Robust optimization

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Introduction

Range and setup existed before IMPT:

➔ Practitioners found ways to deal with it, using the methods available at the time

- avoiding heterogeneities
- compensator smearing
- multiple patch-field combinations
- SFUD treatments with margins

This talk:

Address the new problems that come with IMPT

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Content

1. Motivation: Why margins don't always work

2. Review of robust optimization concepts

- probabilistic approach
- worst case approach

3. Implementations in commercial systems

4. Recent work

- Planning studies for different treatment sites
- Comparison between methods

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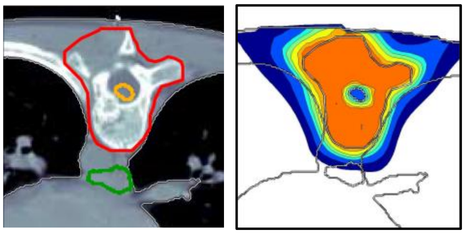
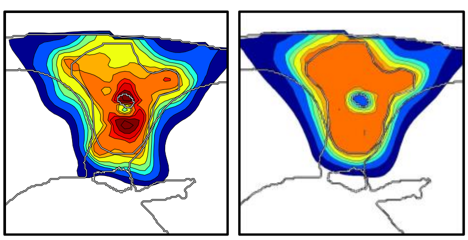
Motivation

Example: spinal metastasis



Motivation

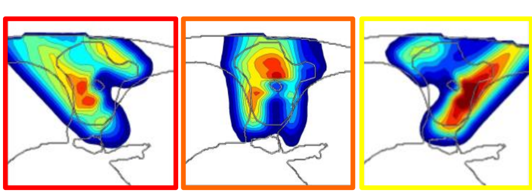
Robustness analysis:



5 mm range overshoot **nominal plan**

Motivation

Dose contributions of individual beams



- steep dose gradients are the problem
- misalignment of dose contributions leads to dose errors

➔ cannot be solved by margins

Solution approach

Robust optimization

Incorporate uncertainty directly into the IMPT treatment plan optimization problem

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Modeling uncertainty

Objective function: function of dose and parameters

$$f(\boldsymbol{d}, q)$$

Dose: linear function of beam intensities

$$d_i = \sum_j D_{ij} x_j$$

Three types of uncertainty

- uncertainty in fluence x_j (implementation error)
- uncertainty in parameters q (TCP/NTCP parameters)
- uncertainty in D_{ij} matrix (dosimetric errors)

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Robust optimization methods

The dose delivered to a voxel depends on a vector of uncertain parameters λ

$$d_i(\boldsymbol{x}, \lambda) = \sum_j D_{ij}(\lambda) x_j$$

λ parameterization of the uncertainty

- range error
- setup error

Discretization: assume K discrete error scenarios

$$D_{ij}^k = D_{ij}(\lambda^k) \qquad d_i^k = \sum_j D_{ij}^k x_j$$

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Robust optimization methods

Dose distribution depends on uncertain parameters

Goal

get a treatment plan that is good for all errors that may occur

Stochastic programming (probabilistic approach)	Mini-max optimization (robust optimization)
optimize the average plan quality	optimize for the worst case that can happen

Probabilistic method

p^k: Assign probability distribution to error scenarios λ_k:

Approach:

Optimize the expected value of the objective function:

$$\underset{x}{\text{minimize}} \quad \sum_k p^k f(\boldsymbol{d}^k)$$

(Unkelbach 2009, Med Phys)

incorporate all possible scenarios into the optimization with a weighting that corresponds to its probability of occurrence

➔ Multi-criteria interpretation

Mini-max method

No probability distribution assigned

Approach:

Minimize (over fluence x) the maximum (over scenarios k) of the objective function

$$\underset{x}{\text{minimize}} \quad \max_k \left[f(\boldsymbol{d}^k) \right]$$

(Fredriksson 2011, Med Phys)

find treatment plan that is as good as possible for the worst error scenario that is anticipated

Worst case variations

Textbook mini-max:

$$\underset{x}{\text{minimize}} \quad \max_k \left[\sum_s \sum_{i \in S} f_i(d_i^k) \right]$$

(Fredriksson 2011, Med Phys)

Objective-wise

$$\underset{x}{\text{minimize}} \quad \sum_s \max_k \left[\sum_{i \in S} f_i(d_i^k) \right]$$

(Chen 2012, PMB)

Voxel-wise:

$$\underset{x}{\text{minimize}} \quad \sum_s \sum_{i \in S} \max_k \left[f_i(d_i^k) \right]$$

(Pflugfelder 2008, PMB)
(Liu 2012, Med Phys)

Spinal metastasis example

3 Scenarios:

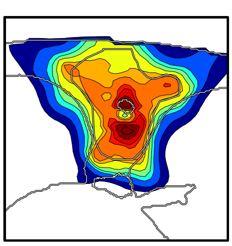
Scenario 1: Nominal scenario, $p^1 = 0.5$
Scenario 2: 5 mm range overshoot, $p^2 = 0.25$
Scenario 3: 5 mm range undershoot, $p^3 = 0.25$

Probabilistic approach:

