A Brief Introduction to Magnetic Resonance Imaging

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**Outline for Today**

1) Introduction
   MRI Case Study; **Caveat!!!**
   “Quasi-Quantum” Nuclear Magnetic Resonance

2) Net magnetization, $m(x,t)$, of the voxel at $x$
   T1 Spin-Relaxation of $m(x,t)$,
   T1 MRI of the 1D patient
   Sketch of the MRI Device

3) ‘Classical’ Approach to NMR
   FID Image Reconstruction, $k$-Space

4) Spin-Echo Reconstruction
   T2 Spin-Relaxation
   T1-$w$, T2-$w$, and PD-$w$ S-MRI
   Spin-Echo / Spin-Warp in 2D
   Some Important Topics Not Addressed Here
Magnetic Resonance Imaging: Mapping the Spatial Distribution of Spin-Relaxation Rates of Hydrogen Nuclei in Soft-Tissue Water and Lipids
Part 1:
“Quasi Quantum” NMR and MRI

Introduction
MRI Case Study; Caveat!!!
“Quasi-Quantum” NMR and MRI
Introduction
Medical Imaging:
differential interactions of probes with different tissues

Beam of ‘probes’: particles or waves

Differential transmission, reflection, emission, *etc.*, of probes

Image receptor
Forms of Contrast Between Good and Bad Apples

produced and detected thru different bio/chemico/physical processes

The Future: Contrast Mechanisms!

**Color**

**Smell**

**Taste**

**Texture**

**Holes**

etc.
## How Imaging Modalities Detect Tissue Contrast

<table>
<thead>
<tr>
<th>Modality</th>
<th>Probe / Signal</th>
<th>Detector</th>
<th>Source of \textit{Contrast}: differences in…</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar R/F X-ray CT</td>
<td>X-rays passing through body</td>
<td>II+CCD, AMFPI; GdO, etc., array</td>
<td>( \int_{S} \mu(\rho, Z, kVp) , ds )</td>
</tr>
<tr>
<td>Nuc Med, SPECT; PET</td>
<td>Gamma-rays from body; 511 keV</td>
<td>NaI single crystal; multiple NaI; LSO array</td>
<td>Radiopharmaceutical uptake, concentration (\textit{e.g.,} (^{99}\text{mTc}), emission)</td>
</tr>
<tr>
<td>US</td>
<td>MHz sound</td>
<td>Piezoelectric transducer</td>
<td>( \rho, \kappa, \mu_{\text{US}} )</td>
</tr>
<tr>
<td>MRI</td>
<td>Magnet, RF</td>
<td>AM radio receiver</td>
<td>T1, T2, PD, [O], blood flow, water diffusion, chemical shift</td>
</tr>
</tbody>
</table>
CT vs. MRI Soft Tissue Contrast

CT

MRI

Posterior reversible encephalopathy syndrome (PRES), edematous changes
Multiple types of *contrast*, created through and reflecting different aspects of soft tissue biophysics.

Selection of contrast type with **pulse sequences**

Most reflect rotations, flow, *etc.*, of water/lipid molecules; yield distinct, unique maps of anatomy, physiology

No ionizing radiation

Risks from intense magnetic fields, RF power

Expensive

Technology complex, challenging to learn; Much of biophysics little understood. But…. 
MRI Case Study; and Caveat!!

with T1, FLAIR, MRS, DTI, or $f$ MRI contrast and MR-guided biopsy studies of a glioma
Case Study

57 year old ♀ medical physicist has had daily headaches for several months. Responds to ½ Advil.

Physical examination unremarkable. Patient appears to be in good general health, apart from mild hypertension, controlled by medication.

Good diet, exercises moderately. Patient reports no major stresses, anxieties.

CT indicates a lesion in the right posterior temporo-occipital region, adjacent to occipital horn of right lateral ventricle. MRI for more information.

Principal concern: Vision for reading.
Lesion: Right Posterior Temporo-Occipital Region, adjacent to occipital horn of right lateral ventricle

T1-w

No enhancement with Gd contrast.

fMRI

visual stimulus

DTI

FLAIR
Chemical Shift and Non-invasive MRS
acetic acid

1.56 ppm

Astrocytoma
MRI-Guided Needle Biopsy

Grade 1-2 Astrocytoma with scattered cellular pleomorphism and nuclear atypia
Caveat!!!

Extreme Danger
How *Not* to Clean a Magnet

strong gradient fields outside bore
MRI Safety

fringes of principal magnetic field rapidly switching (gradient) fields high RF power

FDA: ~ 40 MRI-related accidents/year 70% RF burns, 10% metallic “missiles”

RF Specific Absorption Rate (SAR): $dE/dm$ (W/kg)

no magnetic anything in or entering MRI room restricted access, safe zones, prominent warnings accompany all patients, visitors, non-MRI staff accurate medical history; double-check for metal training for any staff (e.g., cleaners)

possible gadolinium risk: nephrogenic systemic fibrosis (NSF) pregnant patients generally should not have Gd-contrast
MRI Safety – First and Foremost, Restrict Entry!

within / with patient, others

aneurysm clip, shrapnel
cochlear implant, prostheses
artificial heart valve
stent, permanent denture
defibrillator, pacemaker,
electrodes, nerve stimulator
medical infusion pump
drug-delivery patch, tattoo

in / into imaging suite

hemostat, scalpel, syringe
O₂ bottle, IV pole
scissors, pen, phone, laptop
tool, tool chest
wheelchair, gurney
ax, fire extinguisher
gun, handcuffs
cleaning bucket, mop
“Quasi-Quantum” NMR and MRI

Swept-frequency NMR in a tissue voxel
Proton-Density (PD) MRI in the 1D patient
Magnetization, \( m(x,t) \), in the voxel at \( x \)
Two Approaches to Proton NMR/MRI (\textit{incompatible!})

quantum spin-state function for \textit{hydrogen nucleus}

\[ |\psi\rangle \]

