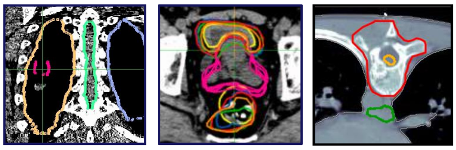


Stochastic programming methods for handling uncertainty and motion in IMRT planning

Jan Unkelbach

Industry collaborations: RaySearch, Philips Medical Systems

Content

A. Stochastic programming in IMRT planning

B. What is the advantage over a PTV approach?

1. Systematic positioning errors
   Balancing target coverage and normal tissue sparing
2. Respiratory motion
   Reducing normal tissue dose through 'horns'
3. Range uncertainty in proton therapy
   Breakdown of the static dose cloud approximation

Fluence map optimization in IMRT

Minimize dose-based objective function

$$\begin{aligned} \text{minimize} \quad & f(d) \\ \text{subject to} \quad & d = Dx \\ & x \geq 0 \end{aligned}$$

Including motion and uncertainty

Assume a discrete set of errors can occur

Delivered dose depends on the error scenario *k*

$$d^k = D^k x$$

Assign probabilities to errors: p_k

Minimize expected value of objective function

$$\text{minimize} \quad \sum_k p_k f\left(d^k\right)$$

Including motion and uncertainty

Quadratic objective function

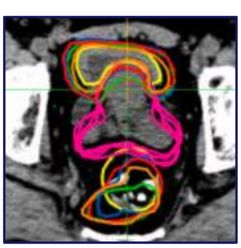
$$f(d) = \sum_{i \in T} \left(d_i - d^{pres}\right)^2 + \sum_{i \in N} \left(d_i\right)^2$$

$$\sum_k p_k \left(d_i^k - d^{pres}\right)^2 = \left(\sum_k p_k d_i^k - d^{pres}\right)^2 + \sum_k p_k \left(d_i^k - \sum_k p_k d_i^k\right)^2$$

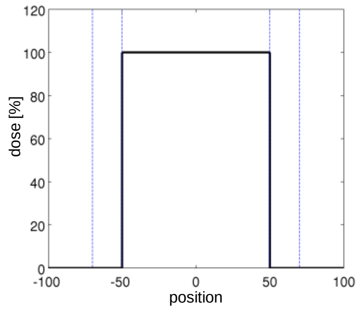
deviation of prescribed and expected dose — variance

Systematic errors

Systematic errors
(Setup errors or internal deformation)



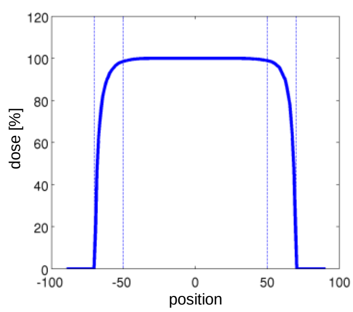
Systematic errors



- Gaussian p_k
- σ = 10
- cutoff at ± 2σ
- 40 scenarios

Ref: Löf 1995, Inv Prob

Systematic errors



- reproduces a PTV-like plan
- may yield a smooth falloff

See also: Sir 2006, PMB

Systematic errors

Benefit:

- Automation: no explicit PTV definition necessary
- Could optimally balance target coverage and OAR sparing

Stochastic programming natural with TCP/NTCP

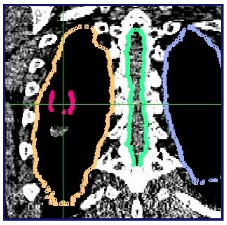
$$\text{minimize} \quad \sum_k p_k \mathrm{TCP}\left(d^k\right)$$

marginalization of a TCP model over the uncertain dose distribution

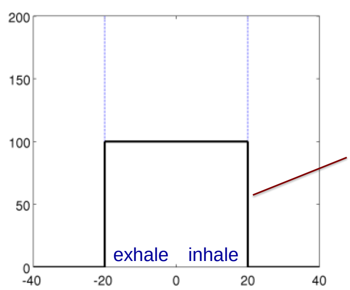
$$\text{subject to} \quad \sum_k p_k \mathrm{NTCP}\left(d^k\right) \le 0.05$$

Motion

Respiratory motion



Respiratory motion



- tumor size = 20
- amplitude = 20

ITV plan

Respiratory motion

Can normal tissue dose be reduced?

