Elasticity Imaging: Principles

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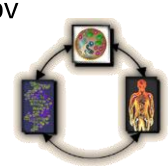
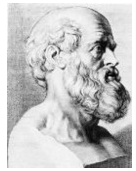


Elasticity Imaging – Goal

Remote
non-invasive (or adjunct to invasive)
imaging (or sensing) of
mechanical properties of
tissue for
clinical applications

Elasticity

Changes in tissue elasticity are related to pathological changes



Hippocrates

... Such swellings as are soft, free from pain, and yield to the finger, ... and are less dangerous than the others.
... then, as are painful, hard, and large, indicate danger of speedy death; but such as are soft, free of pain, and yield when pressed with the finger, are more chronic than these.

THE BOOK OF PROGNOSTICS, Hippocrates, 400 B.C.

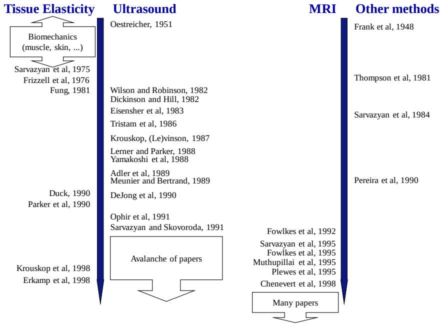
It is the business of the physician to know, in the first place, things similar and things dissimilar; ... which are to be seen, touched, and heard; which are to be perceived in the sight, and the touch, and the hearing, ... which are to be known by all the means we know other things.

ON THE SURGERY, Hippocrates, 400 B.C.

Hippocrates, 400 B.C.

Elasticity Imaging – Glance at History

Hippocrates, circa 460-377 B.C.



Mechanical Properties of Tissue

Elasticity (e.g., bulk and shear moduli)

Viscosity (e.g., bulk and shear viscosities)

Nonlinearity (e.g., strain hardening)

Other (e.g., anisotropy, pseudoelasticity)

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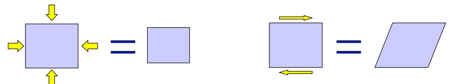
Elasticity

Relations between various elastic constants

Constant	Common Pair		
	λ, μ	E, ν	K, G
λ	λ	$\frac{\nu E}{(1+\nu)(1-2\nu)}$	$K-\frac{2}{3}G$
μ	μ	$\frac{E}{2(1+\nu)}$	G
E	$\frac{\mu(3\lambda+2\mu)}{(\lambda+\mu)}$	E	$\frac{9KG}{3K+G}$
ν	$\frac{\lambda}{2(\lambda+\mu)}=0.5-\frac{\mu}{2(\lambda+\mu)}$	ν	$\frac{3K-2G}{6K+2G}$
K	$\lambda+\frac{2}{3}\mu$	$\frac{E}{3(1-2\nu)}$	K
G	μ	$\frac{E}{2(1+\nu)}$	G

Which Elastic Moduli?



Poisson's ratio (ν)
Bulk modulus (K)

Shear modulus (μ=G)
Young's modulus (E)

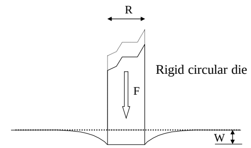
- Most soft tissues are incompressible, i.e., deformation produces <u>no</u> volume change

$$c_t=\sqrt{\frac{G}{\rho}}<c_l=\sqrt{\frac{K+\frac{2}{3}G}{\rho}} \rightarrow G<<K \rightarrow \frac{G}{K}\rightarrow 0 \rightarrow \nu\rightarrow\frac{1}{2}$$

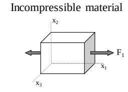
- In addition, due to tissue incompressibility, Young's modulus = 3 · shear modulus

$$E=3\mu$$

Human sense of touch – what do we feel?



Semi-infinite elastic medium
K – bulk modulus
μ – <u>shear</u> modulus



$$F_1=3\mu\varepsilon_0$$

$$E=3\mu$$

$$F=8RWG\left(1+\frac{\frac{G}{K}}{1+\frac{G}{3K}}\right)^{-1}\xrightarrow[\frac{G}{K}\rightarrow 0]{}8RW\mu$$

Static deformation of (nearly) incompressible material is primarily determined by shear or Young's modulus

Sarvazyan et al, 1995

Breast Tissue Elasticity and Pathology

Skovoroda et al., 1995, *Biophysics*, 40(6):1359-1364.

Breast Tissue Type	Normal gland	Infiltrative ductal cancer with alveolar tissue predominating.	Fibroadenomas of glandular origin	Infiltrative ductal cancer with fibrous tissue predominating.	Ductal fibroadenoma
Young's Modulus (kPa)	**0.5-1.5**	**1.0-1.5**	**1.5-2.5**	**2.0-3.0**	**8.0-12.0**

Krouskop et al., 1998, *Ultrasonic Imaging*, 20:260-274.

