Multi-energy CT: Data and image analysis methods

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Outline

• Analysis methods for non-material specific imaging
  – Optimal weighting
  – Virtual monochromatic

• Analysis methods for material specific imaging
  – Basis material and PE-Compton decomposition, K-edge
  – Image space vs. projection space
  – Dimensionality analysis and noise considerations
  – Three material decomposition
  – Material classification

Multi-energy CT

• Non-material specific imaging
  – Virtual monochromatic
  – Optimal weighting

  Reduce artifacts and improve quantitative accuracy
  Improve SNR and dose efficiency

• Material specific imaging
  – Basis material decomposition
  – PE-Compton decomposition
  – K-edge imaging (photon-counting multi-energy)

  Expand CT clinical applications
  • Material quantification (e.g., iodine, bone, high-Z contrast agent)
  • Material classification (e.g., binuric/uric acid/non-uric acid)
Non-material specific imaging

Images at each spectrum or energy bin

Optimal Weighting

• Image space or projection space
• Linear or non-linear
• Optimal weighting depends on
  - Spectra or energy bins
  - Dose partitioning
  - Patient size
  - Material of interest

Yu et al, Med Phys 2009
Virtual Monochromatic Imaging

- Basis material or PE-Corpton map after decomposition
- Monochromatic image by synthesizing

20 keV, 40 keV, 60 keV, 120 keV

Optimal Monochromatic Energy

- Noise vs. Energy
- Iodine CNR vs. Energy

Yu et al, Med Phys 2011

Energy-domain noise reduction

- Each bin $I_b$
- Energy image $I_E$
- Composite image $I_C$
- Low Pass filter $FBL$
- Normalized weighting image $I_{wI} = \frac{I_b}{FBL}$
- HYPRI Image $I_{HYPRI} = I_b \cdot I_{wI}$

Leng, S. et al, Med Phys, 2011
Noise Reduction on Monochromatic images

Original vs. Filtered images at 50 keV and 60 keV, showing noise reduction.

Material-specific imaging

Material Decomposition Methods

<table>
<thead>
<tr>
<th>Factors</th>
<th>Options</th>
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</thead>
<tbody>
<tr>
<td>(\mu)-decomposition</td>
<td>PE-Compton or basis material</td>
</tr>
<tr>
<td>K-edge</td>
<td>with or without K-edge</td>
</tr>
<tr>
<td>Dimension of (\mu)-decomposition</td>
<td>2, 3, 4, ...</td>
</tr>
<tr>
<td>Data space</td>
<td>Projection or image space</td>
</tr>
<tr>
<td>Methods to solve the decomposition</td>
<td>Polynomial; Table look-up; Maximum likelihood estimation (MLE); Linearized MLE</td>
</tr>
<tr>
<td>Prior constraint on material composition</td>
<td>With or without prior assumptions on volume or mass. “3-material decomposition”</td>
</tr>
<tr>
<td>Task</td>
<td>Quantifying material density in a mixture or classifying materials</td>
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</table>
Decomposition of X-ray Linear Attenuation Coefficient

- General form (Alvarez and Macovski, 1976)
  \[ \mu(r, E) = \sum_{k=1}^{K} a_k(r) f_k(E) \]

- In diagnostic energy range and for low-Z material, \( \mu \) can be spanned by 2 independent basis functions
  - PE and Compton (Alvarez and Macovski, 1976)
  - Two basis materials (Lehman et al, 1981)

\[ \mu(r, E) = a_1(r) \cdot \mu_1(E) + a_2(r) \cdot \mu_2(E) \]

Including K-edge for high-Z material

- For high-Z material, the k-edge is well within the diagnostic energy range
  - e.g., Gadolinium (Z=64): 50.2 keV; Gold (Z=79): 80.7 keV

- Expand the 2-basis set to include the k-edge by adding the \( \mu \) of high-Z materials (one or more)

\[ \mu(r, E) = a_1(r) \cdot \frac{1}{\lambda(E)} + a_2(r) \cdot f_{\text{Compton}}(E) + \ldots \]

\[ \mu(r, E) = a_1(r) \cdot \mu_1(E) + a_2(r) \cdot \mu_2(E) + \ldots \]

Measurement Modeling

At each spectrum or energy bin \( S_j(E) (j = 1, ..., N) \):

\[ I_j = \int dE \cdot \Omega_j(E) \cdot e^{-\int dt \mu(r)} = \int dE \cdot \Omega_j(E) \cdot e^{-\int dl a(k)(E) \mu(E)} \]

where \[ A_k(r) = \int a_k(r) dl, k = 1, ..., K \]

Energy integrating:

\[ \Omega_k(E) = \int S_j(E) \cdot D(E) \]

Energy resolving:

\[ \Omega_k(E) = \int S_j(E) \cdot D(E) \]

\[ \Omega_k(E) = 1 \text{ in } j\text{th energy bin, } 0 \text{ elsewhere} \]

Neglecting system non-idealities: charge sharing, pulse pileup, etc.
**Basis material decomposition**

Original subject $\mu(r, E)$

Measured signal at each spectrum or energy bin (before log):

$$\tilde{I}_j = \int dE \cdot \Omega_j(E) \cdot \left[ e^{-A_1 \alpha E} + A_2 \alpha \right], \quad j = 1, \ldots, N$$