\textit{start with this}

Simple QM

\[ |\uparrow\rangle, |\downarrow\rangle \]

transitions between spin-up-, spin-down states

\[ f_{\text{Larmor}}, m_0, T_1 \]

oversimplified; like Bohr atom

Classical Bloch Eqs.

expectation values in voxel

precession, nutation of net voxel magnetization, \( m(t) \)

\[ f_{\text{Larmor}}, T_2, 2D, k\text{-space} \]

exact, for expectation values; from full QM
‘Open’ MRI Magnet

principal magnetic field $B_0$ defines $z$-axis
Moving Charge Produces Magnetic Field (e.g., mag. dipole); Magnetic Dipole Tends to Align in External Field

Mechanical relaxation

simplified QM picture
proton
Energy to Flip Over Needle with Magnetic Moment $\mu$ in $B_z$

$\Delta E_{180^\circ} = \pm 2 \mu B_z$

$E = -\mu \cdot (B_z) = -\mu B_z \cos \theta$
Nuclear Zeeman Splitting for Proton: \( \Delta E = \pm 2 \mu_z B_z \)

\( \mu \) points only along or against \( z \)
Swept-frequency NMR in a tissue voxel
Nuclear Magnetic Resonance

\[ \text{proton } \Delta E_{180^\circ} = 2 \mu_z B_z \]

\[ \text{photon } E = hf \]

\[ hf_{\text{Larmor}}(B_z) = 2 \mu_z B_z \]

\[ 2\pi f_{\text{Larmor}} \equiv \omega = \gamma B_z \]

for \textit{water} protons,

\[ f_{\text{Larmor, } H_2O} = 42.58 \left[ \frac{\text{MHz}}{T} \right] \times B_z [T] \]

\[ B_z(x) = f_{\text{Larmor}}(x) / 42.58 \]

\[ 42.58 \text{ MHz} \]

\[ 63.87 \text{ MHz} \]

\[ \text{Energy (µeV)} \]

\[ f_{\text{Larmor}} [\text{MHz}] \]

\[ B_z (T) \]
SAMS Question

I-1. For protons, $f_{\text{Larmor}} = 42.58 \text{ MHz}$ at 1 T. What is it at 1.5 T?

a. 42.58 MHz  

b. 28.36 MHz  

c. 63.87 MHz  

d. 21.39 MHz  

e. Cannot be determined from this info.
SAMs Q:

I-1. For protons, $f_{\text{Larmor}} = 42.58 \text{ MHz at 1 T}$. What is it at 1.5 T?
(a) 42.58 MHz
(b) 28.36 MHz
(c) 63.87 MHz
(d) 21.39 MHz
(e) Cannot be determined from this info.

Answer: (c). $f_{\text{Larmor}} = 42.58 \times B_z = 42.58 \text{ MHz/T} \times 1.5 \text{ T}$
= 63.87 MHz

NMR Gedanken-Experiment on Water, 1T

monochromatic RF power absorption at (and only at) $f_{\text{Larmor, } \text{H}_2\text{O}}$

Resonance at 1T: 42.58 MHz

Detected RF power

Transmitter $f$ (MHz)
I-2. If the field homogeneity of a 1.5 T scanner is measured to be one part per million (1 ppm), what will be the approximate spread in resonant frequencies?

- 16% a. 1.5 Hz
- 6% b. 42.58 MHz
- 8% c. 42.58 kHz
- 11% d. 63.87 MHz
- 60% e. 63.87 Hz
SAMs Q:

I-2. If the field homogeneity of a 1.5 T scanner is measured to be one part per million (ppm), what will be the approximate spread in resonant frequencies?

(a) 1.5 Hz
(b) 42.58 MHz
(c) 42.58 kHz
(d) 63.87 MHz
(e) 63.87 Hz

Answer: (e). \[ \Delta f_{\text{Larmor}} = (42.58 \times 10^6 \text{MHz/T}) \times (1.5 \text{ T}) \times (10^{-6}) \]
\[ = 63.87 \text{ Hz} \]

Proton-Density (PD) MRI in the 1D patient
1D Phantom in Principal $B_0$ and Gradient $G_x$

\[ B_z(x) - B_0 = G_x \times x \]

\[ G_x \equiv \frac{dB_z(x)}{dx} \]

\[ x = \frac{B_z(x) - B_0}{G_x} \]

(1.002T - 0.998T) / 0.20m = 20 mT/m
1D PD-MRI: \[ f_{\text{Larmor}} = f_{\text{Larmor}}[B_z(x)] \]

1. \[ B_z(x) = f_{\text{Larmor}} / 42.58 \]
2. \[ x = [B_z(x) - B_0] / G_x \]
3. \[ x = [B_z(x) - B_0] / G_x \]
4. \[ G_x \]

Transmitter

sweep the RF \( f \)

\( B(x) \)
1.002 T
1.000 T
0.998 T

\( f(x) \)

\( x = [B_z(x) - B_0] / G_x \)

PD MR Image
Summary: PD MRI on 1D Patient

1. \( f_{\text{Larmor}} \) of NMR Peak

2. \( B_z(x) \)

3. Voxel Position, \( x \)

4. Amplitude of NMR Peak, \( A(f) \)

4'. Pixel Brightness, PD

MRI
Proton-Density MRI

contrast from differences in PD
Two MRI Motion Artifacts

respiration

aortic pulsation
I-3. The net magnetic field $B_z(x)$ is measured to be $0.999 \, \text{T}$ at $x = -0.05 \, \text{m}$ and $1.001 \, \text{T}$ at $+5 \, \text{cm}$. What is $G_x$?

- 15% a. $0.01 \, \text{T/m}$
- 29% b. $0.02 \, \text{T/m}$
- 38% c. $20 \, \text{mT/m}$
- 13% d. $0.2 \, \text{T/cm}$
- 5% e. $0.2 \, \text{T/m}$
SAMs Q:

I-3. The net magnetic field $B_z(x)$ is measured to be 0.999 T at $x = -0.05$ m and 1.001 T at +5 cm. What is $G_x$?

