>
<td>0.12% 0.28% 0.99% 1.12%</td>
<td>0.40% 0.29% 0.40% 1.05% 1.18%</td>
</tr>
<tr>
<td>36 5°</td>
<td>0.27% 0.31% 1.02% 1.16%</td>
<td>2.73% 2.66% 2.69% 3.07% 3.18%</td>
</tr>
<tr>
<td>18 10°</td>
<td>0.50% 0.44% 1.12% 1.25%</td>
<td>7.05% 6.95% 6.96% 7.21% 7.30%</td>
</tr>
<tr>
<td>9 20°</td>
<td>1.07% 0.92% 1.48% 1.60%</td>
<td>14.05% 13.90% 13.88% 14.07% 14.19%</td>
</tr>
</tbody>
</table>

### Gradient Region \(|\nabla D| > 0.3D\)

<table>
<thead>
<tr>
<th># of (\angle)'s</th>
<th>(\Delta\angle)</th>
<th>(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∞ 0°</td>
<td>0.14% 0.31% 0.88% 1.12%</td>
<td>0.47% 0.14% 0.31% 0.88% 1.12%</td>
</tr>
<tr>
<td>360 0.5°</td>
<td>0.16% 0.31% 0.88% 1.14%</td>
<td>1.30% 1.07% 1.17% 1.62% 1.81%</td>
</tr>
<tr>
<td>180 1°</td>
<td>0.18% 0.32% 0.89% 1.14%</td>
<td>2.16% 1.99% 2.05% 2.40% 2.56%</td>
</tr>
<tr>
<td>36 5°</td>
<td>0.63% 0.61% 1.07% 1.25%</td>
<td>7.63% 7.54% 7.53% 7.59% 7.69%</td>
</tr>
<tr>
<td>18 10°</td>
<td>1.24% 1.08% 1.42% 1.52%</td>
<td>11.86% 11.77% 11.74% 11.75% 11.83%</td>
</tr>
<tr>
<td>9 20°</td>
<td>2.31% 2.25% 2.30% 2.33%</td>
<td>17.08% 16.98% 16.91% 16.92% 17.01%</td>
</tr>
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