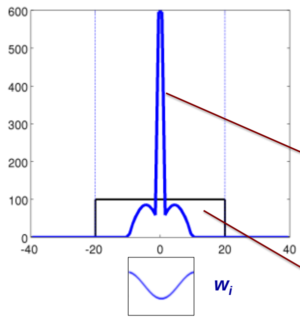
Tumor accumulates dose in different breathing phases

$$d = \sum_{i=1}^{n} w_i D^i x \qquad \sum_{i=1}^{n} w_i = 1$$

Idea:

- reduce dose to regions where the tumor is rarely
- deliver higher dose to regions always occupied by tumor

Respiratory motion



4D optimization

- Assume predictable breathing motion

dose peak where there is tumor most of the time

dose reduction at the edge of ITV

Respiratory motion

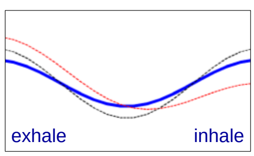
Problem: dose will degrade if breathing pattern varies

Stochastic programming:

Allow different breathing patterns w^k with probability p_k

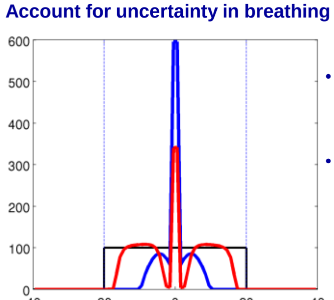
$$d^k = \sum_{i=1}^{n} w_i^k D^i x$$

$$\sum_{i=1}^{n} w_i^k = 1$$



Respiratory motion

Account for uncertainty in breathing pattern

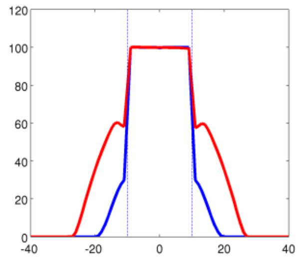


- larger uncertainty in *w* gradually yields more ITV-like plans
- special case $w_i^k = 1$ models systematic error

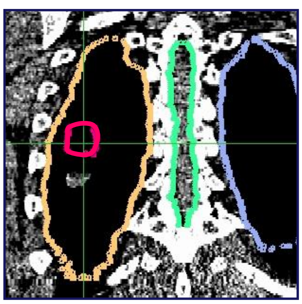
Ref: McQuaid 2011 AAPM summerschool

Respiratory motion

Dose delivered to moving tissue (nominal trajectory)



Realistic cases



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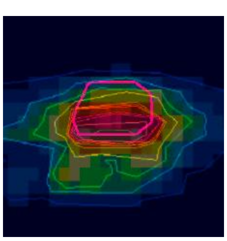
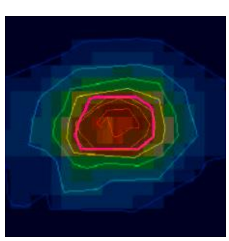
Respiratory motion

Assume predictable motion



Dose on exhale



Accumulated dose

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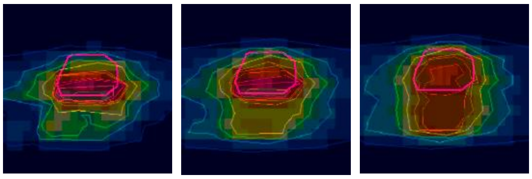
Respiratory motion



No uncertainty | medium motion uncertainty | Motion modeled as systematic error

Ref: Heath 2009 Med Phys

Respiratory motion

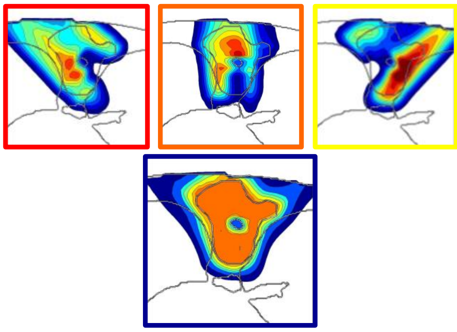
Benefit:

- 4D optimization yields dose horns
  - **Normal tissue dose reduction compared to PTV**
- Stochastic programming can account for breathing variations
  - **Find the balance between robustness and normal tissue sparing through horns**