Breast Tissue Type	**Tissue Elastic Modulus (kPa)**					
	5% precompression			**20% precompression**		
	Loading frequency (Hz)			**Loading frequency (Hz)**		
	0.1	1.0	4.0	0.1	1.0	4.0
Normal fat (n = 8)	18 ± 7	19 ± 7	22 ± 12	20 ± 8	20 ± 6	24 ± 6
Normal glandular tissue (n = 31)	28 ± 14	33 ± 11	35 ± 14	48 ± 15	57 ± 19	66 ± 17
Fibrous tissue (n = 18)	96 ± 34	107 ± 31	116 ± 28	218 ± 87	232 ± 60	244 ± 85
Ductal carcinoma *in situ* (n = 23)	22 ± 8	25 ± 4	26 ± 5	291 ± 67	301 ± 58	307 ± 78
Invasive and infiltrating ductal carcinoma (n = 32)	106 32	93 ± 33	112 ± 43	558 ± 180	490 ± 112	460 ± 178

Breast Tissue Elasticity and Pathology

Wellman et al., 1999, *Harvard BioRobotics Laboratory Technical Report.*

Tissue Type	Elastic Modulus at Strain 0.01	SD	Elastic Modulus at Strain 0.05	SD	Elastic Modulus at Strain 0.10	SD	Elastic Modulus at Strain 0.15	SD
Fat	4.8	2.5	6.6	7	10.4	7.9	17.4	8.4
Gland	17.5	8.6	33	12.0	88.1	66.7	271.8	167.7
Phyllodes Tumor	56.6	0.0	90.8	8.6	164.3	0.0	297.7	0.0
Papilloma	22.2	5.8	54.4	19.7	169.7	80.6	537.8	209.1
Lobular Carcinoma	34.7	0.0	78.9	0.0	221.8	0.0	628.4	0.0
Fibroadenoma	45.5	20.1	100.5	39.6	288.4	110.9	889.2	205
Infiltrating Ductal Carcinoma	47.1	19.8	115.7	42.9	384.5	126.9	1366.5	348.2
Ductal Carcinoma in Situ	71.2	0.0	188.7	0.0	638.7	0.0	2162.1	0.0

Samani et al., 2003, *Physics in Medicine and Biology*, 48:2183-2198

	Adipose tissue	Fibroglandular tissue	High grade ductal carcinoma
s (N m^{-1})	13.8	12.75	87.2
E (kPa)	1.9	1.8	12.0

Elastic Properties of Prostate Gland

Aglyamov S.R. and Skovoroda A.R., 2000, *Biophysics, 2000, 45(6).*

Prostate gland

Tissue		Method		Value	Reference
Canine prostate gland		Uniaxial compression		3–5	Parker et al. 1990
Human prostate gland		Uniaxial compression		1–4	Parker et al. 1990
Human prostate gland *in vitro*	in normalcy	Indentation		1–1.3	Skovoroda et al. 1995
	1*			1.3–1.5	
	2*			1.5–2.0	
	3*			2.3–3.0	
	4*			3.4–3.7	
	5*			3.7–4.5	
	central zone	NMR measurement of shear wave velocity	100 Hz	~18	Dresner et al. 1999
			300 Hz	~60	
			500 Hz	~130	
	peripheral zone		100 Hz	~8	
			300 Hz	~24	
			500 Hz	~65	
	front part in normalcy	Impedance measurements (frequency 0.1–4 Hz)		55 ± 14	Krouskop et al. 1998
	hinder part in normalcy			62 ± 19	
	benign hyperplasia			38 ± 8	
	cancer tumor			96 ± 19	

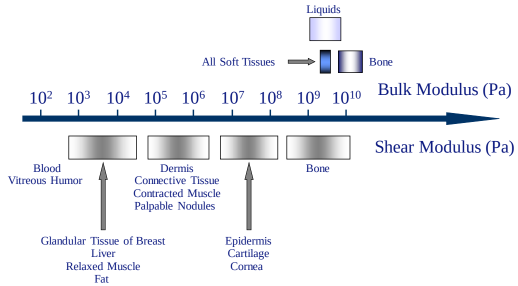
1* Adipose tissue.
2* Ulcerous nodes.
3* Cystically degenerated nodes.
4* Connective tissue membranes of nodes.
5* Hard nodes.