(a) 0.01 T/m
(b) 0.02 T/m
(c) 20 mT/m
(d) 0.2 T/cm
(e) 0.2 T/m

Answer: (c). 20 mT/m. $\Delta B_z(x) / \Delta x = 2$ mT / 0.1 m

Part 2: Magnetization & Relaxation

Net magnetization, $m(x,t)$, of the voxel at $x$
T1 Spin-Relaxation of $m(x,t)$,
T1 MRI of the 1D Patient
Sketch of the MRI Device
Net magnetization, $m(x,t)$, of the voxel at $x$
Net Magnetization, $m(x,t)$, for the Single Voxel at Position $x$: magnetic field from the ensemble of protons or needles themselves

Single voxel at position $x$

$B_z > 0$

Gentle agitation (noise energy)
Voxel’s MRI Signal Proportional to Its Magnetization, $m(x,t)$

$i)$ What is the magnitude of voxel magnetization at dynamic thermal equilibrium, $m_0$?

$ii)$ How long does it take to get there (T1)?

$iii)$ What is the mechanism?
Switching $B_z(x)$ On at $t = 0$ Induces Magnetization $m(x,t)$ from protons in voxel at $x$.
Filling Four Energy Levels of Marbles vs. Noise Level

equilibrium from battle between energy and entropy

(all slippery; black balls denser)

Shaking (Noise) Energy:

too much  too little  just right
Magnetization in Voxel at \( x \), under Dynamic Equilibrium:

\[
m_0(x,t) = [N_-(x,t) - N_+(x,t)] \times \mu, \quad t \to \infty
\]

collective magnetic field produced by all the protons in it

<table>
<thead>
<tr>
<th>( B_z )</th>
<th>( m_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 tesla</td>
<td>( 0 )</td>
</tr>
<tr>
<td>0.01 T \text{ or } ( t = 0^+ )</td>
<td>( \sim 0 )</td>
</tr>
<tr>
<td>( \gg 1.5 ) T</td>
<td>( N\mu_z )</td>
</tr>
<tr>
<td>1.5 T \text{ or } ( t \gg 0 )</td>
<td>( 5 \times 10^{-6} N\mu_z )</td>
</tr>
</tbody>
</table>

* after abruptly turning on \( B_z \), or after a 90° pulse.
$|m_0(x)|$ and Signal from it Increase with $B_0$

p.s., trade-off: SNR, resolution, acquisition-time
T1 Spin-Relaxation of $m(x,t)$
Voxel’s MRI Signal Proportional to Its Magnetization, $m(x,t)$

1. What is the magnitude of voxel magnetization at thermal equilibrium, $m_0$?
2. How long does it take to get there (T1)?
3. What is the mechanism?
Disturbed System Moving toward Up-Down Equilibrium

energy imparted to individual ‘down’ spins from ‘outside’

Spin ‘tickled’ down by $f_{\text{Larmor}}$ component of magnetic noise; emits RF photon, phonon. A few can be kicked ‘up’, as well.
Decay of Tc-99m Sample to Final Value, $n(\infty) = 0$

$n(0)$ radionuclei at initially
$n(t)$ remaining after time $t$.

$n(\infty)$ at ‘equilibrium’  
$n(\infty) = 0$

$\frac{dn}{dt}(t) = -\lambda n(t)$

$n(t) = n(0) e^{-\lambda t}$

$dn/dt \sim [n(t) - n(\infty)]$

scint. decay ($t$)  pop. growth ($t$)  photon atten. ($x$)
tracer conc. ($t$)  cell killing ($D$)  ultrasound atten. ($x$)
Return of Voxel’s $m_z(t)$ to Final Value, $m_z(\infty) \equiv m_0$

\[
\frac{d[m_0 - m_z(t)]}{dt} = -\left(\frac{1}{T_1}\right) [m_0 - m_z(t)]
\]

\[
m_z(t)/m_0 = 1 - e^{-t/T_1}
\]

Actually, dynamic equilibrium more complicated: Spins tickled down and up!
T1 MRI of the 1D Patient
T1 for 2 Voxels: Lipid vs. CSF
both at $f_{\text{Larmor}}$ of H$_2$O for local field

$PD(x)$

$f(x)$

42.58

42.62
Recovery of $m_z(t)$ over Time, from $m_z(\text{TR}_i)$ Measurements

longitudinal component of net voxel magnetization, $m_z(t)$

1. First, do lipid at $x = 0$.
2. Strong burst of RF over range $42.58 \text{ MHz} \pm 250 \text{ Hz}$ causes $N_- \sim N_+$
3. After $\text{TR}_1$ delay, sweep through $f_{\text{Larmor}}$, record peak’s amplitude.
4. Repeat for several other $\text{TR}_i$ values.
5. Plot.
\[ \frac{m_z(\text{TR})}{m_0} = (1 - e^{-\frac{\text{TR}}{T_1}}) \]

curve-fit for set of \{\text{TR}_i\} for voxel at \(x = 0\)

by convention for T1-imaging: pixel white for fast-T1 tissue 
Spritely Brightly
\[ m_z(x,t)/m_0(x) = (1 - e^{-t/T1(x)}) \]

repeat for CSF voxel at \( x = 5 \) cm, \( f_{\text{Larmor}} = 42.62 \) MHz…

Lipid: \( T1 = 250 \) ms
\( x = 0 \)

CSF: \( T1 = 2,000 \) ms
\( x = +5 \) cm

\( T1 \) MR image

\( \{ \begin{align*} 
\text{lipid: } & \sim 250 \text{ ms} \\
\text{CSF: } & \sim 2,000 \text{ ms} 
\end{align*} \)
Gd sub-cut. fat
T1 \sim 260\,\text{ms}