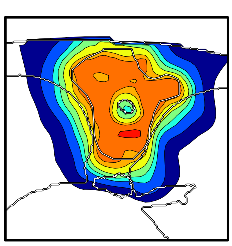
$$\underset{x}{\text{minimize}} \quad \sum_{k=1}^{3} p^k \left[\underbrace{w_T \sum_{i \in T} \left(d_i^k - D^{pres} \right)^2}_{\text{target}} + \underbrace{w_{SC} \sum_{i \in SC} \left(d_i^k \right)^2}_{\text{spinal cord}} \right] + \underbrace{f_{HT}(d^1)}_{\text{healthy tissue}}$$

Sensitivity analysis

5 mm range overshoot



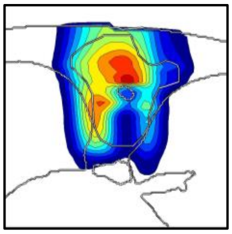
conventional plan



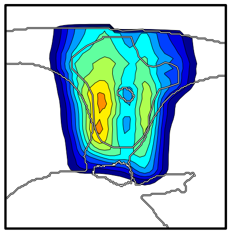
generated using robust optimization

Motivation

How is robustness achieved?



conventional plan



generated using robust optimization

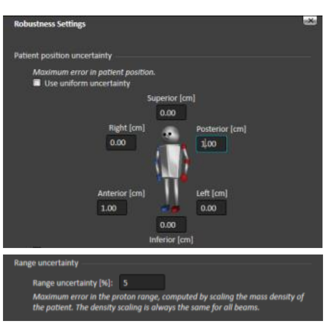
Commercial implementations

Status of Robust optimization in Commercial TPS

Multiple vendors work on Robust Optimization

Examples:
- **Pinnacle** (in development)
  (implements a probabilistic approach)
- **RayStation v4.5** (released in Europe)
  (implements a worst case approach)
- **Others**

Raystation 4.5

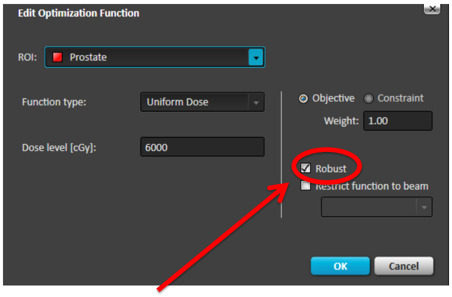


User interface

Setup uncertainty

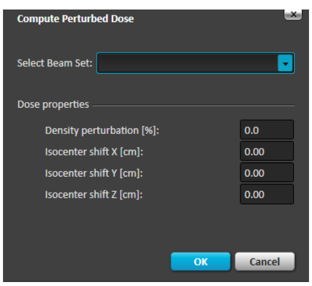
Range uncertainty

Raystation 4.5



User can robustify important objectives

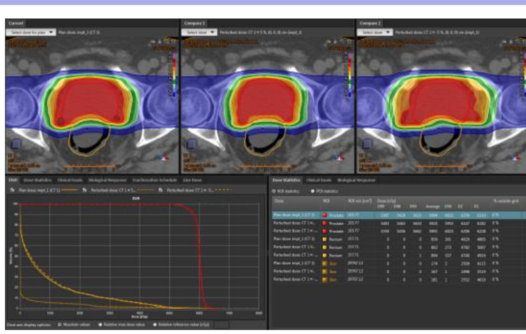
Raystation 4.5



Plan evaluation

define error scenario

Raystation 4.5



Current research topics

Robust optimization in practice

Early papers: Focus on methodology
(demonstration for extreme cases, e.g. spinal tumors)

Recent work: evaluation for different treatment sites
(when and how to use robust optimization)

Example: Liu et al, 2013, Med Phys.
"Effectiveness of robust optimization in intensity-modulated proton therapy planning for head and neck cancers"

Range/Setup errors as surrogate for anatomical variations
(e.g. evaluate plans on CTs acquired during treatment)

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Current research topics

Which robust method is best?

It will depend on the case, planning goals, and evaluation criteria

All methods are the same to first order
(expansion of the target, smoother gradient)

There are differences in the details
(but unclear how much that matters)

Mini-max only considers the worst case scenario
➔ plan might be suboptimal in easy scenarios

Probabilistic approach considers average plan quality
➔ plan might not be as robust in the worst case scenario

Recent paper: Fredriksson and Bokrantz, 2014, Med Phys.
"A critical evaluation of worst case optimization methods for robust IMPT planning"

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Current research topics

Modeling uncertainty

Common approach: discrete error scenarios

Recent paper: Bangert et al, 2013, PMB.
"Analytical probabilistic modeling for radiation therapy planning"

Analytical calculation of expected dose and variance

- assuming Gaussian range and setup errors
- pencil beam parameterization that can be convolved with Gaussians

➔ may allow for efficient implementation of stochastic programming in specific cases (e.g. quadratic objective function)

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Conclusions

In IMPT, safety margins don't always work

Robust optimization can overcome these limitations

First commercial implementations available

- allows practitioners to gain experience
- characterize robust optimization for specific sites

Research topics

- IMPT for Lung
- disease site specific uncertainty models, anatomical variations
- efficient implementations
- Comparison between different formulations

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