Elastic Properties of Arteries

Sarvazyan A.P., 2001," *In: Handbook of Elastic Properties of Solids, Liquids and Gases.*

Type of soft tissue	E, kPa	Comments	Reference
Artery			
human, in vitro			
Thoracic aorta	300-940	For normal physiological conditions of longitudinal tension and distending blood pressure, below 200 mm Hg.	McDonald, 1974
Abdominal aorta	980-1,420		
Iliac artery	1,100-3,500		
Femoral artery	1,230-5,500		
Ascending aorta	183-582	M-mode echocardiography in different age groups.	Alessandri et al., 1995
Coronary artery	1,060-4,110	Represents values for various ages (0-80 y.o.) and atherosclerosis conditions in right and left arteries.	Ozolanta et al., 1998
rat, in vitro			
mesenteric small arteries	100-1,500	Equilibrated at intraluminal pressure of 45 mmHg, media thickness and lumen diameter were measured in the applied 3 -140 mmHg intraluminal pressure range.	Intengan et al., 1998
aortic wall	700-1,600	Range includes measurements for normal vs spontaneously hypertensive animals.	Marque et al., 1999
porcine, in vitro			
thoracic aorta			
intima-medial layer	43		
adventitia layer	4.7		
ascending aorta			
intima-medial layer	447	Bending experiments were used to impose various strains on different layers of tested blood vessels.	Fung, 1993
adventitia layer	112		
descending aorta			
intima-medial layer	248		
adventitia layer	69		

Contrast in Elasticity Imaging



Sarvazyan et al, 1995

Contrast Mechanism in Other Imaging Modalities

Computerized Tomography:
spatial distribution of the absorption (density)

MRI:
proton spin density and relaxation time constants

Ultrasound Imaging:
variation in acoustical impedance
(bulk modulus and density)

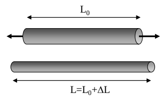
Optical Imaging:
refraction index, absorption/scattering

Theory of Elasticity

- Strain
- Stress
- Constitutive relationships
- Equations of equilibrium
- Equations of motion (wave equation)
- References

Strain

1-D deformation



<u>Lagrangian</u> strain:

$$\varepsilon = \frac{1}{2}\left(\frac{L^2 - L_0^2}{L_0^2}\right)$$

Other strains:

$$\frac{L-L_0}{L_0};\frac{L-L_0}{L};\frac{1}{2}\left(\frac{L^2-L_0^2}{L^2}\right)$$

Small deformations:

$$\frac{\Delta L}{L_0} << 1 \rightarrow \frac{1}{2}\left(\frac{L^2-L_0^2}{L_0^2}\right) \approx \frac{L-L_0}{L_0} \approx \frac{1}{2}\left(\frac{L^2-L_0^2}{L^2}\right)$$

Strain Tensor

$$\begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix}$$

General:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right)$$

Symmetry:

$$\varepsilon_{ij} = \varepsilon_{ji}$$

Normal strain components:

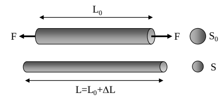
$$\varepsilon_{11};\varepsilon_{22};\varepsilon_{33}$$

Shear strain components:

$$\varepsilon_{12} = \varepsilon_{21};\varepsilon_{23} = \varepsilon_{32};\varepsilon_{31} = \varepsilon_{13}$$

Stress

1-D example



<u>Cauchy</u> stress:

$$\sigma = \frac{F}{S}$$

Other stresses:

$$\frac{F}{S_0};\frac{F}{S_0}\frac{L_0}{L}$$

Small deformations:

$$\frac{\Delta L}{L_0} << 1 \rightarrow \frac{F}{S} \approx \frac{F}{S_0} \approx \frac{F}{S_0}\frac{L_0}{L}$$

Stress Tensor

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

Symmetry:

$$\sigma_{ij} = \sigma_{ji}$$

Normal and Shear Stresses

Normal stress components:

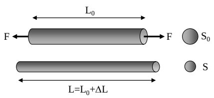
$$\sigma_{11};\sigma_{22};\sigma_{33}$$

Shear stress components:

$$\sigma_{12} = \sigma_{21};\sigma_{23} = \sigma_{32};\sigma_{31} = \sigma_{13}$$

Constitutive Relations

Uniaxial Deformation



$\sigma_{11} = E\varepsilon_{11}$

E – Young's modulus

$\nu = -\frac{\varepsilon_{22}}{\varepsilon_{11}}$

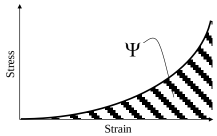
ν – Poisson's ratio

This is not a definition of a Poisson's ratio

General Concept

$$\sigma_{ij} = \frac{\partial \Psi}{\partial \varepsilon_{ij}}$$

Ψ – strain energy function or potential



Examples:

Rubber – Mooney-Rivlin function
Foam – Blatz-K(a)o potential
Tissue – ??? (Fung 1981)

Constitutive Relations

Generalized Hooke's Law

Strain energy (linear, anisotropic material):

$$\Psi = \Psi_0 + \frac{1}{2} C_{ijmn} \varepsilon_{ij} \varepsilon_{mn}$$

Stress-strain relations:

$$\sigma_{ij} = C_{ijmn} \varepsilon_{mn}$$

81 constants → $C_{ijmn} = C_{jimn}$, $C_{ijmn} = C_{ijnm}$, $C_{ijmn} = C_{mnij}$ → 21 constants