CSF
T1 \sim 2000\,\text{ms}

Gd

T1-w MR Image
<table>
<thead>
<tr>
<th>Tissue</th>
<th>PD $p^+$/mm$^3$, rel.</th>
<th>T1, $1T$ (ms)</th>
<th>T1, $1.5T$ (ms)</th>
<th>T1, $3T$ (ms)</th>
<th>T2 (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pure H$_2$O</td>
<td>1</td>
<td>4000</td>
<td>4000</td>
<td>4000</td>
<td>4000</td>
</tr>
<tr>
<td>brain</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CSF</td>
<td>0.95</td>
<td>2000</td>
<td>2000</td>
<td>2000</td>
<td>200</td>
</tr>
<tr>
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<td>850</td>
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<td>900</td>
<td>1300</td>
<td>100</td>
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<td>edema</td>
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<tr>
<td>glioma</td>
<td></td>
<td>930</td>
<td>1000</td>
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<td>500</td>
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<td>40</td>
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<td>hepatoma</td>
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<td></td>
<td>1100</td>
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<td>85</td>
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<td>muscle</td>
<td>0.9</td>
<td>700</td>
<td>900</td>
<td>1800</td>
<td>45</td>
</tr>
<tr>
<td>adipose</td>
<td>0.95</td>
<td>240</td>
<td>260</td>
<td></td>
<td>60</td>
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</tbody>
</table>
Voxel’s MRI Signal Proportional to Its Magnetization, \( m(x,t) \)

- \( i \) What is the magnitude of voxel magnetization at thermal equilibrium, \( m_0 \)?
- \( ii \) How long does it take to get there (T1)?
- \( iii \) What is the mechanism?
In MRI, the **only** thing a proton is **ever** aware of, or reacts to, is the *local* magnetic field, \( B_{\text{local}}(t) \).

!!!

But the source of \( f_{\text{Larmor}}(B_{\text{local}}(t)) \) can be either **external** (\( B_{\text{RF}} \)) or **internal** (e.g., moving partner).

!!!!!
Random $B_{RF}(t)$ at $f_{Larmor}$ Causes T1-Transitions

variations in proton magnetic dipole-dipole interactions

each water proton produces magnetic field fluctuations
of various frequencies, including local $f_{Larmor}$, at its partner proton
Restrictions on Rotations of Water Molecules

more or less hydration free
bound layer
Rotational Power Spectra, \( J(f) \), of Water for several water populations

- Water on large biomolecule
- Water on small molecule
- Pure water

Magnetic noise (water tumbling, etc.) frequency

Probability / relative number

T1 spin flips

Slow motion \( \rightarrow \) Rapid
Average Water T1 Determined by Noise Power Spectrum,

\[ J(f_{\text{Larmor}}) \]

Relaxation Time, T1

Water on intermediate sized molecule

Water on large, slow biomolecule

Pure water

1.5 T

Magnetic noise (water tumbling, etc.) frequency

Slow motions ← Rapid

\[ f_{\text{Larmor}} \]
MRI:

Spin-Relaxation Times (T1, T2) – their Spatial Distributions!
Sketch of the MRI Device
Damadian’s Indomitable, 1977 (Smithsonian)

Nobel Prize, 2003
Paul Lauterbur
Peter Mansfield
Major Components of a Superconducting MRI System
MRI Magnets

Open (most of this presentation) electromagnetic or permanent

Superconducting
e.g., niobium-titanium wire

$B_0$: < 0.5 T, 1.5 T, 3.0 T, (7 T)
Homogeneity: < 10 ppm
Shielding: passive and active
Cryogen: 0.1 liter He/y
Weight: 4 tons (supercond.)
Three External Magnetic Fields in Open Magnet MRI

$B_0$, $G_z$, and $G_x$ all point along $z$!

$G_z \equiv \frac{\Delta B_z(z)}{\Delta z}$

$G_x \equiv \frac{\Delta B_z(x)}{\Delta x}$
**$x$-Gradient Magnet Winding for Superconducting Magnet**

$x$-Gradient \(dB_z/dx\) \(20 - 60\) mT/m

Rise time \(0.3\) ms (to reach 10 mT/m)

Slew rate \(50 - 200\) mT/m/ms

one layer of $x$-gradient coil
Artifact: Gradient Non-Linearity
Gradient coils affect the strength of the magnetic field that, at all locations in space, point…:

a. along the x direction of the scanner
b. along the y direction of the scanner
c. along the z direction of the scanner
d. along all directions of the scanner

not enough information given here to answer
Gradient coils affect the strength of the magnetic field that, at all locations in space, point…:

(a) along the $x$ direction of the scanner
(b) along the $y$ direction of the scanner
(c) along the $z$ direction of the scanner
(d) along all directions of the scanner
(e) not enough information given here to answer

Answer: (c). Along the principal magnetic field, $B_0$, hence the $z$-axis

RF Coils

\[ B_{RF} : 20 \ \mu T \]
Pulse on-time: 3 msec
RF power: 15 – 25 kW
SAR: 2 – 20 W/ kg
‘Parallel’ RF Receiving Coils for Much Faster Imaging

transmit coils coming
II-5  The acronym used to describe the amount of RF radiation uptake in patient tissue is:

a. REM
b. SAR
c. CTDI
d. STIR
e. RARE
SAMs Q:

II-5. The acronym used to describe the amount of RF radiation uptake in patient tissue is:

(a) REM
(b) SAR
(c) CTDI
(d) STIR
(e) RARE

Answer: (b). Specific Absorption Rate

Part 3: “Classical” MRI

‘Classical’ Approach to NMR FID Image Reconstruction, $k$-Space
‘Classical’ Approach to NMR; FID Image Reconstruction, $k$-Space