Proton therapy

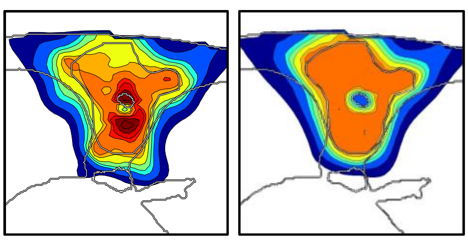
Range uncertainty in IMPT

Proton therapy



Proton therapy

Robustness analysis:



5 mm range overshoot **nominal plan**

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Proton therapy

Stochastic programming:

Assume 3 scenarios:

- nominal scenario $p_1 = 0.5$
- 5 mm range overshoot $p_2 = 0.25$
- 5 mm range undershoot $p_3 = 0.25$

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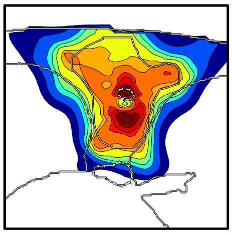
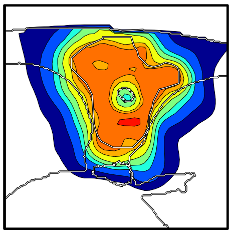
Sensitivity analysis

5 mm range overshoot



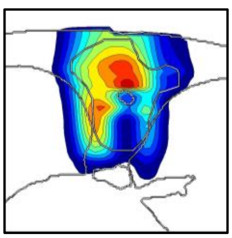
conventional plan



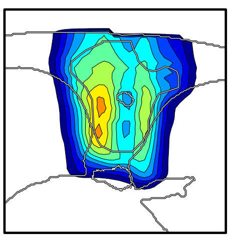
generated using stochastic programming

Motivation

How is robustness achieved?



conventional plan



generated using stochastic programming

Commercial implementations

Proton therapy led to the first implementation of probabilistic / robust planning in commercial TPS

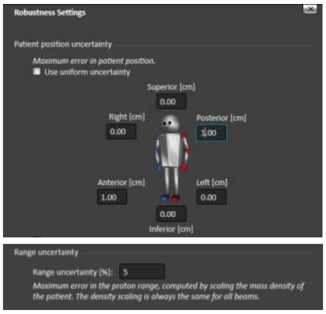
Examples:

- **IMPT Pinnacle** (in development)
  (implements a probabilistic approach)
- **RayStation v4.5**
  (implements a minimax approach)
  (Ref: Fredriksson 2011, Med Phys)

Before that:

- **Hyperion** (in-house TPS in Tübingen, Germany)
  (coverage probability method to account for positioning errors in prostate treatments)
  (Ref: Baum 2006, R&O)

Raystation 4.5

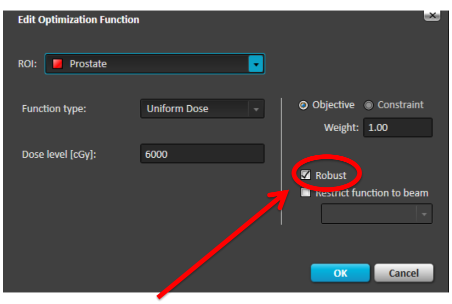


User interface

Setup uncertainty

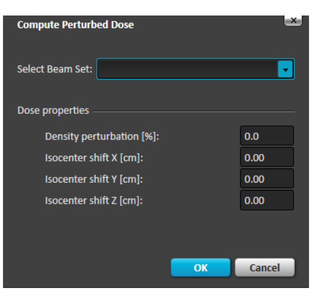
Range uncertainty

Raystation 4.5



User can robustify important objectives

Raystation 4.5



Plan evaluation

define error scenario

Summary

Stochastic programming for handling uncertainty:

- optimize expected value of the objective function
- general purpose method applicable to many uncertainties

Advantage over a PTV depends on type of uncertainty:

- **Automating target expansions**
  (systematic positioning errors)
- **Normal tissue dose reduction through horns**
  (respiratory motion)
- **Mitigate beam misalignments risks**
  (IMPT)

Status in practice

Range and setup uncertainty in IMPT:

- Fundamental limitations of the PTV concept
- led to the first commercial implementations

Respiratory motion

- Dose accumulation relies on deformable registration
- Computationally intensive

Setup errors, inter-fraction organ motion

- Qualitatively similar to PTV plans
- Magnitude of the error reduced through image guidance