Symmetry:

one plane (monoclinic) – 13
two orthogonal planes (orthotropic) – 9
one axis (transversely isotropic) – 5
(cubic) – 3

Hooke's Law

Strain energy (linear, isotropic material):

$$\Psi = \Psi_0 + \frac{\lambda}{2} \varepsilon_{ii}^2 + \mu \varepsilon_{ij}^2$$

Stress-strain relations:

$$\sigma_{ij} = \lambda \varepsilon_{ii} \delta_{ij} + 2\mu \varepsilon_{ij}$$

Two elastic constants (λ and μ, for example) are needed to describe linear isotropic material

Nonlinearity

Strain energy:

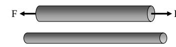
$$\Psi = \Psi(I_1, I_2, I_3)$$

Stress-strain relations:

Nonlinear

Equations of Equilibrium

1-D example



$$\sigma_{11}(x) = const \rightarrow \frac{\partial \sigma_{11}}{\partial x_1} = 0$$

Remarks:

- Generally, both elastic moduli are functions of coordinate: $\lambda=\lambda(x_1,x_2,x_3)$ and $\mu=\mu(x_1,x_2,x_3)$
- In forward problem formulation, there are 15 equations and 15 unknowns (σ_{ij}, ε_{ij}, u_i)
- The existence and uniqueness of the solution can be demonstrated

General

Equilibrium (Newton's 2nd law)

$$\sum_i \frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0$$

f_i – body (volumetric) forces, e.g., gravity

⇑

Constitutive (stress-strain) relations:

$$\sigma_{ij} = \frac{\partial \Psi}{\partial \varepsilon_{ij}} \left(\text{e.g., } \sigma_{ij} = \lambda \varepsilon_{ii} \delta_{ij} + 2\mu \varepsilon_{ij}\right)$$

⇑

Strain-displacement relations:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right)$$

Derive 1-D equation of equilibrium for E=const

Equations of Equilibrium

Small deformations of linear, isotropic (i.e., Hookean) material

$$\frac{\partial}{\partial x_1}(\lambda(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}))+\frac{\partial}{\partial x_1}(2\mu\frac{\partial u_1}{\partial x_1})+\frac{\partial}{\partial x_2}(\mu(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}))+\frac{\partial}{\partial x_3}(\mu(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}))+f_1=0$$

$$\frac{\partial}{\partial x_2}(\lambda(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}))+\frac{\partial}{\partial x_1}(\mu(\frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}))+\frac{\partial}{\partial x_2}(2\mu\frac{\partial u_2}{\partial x_2})+\frac{\partial}{\partial x_3}(\mu(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}))+f_2=0$$

$$\frac{\partial}{\partial x_3}(\lambda(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}))+\frac{\partial}{\partial x_1}(\mu(\frac{\partial u_3}{\partial x_1}+\frac{\partial u_1}{\partial x_3}))+\frac{\partial}{\partial x_2}(\mu(\frac{\partial u_3}{\partial x_2}+\frac{\partial u_2}{\partial x_3}))+\frac{\partial}{\partial x_3}(2\mu\frac{\partial u_3}{\partial x_3})+f_3=0$$

Boundary (<u>force</u> and/or <u>displacement</u>) conditions

$$\left(\sum_j \sigma_{ij} n_j - F_i\right)\delta(u_i - u_i^0) = 0$$

n=(n_1,n_2,n_3) – unit normal vector at the surface

u_i^0 – surface displacement components

Free surface (i.e., F_i=0): $\sum_j \sigma_{ij} n_j = 0 \Rightarrow \begin{matrix} \sigma_{11}n_1+\sigma_{12}n_2+\sigma_{13}n_3=0 \\ \sigma_{21}n_1+\sigma_{22}n_2+\sigma_{23}n_3=0 \\ \sigma_{31}n_1+\sigma_{32}n_2+\sigma_{33}n_3=0 \end{matrix}$

Mixed boundary conditions:

$$\begin{matrix} \sigma_{11}n_1+\sigma_{12}n_2+\sigma_{13}n_3=F_1 \\ u_2=0 \\ u_3=0 \end{matrix}$$

Displaced surface: $\delta(u_i-u_i^0)=0 \Rightarrow \begin{matrix} u_1=u_1^0 \\ u_2=u_2^0 \\ u_3=u_3^0 \end{matrix}$

To describe elastic properties of linear isotropic material, how many moduli of elasticity are required:

21%	1. One modulus
7%	2. Two moduli
29%	3. Three moduli
21%	4. Four moduli
21%	5. Five moduli

To describe elastic properties of linear isotropic material, how many moduli of elasticity are required:

Generalized Hooke's Law

Strain energy (linear, anisotropic material):

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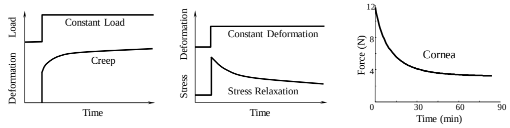
Other (e.g., anisotropy, pseudoelasticity)

Viscosity

- Shear viscosity: shear waves
- Bulk viscosity: longitudinal ultrasound waves
- Viscoelastic models:
  Maxwell, Voigt, Kelvin, KVFD, etc.