Normal modes, resonance, precession, nutation
Free Induction Decay (FID) in a voxel, without the decay
e.g., very slow T1 relaxation
Untangling the FID RF signals from a row of voxels:
   Temporal Fourier Series approach
FID imaging of the two-voxel 1D patient
FID Imaging via $k$-Space; Spatial FT
The Two Approaches to NMR/MRI *(incompatible!)*

quantum state function

\[ |\psi\rangle \]

Simple QM

\[ |\uparrow\rangle, |\downarrow\rangle \]

transitions between spin-up, spin-down states

\[ f_{\text{Larmor}}, m_0, T1 \]

oversimplified

Classical Bloch Eqs.

for expectation values

precession, nutation of voxel magnetization, \( m(t) \)

\[ f_{\text{Larmor}}, T2, k\text{-space} \]

exact; from full QM
Normal Modes, Resonance, Precession, Nutation
Normal Mode, at $f_{\text{normal}}$
A Normal Mode of a 2-D Pendulum

cone of rotation

$f_{normal}$
Normal Mode *Precession* about External Gravitational Field

\[ J(t) : \text{Angular Momentum} \]

With torque, \( \tau \):
\[ \frac{dJ}{dt} = \tau \]

*Equation of Motion*

Angular acceleration is just like
\[ \frac{dp}{dt} = F \]
but here, \( J(t) \) changes \textit{direction only}:

*Precession* at \( f_{\text{normal}} \)
Normal Mode Precession of Voxel \( m(x,t) \) in Magnetic Field can be derived rigorously from quantum mechanics.
Classical Bloch Equations of Motion for $m(x,t)$ in $B_0$

Equation of motion for spinning body

$$\frac{dJ}{dt} = \tau \quad \text{(external torque)}$$

recall: $\mu = \gamma J$

$$\frac{d(\mu/\gamma)}{dt} = \tau$$

**Lorentz torque** on spins at $x$ with magnetic moment $\mu$ in $B_z(x)$:

$$\tau = \mu \times B_z(x) \quad \text{(vector cross product)}$$

Equation of motion becomes:

$$\frac{d\mu(x,t)}{dt} = \gamma \mu(x,t) \times B_z(x)$$

Sum/average over all protons in voxel:

$$\frac{d\langle m(x,t) \rangle}{dt} = \gamma \langle m(x,t) \rangle \times B_z(x)$$

With T1 relaxation along z-axis:

$$\frac{d\langle m(x,t) \rangle}{dt} = \gamma \langle m(x,t) \rangle \times B_z(x) + [\langle m(x,t) \rangle - m_0(x)]\hat{z} / \text{T1}$$

Expectation Value, $\langle m(x,t) \rangle$, behaves \textit{classically}
Precession of $m(x,t)$ about $B_0$ at $f_{\text{Larmor}} = (\gamma/2\pi) B_z(x)$ as seen from a frame that is

**Fixed**

**Rotating at $f_{\text{Larmor}}(x)$**

as if $B_0 = 0$

The ponies don’t advance when you’re *on* the carousel; It’s as if $B_0 = 0$!
Resonance Energy Transfer when $f_{\text{driving}} = f_{\text{normal}}$
Normal Mode and Resonance of a 2-D Pendulum

Precession

Nutation

\[ f_{\text{driving}} = f_{\text{normal}} \]
And Now for Something Completely Different: **Nutation**

Cone of Precession

Nutation at Resonance
**Nutation of the Voxel’s Magnetization, \( m(x,t) \)**

\[
\frac{B_{RF}}{B_0} \sim 10^{-5} - 10^{-4}
\]

\[f_{\text{Nutation}} = \left(\frac{\gamma}{2\pi}\right)B_{RF}\]

**Precesses at**

\[f_{\text{Larmor}} = \left(\frac{\gamma}{2\pi}\right)B_z(x)\] (always!)

**Nutates at**

\[f_{\text{Nut}} = \left(\frac{\gamma}{2\pi}\right)B_{RF}\] (only when \( B_{RF} \) is on!)
Free Induction Decay (FID) in a voxel, without the decay
  *e.g.*, very slow T1 relaxation
$m(x,t)$ for a *Single Voxel at $x$ and Precessing in the $x$-$y$ Plane following a 90° pulse*
FID Precession, Reception, Fourier Analysis (single voxel)

*n.b.* measure induced $V(t)$, not power absorption (as before)
In MRI, the only signal you ever see comes from the set \( \{ m(x,t) \} \) all precessing in the \( x-y \) plane!!!
III-6. The rate of magnetization nutation depends upon:

a. the magnitude of the magnetization, $|m(t)|$

b. the strength of the principal magnetic field, $|B_0|$

c. strength of the $x$-gradient field, $|x \times G_x|$

d. the strength of the RF magnetic field, $|B_{RF}(t)|$

e. $T1$
SAMs Q:

III-6. The rate of magnetization nutation depends upon:
   a) the magnitude of the magnetization, $|m(t)|$
   b) the strength of the principal magnetic field, $|B_0|$
   c) strength of the $x$-gradient field, $|x \times G_x|$ \
   d) the strength of the RF magnetic field, $|B_{RF}(t)|$
   e) T1

Answer: (d).

Untangling the FID RF Signal from Two Voxels: temporal Fourier series approach
Wave Interference in Time or Space

positive (in-phase) interference

negative interference
Wave Interference and Spectral Analysis

Separate signals:

Composite beat signal:

Decomposition via Fourier analysis, \( \mathcal{F} \):

Time or space:

\[ t \text{ (sec)} \]
\[ x \text{ (cm)} \]

\[ f \text{ (Hz)} \]
\[ k \text{ (cm}^{-1}) \]
Fourier Decomposition of Periodic Temporal Signal

\[ S(t) \sim \frac{1}{2} + \frac{2}{\pi}\left\{ \sin(2\pi f_1 t) + \frac{1}{3} \sin(6\pi f_1 t) + \frac{1}{5} \sin(10\pi f_1 t) + \ldots \right\} \]

*fundamental:* \( f_1 \) Hz (cycles/sec)

**orthonormal basis vectors**

- fundamental \( f_1 \)
- 3rd harmonic \( f_3 \)
- 5th \( f_5 \)

**spectrum**

- Component amplitude
- \( f \) Hz
- \( f \)
FID imaging of the two-voxel 1D patient
\[
 f_{\text{Larmor}}(x) = \left(\gamma / 2\pi\right) \left(x \times G_x\right) \quad \text{(in rotating frame)}
\]
e.g., FID from 2 water slices, at \(x = 0\) and 5 cm; *little* decay!
FID Following *Wide-Bandwidth* 90° RF Pulse

covers $f_{\text{Larmor}}(x)$ for all $N$ voxels while $G_x$ is being applied!