Basis material maps in sinogram:

$$A_k(E) = \int \mu_k(r) \, dr$$

Basis material density map after reconstruction:

$$\hat{\rho}_k(r) = \text{Recon}(A_k(r))$$

**PE-Compton Decomposition**

Measured signal:

$$I_j = \int dE \cdot \Omega_j(E) \cdot e^{-A_1 \alpha E - A_2 \alpha E} = \left[ e^{-A_1 \alpha E} + A_2 \alpha \right], \quad j = 1, \ldots, N$$

PE and Compton maps in sinogram:

$$A_k(E) = \int \mu_k(r) \, dr$$

PE and Compton maps:

$$\hat{a}_k(r) = \text{Recon}(A_k(r)), \quad k = \text{PE}, \text{Compton}$$

Solve for effective $\rho, Z$:

$$\begin{align*}
A_{\text{PE}} &= \frac{Z}{A} \cdot \rho \\
A_{\text{Compton}} &= \frac{Z}{A} \cdot \rho
\end{align*}$$

Electron density $\rho_e$:

$$\rho_e = N \cdot \frac{Z}{A} \cdot \rho$$

**Methods to Solve Decomposition**

- Polynomial + iterative solver
- Table look-up procedure
- Maximum likelihood estimation (MLE)
- Linearized MLE*
  - Non-iterative estimator, faster than MLE.
  - Calibration-based and does not require spectrum and detector response information.
  - Low bias and approximately achieves CRLB.

*Álvarez, Med Phys 2011
Image-based Material Decomposition

- **Pros:**
  - Easy to implement
  - Computationally fast
  - No raw data mismatch problem (dual-source or fast kV switching)

- **Cons:**
  - Difficult to incorporate detailed data or system model (careful calibration is needed)
  - May suffer from beam hardening effect (in practice might not matter)

Dimensionality analysis for low-Z material

- **Two basis functions**
  - Tradeoff among accuracy, noise, and dose
  - Limited by dual-energy measurements historically

- Can multi-energy bin measurements allow >2 material decomposition?
  - Depends on intrinsic dimensionality of $\mu$

- Intrinsic dimensionality of $\mu$ (Z=1-20) could be as high as 4

Decomposition dimension and noise

- Cramér-Rao lower bound (CRLB) to quantify the increase in noise with dimensionality (Alvarez, Med Phys 2013)

<table>
<thead>
<tr>
<th>Two basis material decomposition</th>
<th>3-basis: Adipose as a third material</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>$10^3$</td>
</tr>
<tr>
<td>$10^6$</td>
<td>$10^6$</td>
</tr>
<tr>
<td>$10^9$</td>
<td>$10^9$</td>
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</tbody>
</table>

Bornefalk, Med Phys 2012
Material-dependent noise amplification

- Whether to use a higher dimension depends on if the increased noise (and dose) acceptable clinically

Bone: $1.4 \times 10^3$; Soft tissue: $2.7 \times 10^4$

Variance amplification

Bone: 1.03; Soft tissue: 7.4

Three material decomposition in dual-energy

- 3-material mixture:
  \[ \mu = \alpha_1 \cdot \mu_1 + \alpha_2 \cdot \mu_2 + \alpha_3 \cdot \mu_3 \]
  - 3 unknowns
  - 2 measurements in dual-energy cannot solve the problem

A third independent condition is needed.

Assumptions on 3-material mixture

- First order approximation:
  \[ \rho_w (\rho_1, \rho_2) = \rho_{w_0} + \rho_1 \frac{\partial \rho_w}{\partial \rho_1} + \rho_2 \frac{\partial \rho_w}{\partial \rho_2} \]

- Volume conservation (incompressible)
  \[ \alpha_1 + \alpha_2 + \alpha_3 = 1 \]
  \[ \alpha_i = \frac{\rho_i}{\rho_{w_0}} \text{ volume fraction} \]

- Mass conservation (need to calculate effective density at first)

Kelcz et al, Med Phys 1978
Yu et al, SPIE 2009
Liu et al, Med Phys 2009
Three material decomposition in dual-energy

- Material decomposition in multi-energy CT
  - Is it necessary to include an additional constraint in ME?
    - Compare DE and ME w/ & w/o prior assumption on volume conservation
    - Figure of merit: CRLB of standard deviation in iodine
  - Prior assumption on volume conservation improves noise, even for multi-energy!

Bornefalk & Persson, IEEE-TMI, 2014

Dose Consideration in Virtual Non-contrast Imaging

- Dose saving by VNC should be quantified as the dose that is required to generate the image quality equivalent to VNC.
- Image quality (dose saving) of VNC depends on
  - Dose partitioning
  - Spectra separation
  - Patient size
Impact from Spectrum Separation

Noise Reduction in Material Decomposition

Material Classification

• Iterative image-domain decomposition*
  - Formulated in the form of least-square estimation with smoothness regularization
  - Include a weight in the least-square term to incorporate correlation of DE images


Material Classification

• Common clinical questions related to material classification
  - Uric acid vs. non-uric stones
  - Bone vs. iodine
  - Uric acid crystals vs. calcium-containing crystals

\[ \text{DE ratio} = \frac{C_{\text{LH}}}{C_{\text{TH}}} = f(Z) \]

Independent of concentration
Summary

- Dimensionality of material decomposition could be higher than 2 for low-Z materials
- Multi-energy CT with energy bins >2 provides solution to higher dimension, including k-edge imaging
- Prior constraints on material composition are useful, even for multi-energy CT with energy bins >2
- Noise amplification is one of the main limits for reliable quantification of multi-material mixture.

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http://www.mayo.edu/research/centers-programs/ct-clinical-innovation-center