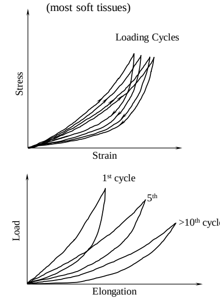
Creep Stress Relaxation



- Is there a characteristic time?

Viscosity → Equations of Motion

- Hysteresis loops:
  independent of the rate of loading (most soft tissues)



Equilibrium (Newton's 2nd law)

$$\sum_i \frac{\partial \sigma_{ij}}{\partial x_j} + f_i = \rho \frac{\partial^2 u_i}{\partial t^2}$$

Constitutive (stress-strain) relations:

$$\sigma_{ij} = \lambda \varepsilon_{ii} \delta_{ij} + 2\mu \varepsilon_{ij} + \xi \frac{\partial \varepsilon_{ii}}{\partial t} \delta_{ij} + 2\eta \frac{\partial \varepsilon_{ij}}{\partial t}$$

η – shear viscosity
ξ – bulk viscosity

- Bulk and shear viscosities are introduced
- Four (4) parameters are needed to describe mechanical properties of the viscoelastic material
- There are various models to describe constitutive relations, e.g., Voight model, Maxwell model, Kelvin model, etc.

To describe viscoelastic properties of linear isotropic material, how many parameters are required:

0%	1.	One parameter
23%	2.	Two parameters
18%	3.	Three parameters
0%	4.	Four parameters
18%	5.	Five parameters

To describe viscoelastic properties of linear isotropic material, how many parameters are required:

Equilibrium (Newton's 2nd law)

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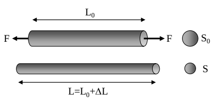
Other (e.g., anisotropy, pseudoelasticity)

Nonlinearity → Constitutive Relations

Uniaxial Deformation



$$\sigma_{11} = E\varepsilon_{11}$$

E – Young's modulus

$$\nu = -\frac{\varepsilon_{22}}{\varepsilon_{11}}$$

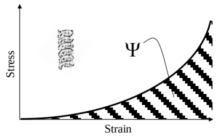
ν – Poisson's ratio

This is <u>not</u> a definition of a Poisson's ratio

General Concept

$$\sigma_{ij} = \frac{\partial \Psi}{\partial \varepsilon_{ij}}$$

Ψ – strain energy function or potential



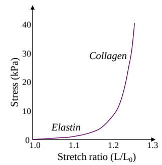
Examples:

Rubber – Mooney-Rivlin function
Foam – Blatz-K(a)o potential
Tissue – ??? (Fung 1981)

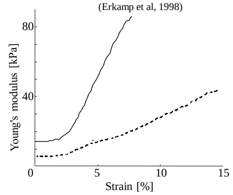
Strain hardening

- Most soft tissues exhibit strain hardening
- Tissues of organs with primary "mechanical" functions (muscle, skin, tendon, etc.)
- Other tissues (kidney, brain, blood clot, etc.)
- Strain hardening is more common although strain softening is possible

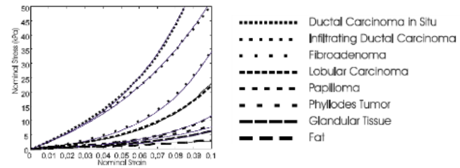
Vessel Composition (Fung, 1988): muscle (soft), elastin (soft), collagen (hard)



Strain hardening in kidney samples (Erkamp et al, 1998)



Strain hardening



		Ratio to Fat at		
Tissue Type	Strain = 0.01	Strain = 0.05	Strain = 0.10	Strain = 0.15
Gland	4	5	8	16
Phyllodes Tumor	12	14	16	17
Papilloma	5	8	16	31
Lobular Carcinoma	7	12	21	36
Fibroadenoma	9	15	28	51
Infiltrating Ductal Carcinoma	10	18	37	79
Ductal Carcinoma in Situ	15	29	61	124

Table 4: The ratio of elastic modulus of each tissue type to fat at 4 different strain levels.

Wellman et al., 1999

Strain hardening

This and other graphs as well as other literature data suggest that tissue strain hardening (or nonlinearity in stress-strain relations) can be used for tissue analysis including composition, differentiation, etc.

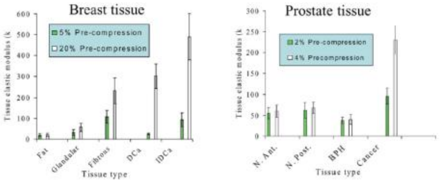


Figure 1. Tissue elastic moduli obtained from normal and abnormal breast and prostate tissues [26]. DCa = ductal carcinoma, IDCa = intraductal carcinoma, N. Ant. = anterior portion of the normal prostate, N. Post. = posterior portion of the normal prostate, BPH = benign prostatic hypertrophy.