* FID Following
* Wide-Bandwidth
* 90° RF Pulse

while gradient is on…
**FID: After Wide-Band RF 90° Pulse Centered on $f_{\text{Larmor}}(B_0)$**

**n.b.:**  
$m(x,t) = m(x,0)e^{-2\pi i f(x)t}$

\[ m_{xy}(x=0) \]
\[ m_{xy}(x=5) \]

**Separate RF signals**

**Temporal $\mathcal{F}$**

**Fourier spectrum**

**Detected FID signal**

\[ x = \frac{f_L(x)}{(\gamma/2\pi)} G_x \]
To Summarize What We Have Done So Far with FID

follow temporal $\mathcal{F}$ with isomorphism of $A(f)$ to real-space, $R(x)$

$$A(f) = \int S(t) e^{-2\pi i f(t) t} dt$$

$$F$$

MRI signal

$S(t)$

$t \leftrightarrow f$

Signal spectrum

$42.58$ MHz $\rightarrow 42.62$ MHz

$$x = \frac{2\pi f(x)}{\gamma G_x}$$

Real-space representations

$R(x)$
FID Imaging \textit{via} $k$-Space; \textit{Spatial} FT
Again, What We Have Done So Far:

Detected FID signal

Temporal: $t \leftrightarrow f$

Signal spectrum

$x = 2\pi f_{\text{Larmor}}(x) / \gamma G_x$

Real-space representation
Now Consider 1D *Spatial* Fourier Analysis

Fundamental: $k_1$ (cycles/mm)

$$S(x) \sim \frac{1}{2} + \frac{2}{\pi} \left[ \sin \left( 2\pi k_1 x \right) + \frac{1}{3} \sin \left( 6\pi k_1 x \right) + \frac{1}{5} \sin \left( 10\pi k_1 x \right) + \ldots \right]$$
Experiment: Continue on with a FT from $x$- to $k_x$-Space

Detected FID signal

Temporal: $t \leftrightarrow f$

$S(t)$

$t_{sec}$

Signal spectrum

Frequency: $f$

42.58 MHz

42.62 MHz

$A(f)$

Spatial: $k_x \leftrightarrow x$

$k_{x}$-space representation

$k$-

$k_x = 2\pi f_{\text{Larmor}}(x) / \gamma G_x$

$x = 2\pi f_{\text{Larmor}}(x) / \gamma G_x$

$x$ = 0

$x$ = +5 cm

Real-space representation

$R(x)$

$\alpha(k) = \int R(x) e^{-2\pi i k_x x} dx$

$\alpha(k)$

$k$-

$k$-space representation
And Complete the Loop Linking $t$-, $f$-, $x$- and $k_x$-Spaces

Detected FID signal

$S(t)$

$t_{sec}$

$\alpha(k)$

$k_x(t) \equiv \gamma G_x t / 2\pi$

$k$-space representation

Temporal:

$t \leftrightarrow f$

$k_x \leftrightarrow x$

Spatial:

$k \leftrightarrow x$

Signal spectrum

$A(f)$

$42.58$ MHz $\quad 42.62$ MHz

$x = 2\pi f_{\text{Larmor}}(x) / \gamma G_x$

$x$ cm

0 $\quad +5$ cm

Real-space representation
So, We Can Get from $S(t)$ to $R(x)$ in Either of Two Ways!

but only the approach via $k$-space works in 2D and 3D

Detected FID signal

Signal spectrum

$k$-space representation

Real-space representation

$S(t)$

$R(x)$

$f(t)$

$\alpha(k)$

$\mathcal{F}$

$f_{\text{Larmor}}(x) t = k_x(t) x$

$\mathcal{F}^{-1}$

$k_x(t) \equiv \gamma G_x t / 2\pi$

$x = 2\pi f_{\text{Larmor}}(x) / \gamma G_x$
$k$-Space Procedure for FID in 1D Phantom

$S(t)$ comprised of contributions from the $m(x)$ for all $x$

Detected FID signal

\[
m(x,t) = m(x,0) e^{-2\pi i f(x) t} = m(x,0) e^{-2\pi i k_x(t) x}
\]

\[
S(t) = S(k(t)) \sim \int m(x) e^{-2\pi i k_x(t) x} \, dx.
\]

Therefore, taking the inverse FT, $\mathcal{F}^{-1}$:

\[
m(x) = \int S(t) e^{+2\pi i k_x(t) \cdot x} \, d^3k
\]

$k$-space representation

Real-space representation
Spatial Waves, and Points in $k$-Space
Point in $k$-Space and Its Associated Wave in Real Space
Points in \( k \)-Space and Their Waves in Real Space
Composite Patterns in $k$-Space and Real Space

linearity

$y$

$x$

Real Space

$k$-Space

$k_x$

$k_y$
Fourier Transform of $k$-Space Pattern to Real Space
Fourier Transform of MRI Data in $k$-Space to Real Space
Fourier Transform from Parts of $k$-Space to Real Space
Herringbone Artifact
noise spike during data acquisition
III-7 Information about the structural detail in an image is:

a. dependent on the Fourier transform.
b. contained in the periphery of $k$-space.
c. blurred by increasing separation of lines in $k$-space.
d. contained in the center of $k$-space.
e. related to gradient duration.