Krouskop et al, 1998
Ophir et al., 2001

Mechanical Properties of Tissue

Elasticity (e.g., bulk and shear moduli)

Viscosity (e.g., bulk and shear viscosities)

Nonlinearity (e.g., strain hardening)

Other (e.g., anisotropy, pseudoelasticity)

Anisotropy → Constitutive Relations

Generalized Hooke's Law

Strain energy (linear, anisotropic material):

$$\Psi = \Psi_0 + \frac{1}{2} C_{ijmn} \varepsilon_{ij} \varepsilon_{mn}$$

Stress-strain relations:

$$\sigma_{ij} = C_{ijmn} \varepsilon_{mn}$$

81 constants → $C_{ijmn} = C_{jimn}$, $C_{ijmn} = C_{ijnm}$, $C_{ijmn} = C_{mnij}$ → 21 constants

Symmetry:

one plane (monoclinic) – 13
two orthogonal planes (orthotropic) – 9
one axis (transversely isotropic) – 5
(cubic) – 3

Hooke's Law

Strain energy (linear, isotropic material):

$$\Psi = \Psi_0 + \frac{\lambda}{2} \varepsilon_{ii}^2 + \mu \varepsilon_{ij}^2$$

Stress-strain relations:

$$\sigma_{ij} = \lambda \varepsilon_{ii} \delta_{ij} + 2\mu \varepsilon_{ij}$$

Two elastic constants (λ and μ, for example) are needed to describe linear isotropic material

Nonlinearity

Strain energy:

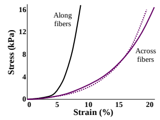
$$\Psi = \Psi(I_1, I_2, I_3)$$

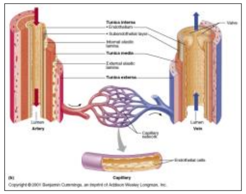
Stress-strain relations:

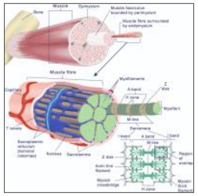
Nonlinear

Anisotropy

- Arterial wall: orthotropic material – 9 constants two orthogonal planes of symmetry
- Muscle: transversely isotropic – 5 constants axis of symmetry
- Isotropic – 2 constants







To describe elastic properties of linear anisotropic material, how many parameters are required:

8%	1.	One parameter
4%	2.	Two parameters
4%	3.	Nine parameters
8%	4.	Twenty one parameters
21%	5.	Infinite set of parameters

To describe elastic properties of linear anisotropic material, how many parameters are required

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Theory of Elasticity: References

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- Other text books and archival publications

Elasticity Imaging – Approaches

Static (strain-based, or reconstructive)
imaging internal motion under static deformation

Dynamic (wave-based)
imaging shear wave propagation

Mechanical (stress-based, also reconstructive)
measuring tissue response at the surface

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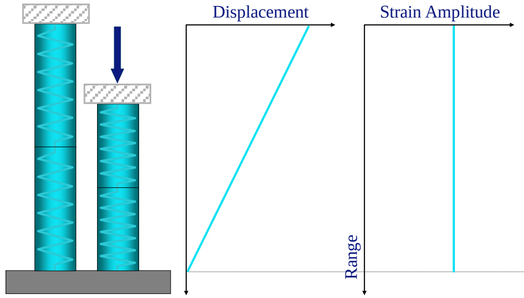
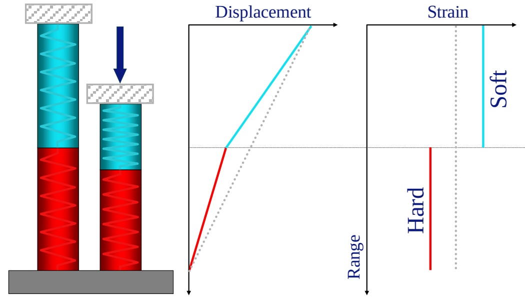
How … Static Elasticity Imaging?



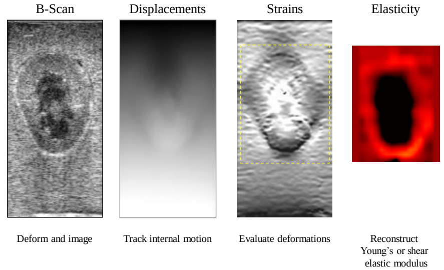
How … Static Elasticity Imaging?



How … Static Elasticity Imaging?

- Capture data during <u>deformation</u>
- Estimate <u>displacement</u> vector (3-D)
- Compute <u>strain</u> tensor (3-D)
- Reconstruct <u>mechanical</u> properties

Static or Reconstructive Ultrasound Elasticity Imaging



Elasticity Reconstruction: back to Equations of Equilibrium

Small deformations of linear, isotropic (i.e., Hookean) material

$$\frac{\partial}{\partial x_1}(\lambda(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}))+\frac{\partial}{\partial x_1}(2\mu\frac{\partial u_1}{\partial x_1})+\frac{\partial}{\partial x_2}(\mu(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}))+\frac{\partial}{\partial x_3}(\mu(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}))+f_1=0$$

$$\frac{\partial}{\partial x_2}(\lambda(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}))+\frac{\partial}{\partial x_1}(\mu(\frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}))+\frac{\partial}{\partial x_2}(2\mu\frac{\partial u_2}{\partial x_2})+\frac{\partial}{\partial x_3}(\mu(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}))+f_2=0$$

$$\frac{\partial}{\partial x_3}(\lambda(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}))+\frac{\partial}{\partial x_1}(\mu(\frac{\partial u_3}{\partial x_1}+\frac{\partial u_1}{\partial x_3}))+\frac{\partial}{\partial x_2}(\mu(\frac{\partial u_3}{\partial x_2}+\frac{\partial u_2}{\partial x_3}))+\frac{\partial}{\partial x_3}(2\mu\frac{\partial u_3}{\partial x_3})+f_3=0$$

Eliminating "non-measurable" parameters and assuming "2-D" case:

$$\frac{\partial}{\partial x_2}\left[\frac{\partial}{\partial x_1}(2\mu\frac{\partial u_1}{\partial x_1})+\frac{\partial}{\partial x_2}(\mu(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}))\right]-\frac{\partial}{\partial x_1}\left[\frac{\partial}{\partial x_1}(\mu(\frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}))+\frac{\partial}{\partial x_2}(2\mu\frac{\partial u_2}{\partial x_2})\right]=0$$

<u>Second-order</u> hyperbolic partial differential equation:

mechanical boundary conditions are required along <u>two intersecting characteristics</u>

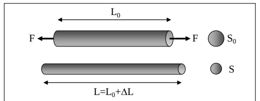
<u>no assumptions</u> are made regarding either $\lambda(x_1,x_2)$ or Poisson's ratio

Elasticity Imaging – Approaches

Static (strain-based, or reconstructive)
imaging internal motion under static deformation

Dynamic (wave-based)
imaging shear wave propagation

Mechanical (stress-based, also reconstructive)
measuring tissue response at the surface



- Strain

$$\varepsilon_{11} = \frac{L - L_0}{L_0}$$

- Stress

$$\sigma_{11} = \frac{F}{S}$$

- Constitutive relationships

$$\sigma_{11} = E\varepsilon_{11} \qquad \text{E – Young's modulus}$$

- Equations of motion

$$\frac{\partial \sigma_{11}}{\partial x_1} = \rho \frac{\partial^2 u_1}{\partial t^2}$$

Theory of Elasticity (Dynamic Approach)

General (3-D) case

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right)$$

$$\Downarrow$$

$$\begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix} \longleftrightarrow \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

$$\Downarrow$$

$$\sigma_{ij} = \lambda \varepsilon_{ii}\delta_{ij} + 2\mu\varepsilon_{ij} + \xi \frac{\partial \varepsilon_{ii}}{\partial t}\delta_{ij} + 2\eta \frac{\partial \varepsilon_{ij}}{\partial t}$$

$$\Downarrow$$

$$\sum_i \frac{\partial \sigma_{ij}}{\partial x_j} + f_i = \rho \frac{\partial^2 u_i}{\partial t^2}$$

Example: plane waves

- Infinite homogeneous (λ,μ=const) elastic medium (i.e., ignore bulk and shear viscosities), no body forces (f_i=0)
- Assume that $u_1=u_1(x_1,t)$, $u_2=u_2(x_1,t)$, and $u_3=u_3(x_1,t)$

$$\frac{\partial}{\partial x_1}(\lambda(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}))+\frac{\partial}{\partial x_1}(2\mu\frac{\partial u_1}{\partial x_1})+\frac{\partial}{\partial x_2}(\mu(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}))+\frac{\partial}{\partial x_3}(\mu(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}))=\rho\frac{\partial^2 u_1}{\partial t^2}$$

$$\frac{\partial}{\partial x_2}(\lambda(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}))+\frac{\partial}{\partial x_1}(\mu(\frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}))+\frac{\partial}{\partial x_2}(2\mu\frac{\partial u_2}{\partial x_2})+\frac{\partial}{\partial x_3}(\mu(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}))=\rho\frac{\partial^2 u_2}{\partial t^2}$$

$$\frac{\partial}{\partial x_3}(\lambda(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}))+\frac{\partial}{\partial x_1}(\mu(\frac{\partial u_3}{\partial x_1}+\frac{\partial u_1}{\partial x_3}))+\frac{\partial}{\partial x_2}(\mu(\frac{\partial u_3}{\partial x_2}+\frac{\partial u_2}{\partial x_3}))+\frac{\partial}{\partial x_3}(2\mu\frac{\partial u_3}{\partial x_3})=\rho\frac{\partial^2 u_3}{\partial t^2}$$

$$(\lambda+2\mu)\frac{\partial^2 u_1}{\partial x_1^2}-\rho\frac{\partial^2 u_1}{\partial t^2}=0 \rightarrow \frac{\partial^2 u_1}{\partial x_1^2}-\frac{1}{c_l^2}\frac{\partial^2 u_1}{\partial t^2}=0 \text{ where } c_l=\sqrt{\frac{\lambda+2\mu}{\rho}}$$