[Bar chart showing the following percentages: 60% for b, 28% for d, 7% for a, 3% for c, and 2% for e.]
SAMs Q:

III-7 Information about the structural detail in an image is:
  a) dependent on the Fourier transform.
  b) contained in the periphery of $k$-space.
  c) blurred by increasing separation of lines in $k$-space.
  d) contained in the center of $k$-space.
  e) related to gradient duration.

Answer: (b).

Part 4: Spin-Warp; 2D

Spin-Echo Reconstruction (SE vs. FID)
T2 Spin-Relaxation (Static vs. Random Dephasing)
T1-\(w\), T2-\(w\), and PD-\(w\) S-E MR Imaging
Spin-Echo / Spin-Warp in 2D (2X2 illustration)
Spin-Echo Reconstruction

Spin Echo vs. FID
Static Field de-Phasing of FID
in x-y Plane of Spins in Each Voxel

$SFD-P$ from static-field inhomogeneities & susceptibility effects

This de-phasing is **REVERSABLE!!!!**

$m_{xy}(x,t)$

$\Delta f_{\text{Larmor}} \sim \Delta B_0$

Rate of $SFD-P$ decay:

$$\frac{1}{T_{SFD-P}} = \kappa \Delta B_0 + \kappa' \Delta \chi$$

n.b., $m_{xy}(x,t)$.... but still 1D phantom!
Spin-Echo Generates an Echo and Eliminates Effects of SFd-P

Local magnetic environment for each spin packet is static. So this de-phasing can be REVERSED!!!

T2 dephasing, by contrast, is random over time, and can NOT be undone!!
S-E Sequence Using 180° RF Pulse and Readout Gradient with 256 voxels, e.g., sample/digitize echo signal 256-512 times

$G_x \cdot f_{\text{Larmor}}(B_0 \pm G_x \times \frac{1}{2} \text{FOV})$

readout
**k-Space (spatial frequency) Procedure for SE in 1D Phantom**

Sampled SE signal

\[ S(t) = S(k(t)) \sim \int m(x) e^{-2\pi i k_x(t) x} dx \]

\( S(t) \) made up of contributions from \( m(x) \) for all \( x \);
\( m(x) \) can be extracted from \( S(t) \) with \( k_x \rightarrow x \) \( \mathcal{F} \):

\[ m(x) = \int S(t) e^{-2\pi i k(t) \cdot x} d^3 k \]

\[ R(x) = \int \alpha(k) e^{+i k x} dk \]

**k-space representation**

\( \alpha(k) \): Amplitude and phase of spatial frequency (k)

**Real-space representation**
During Readout, $k_x$ Increases Linearly with $t$

\[ S(t) = S(k(t)) \sim \int m(x) e^{-2\pi i k_x(t) x} \, dx. \]

$k_x(t)$ for all voxels increases linearly with $t$ while the echo signal is being received and read. With signal sampled at 512 adjacent instances spaced $\Delta t$ apart, $t$ represents the exact sampling time: the $n^{th}$ sample is taken a time $t_n = n \times \Delta t$ after $G_x$ was turned on. Different values of $k$ correspond to different sampling times. In particular, for $t_n$,

\[ k_n = \left[ G_x \gamma / 2\pi \right] t_n \]

**Larger $k$-values** correspond to **greater spatial frequencies**!
During Readout, $k_x$ Increases Linearly with $t$

Signal is sampled sequentially at 256-512 times spaced $\Delta t$ apart; $t_n$ is the exact sampling time after $G_x$ is turned on.

$k(t)$ for all voxels increases linearly with $t$ while the echo signal is being received and read: $k_n = \left[ G_x \frac{\gamma}{2\pi} \right] t_n$. Later sampling times (and larger $k$-values) correspond to greater spatial frequencies.

S-E reads out from $k_n = -k_{max}$ to $k = 0$ to $k = k_{max}$
T2 Spin-Relaxation

T2 Relaxation refers to the rate at which the transverse magnetization decays (disappears).

T2 relaxation results from T1-Events and *Non-Static, Random, Non-Reversible Proton-Proton Dipole Interactions*.

*Both* Contribute to the decay rate (1/T2).

(Specifically, any process that either reduces the number of transverse spins or their relative phase relationship will add to T2 relaxation.)
Thus, the Spin-Echo Signal Intensity $S(t)$ **Does** Decay, and decays much Faster than 1/T1!

Spin-Echo Spin De-Phasing Is Caused by T1 Events *as well as* By **Random**, Proton-Proton Dipole Interactions

Spin Multi-Echo: we **might** expect $S(t)$

However, **Harsh Reality**

$m_{xy}(x,t) \sim e^{-t/T2}$
Spin-Spin (Secular) De-Phasing in Bound Water with its protons precessing in the $x$-$y$ plane

Quasi-static spin-spin interactions does not involve exchange of energy like T1 relaxation!
Exponential T2-Caused De-Phasing of $m_{xy}(t)$ in x-y Plane

$$m_{xy}(t) / m_{xy}(0) = e^{-t/T2}$$

$$\frac{dm_{xy}(t)}{dt} = -\frac{1}{T2} m_{xy}(t)$$

Classical Bloch Equation

$$\frac{dm(x,t)}{dt} = \gamma m(x,t) \times B_z(x) - \frac{[m(x,t) - m_0(x)] \hat{z}}{T1} - \frac{[m_x \hat{x} + m_y \hat{y}]}{T2}$$
One Last Member of the Spin-Relaxation Family Tree: T2*

**fastest**

- **T2***
  - External field inhomogeneity contribution to T2* \((T_{SFd-P})\)
  - **T1**
    - T1 contribution to T2
    - Discrete spin-dephasing: involves \(\Delta E\)
  - **T2**
    - Secular contribution to T2
    - Continuous spin-dephasing: *no \(\Delta E\)!*

**FID, Gradient-Echo**

- G-E generally much faster than S-E T2
T2 relaxation refers to:

a. The rate at which longitudinal magnetization recovers.

b. The rate at which longitudinal magnetization disappears.

c. The rate at which transverse magnetization recovers.

d. The rate at which transverse magnetization disappears.

e. The rate at which tissue is magnetized.
T2 relaxation refers to:

a) the rate at which longitudinal magnetization recovers.
b) the rate at which longitudinal magnetization disappears.
c) the rate at which transverse magnetization recovers.
d) the rate at which transverse magnetization disappears.
e) the rate at which tissue is magnetized.