Longitudinal wave (ultrasound)

$$\mu\frac{\partial^2 u_{2(3)}}{\partial x_1^2}-\rho\frac{\partial^2 u_{2(3)}}{\partial t^2}=0 \rightarrow \frac{\partial^2 u_{2(3)}}{\partial x_1^2}-\frac{1}{c_t^2}\frac{\partial^2 u_{2(3)}}{\partial t^2}=0 \text{ where } c_t=\sqrt{\frac{\mu}{\rho}}$$

Shear waves !

Speed of shear waves is most closely related to:

18%	1.	Poisson's ratio
11%	2.	Bulk modulus
7%	3.	Young's modulus
11%	4.	Compressibility
11%	5.	Shear viscosity

Speed of shear waves is most closely related to

- Infinite homogeneous (λ,μ=const) elastic medium (i.e., ignore bulk and shear viscosities), no body forces ($f_i=0$)
- Assume that $u_1=u_1(x_1,t)$, $u_2=u_2(x_1,t)$, and $u_3=u_3(x_1,t)$

$$\frac{\partial}{\partial x_1}(\lambda(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}))+\frac{\partial}{\partial x_1}(2\mu\frac{\partial u_1}{\partial x_1})+\frac{\partial}{\partial x_2}(\mu(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}))+\frac{\partial}{\partial x_3}(\mu(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}))=\rho\frac{\partial^2 u_1}{\partial t^2}$$

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$$\frac{\partial}{\partial x_3}(\lambda(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}))+\frac{\partial}{\partial x_1}(\mu(\frac{\partial u_3}{\partial x_1}+\frac{\partial u_1}{\partial x_3}))+\frac{\partial}{\partial x_2}(\mu(\frac{\partial u_3}{\partial x_2}+\frac{\partial u_2}{\partial x_3}))+\frac{\partial}{\partial x_3}(2\mu\frac{\partial u_3}{\partial x_3})=\rho\frac{\partial^2 u_3}{\partial t^2}$$

$$(\lambda+2\mu)\frac{\partial^2 u_1}{\partial x_1^2}-\rho\frac{\partial^2 u_1}{\partial t^2}=0 \rightarrow \frac{\partial^2 u_1}{\partial x_1^2}-\frac{1}{c_l^2}\frac{\partial^2 u_1}{\partial t^2}=0 \text{ where } c_l=\sqrt{\frac{\lambda+2\mu}{\rho}}$$

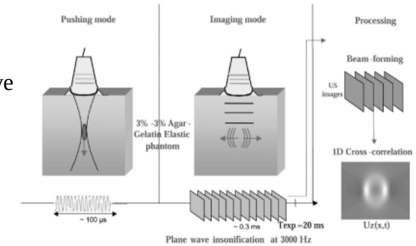
Longitudinal wave (ultrasound)

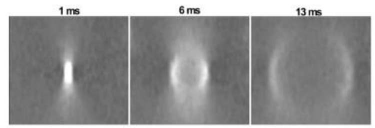
$$\mu\frac{\partial^2 u_{2(3)}}{\partial x_1^2}-\rho\frac{\partial^2 u_{2(3)}}{\partial t^2}=0 \rightarrow \frac{\partial^2 u_{2(3)}}{\partial x_1^2}-\frac{1}{c_t^2}\frac{\partial^2 u_{2(3)}}{\partial t^2}=0 \text{ where } c_t=\sqrt{\frac{\mu}{\rho}}$$

Shear waves !

$$E=3\mu$$

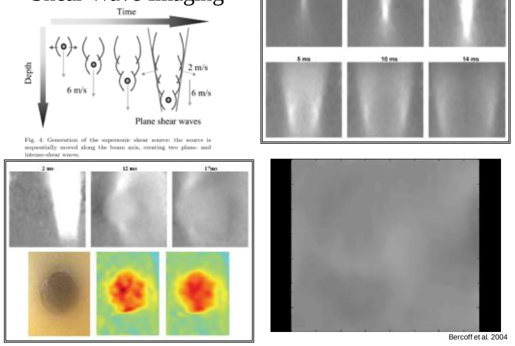
Shear Wave Imaging





Bercoff et al. 2004

Supersonic
Shear Wave Imaging



Bercoff et al. 2004

Elasticity Imaging: Principles

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