Answer: (d).

Tissue Contrast Weighting (w) in S-E MRI
T1-w, T2-w, and PD-w
<table>
<thead>
<tr>
<th>Tissue</th>
<th>PD $p^+$/mm³, rel.</th>
<th>T1, 1T (ms)</th>
<th>T1, 1.5T (ms)</th>
<th>T1, 3T (ms)</th>
<th>T2 (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pure H₂O</td>
<td>1</td>
<td>4000</td>
<td>4000</td>
<td>4000</td>
<td>4000</td>
</tr>
<tr>
<td>brain</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CSF</td>
<td>0.95</td>
<td>2500</td>
<td>2500</td>
<td>2500</td>
<td>200</td>
</tr>
<tr>
<td>white matter</td>
<td>0.6</td>
<td>700</td>
<td>800</td>
<td>850</td>
<td>90</td>
</tr>
<tr>
<td>gray matter</td>
<td>0.7</td>
<td>800</td>
<td>900</td>
<td>1300</td>
<td>100</td>
</tr>
<tr>
<td>edema</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>glioma</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>liver</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hepatoma</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>muscle</td>
<td>0.9</td>
<td>700</td>
<td>900</td>
<td>1800</td>
<td>45</td>
</tr>
<tr>
<td>adipose</td>
<td>0.95</td>
<td>240</td>
<td>260</td>
<td></td>
<td>60</td>
</tr>
</tbody>
</table>

**Relaxation Rates:** $1/T2 \sim 10 \times (1/T1)$
Three Different Forms of MRI Contrast
created by, and reflecting, three quite different physical properties
Multiple Spin-Echo Pulse Sequence

RF

$90^\circ$  $180^\circ$  Echo  $90^\circ$  $180^\circ$  Echo

$G_x$ readout

$0$  $\text{TE/2}$  $\text{TE}$  $0$  $\text{TE/2}$  $\text{TE}$

$\text{TR}$  $\text{TR}$
MRI Signal Strength at $t = TE$ Depends on....

<table>
<thead>
<tr>
<th>Inherent Parameters</th>
<th>Operator Settings</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>TR</td>
</tr>
<tr>
<td>T2</td>
<td>TE</td>
</tr>
<tr>
<td>PD</td>
<td></td>
</tr>
</tbody>
</table>

\[ S(t = TE) \sim PD (1 - e^{-TR/T1}) e^{-TE/T2} \]

- proton density
- prior regrowth along z-axis
- current dephasing in x-y plane

**bright regions:**

\[ m_{xy} \text{ large at } t = TE \]
In a spin-echo pulse sequence, a short TE will:

| 13% | a. Minimize T1 image contrast |
| 83% | b. Eliminate T2 effects       |
| 1%  | c. Result in a lower SNR      |
| 2%  | d. Increase the effects of static-field de-phasing |
| 2%  | e. Enhance tissue susceptibility differences |
In a spin-echo pulse sequence, a short TE will:

(a) Minimize T1 image contrast
(b) Eliminate T2 effects
(c) Result in a lower SNR
(d) Increase the effects of static-field dephasing
(e) Enhance tissue susceptibility differences

Answer: (b).

**T1-weighted**  –  Short TE to Eliminate T2 Contribution

\[ S(t \sim \text{TE}) \sim \text{PD} \left( 1 - e^{-\frac{TR}{T_1}} \right) e^{-\frac{TE}{T_2}} \]

- **TR \sim T_{1_{av}}** to maximize contrast
- Short TE to minimize T2 impact

**proton density**

**prior regrowth along z-axis**

**current de-phasing in x-y plane**

**T1-w**

- Lipid
- CSF

**Short-T1 tissues bright**

(sprightly brightly)
**T1-w** – Short TE to *Eliminate* T2 Contribution

![Diagram](image)

- TE Short (20 ms);
- TR ~ $T_{1\,av}$ (500 ms)

Short T1 ➔ Bright

- $m_0$
- $m_z(TR)$
- $m_{xy}(TE)$
- $m_z(t)$

**Key abbreviations:**
- TR: Repetition Time
- TE: Echo Time
- CSF: Cerebrospinal Fluid
- Lipid

**Explanations:**
- Short TE is used to eliminate T2 contribution.
- TR is approximately $T_{1\,av}$ to ensure appropriate signal from different tissue types.
For T2-weighed imaging:

\[ S(t \sim TE) \sim PD \left( 1 - e^{-\frac{TR}{T1}} \right) e^{-\frac{TE}{T2}} \]

**Long TR**
- Eliminate T1 impact
- Maximize contrast

**TE at mid-T2**
- Maximize contrast

**proton density**

**prior regrowth along \( z \)-axis**

**current de-phasing in \( x-y \) plane**

**T2-weighed (T2-w)**
- Lipid
- CSF

**Long-T2 tissues bright**
- T2 long
**T2-w** – Long TR to Eliminate T1 Contribution

TE ~ mid-T2 to maximize T2 contrast

- **TR**
- **CSF**
- **lipid**

- **M_z(t)**
- **M_{xy}(t)**
- **M_{xy}(TR+)**
- **M_{xy}(TE)**

- **time**

**TR**

- 90°
- **2000 ms**

**CSF**

**lipid**

**TE**

- 90°
- 180°
- **80-100 ms**
**PD-, T1-, & T2-Weighted Spin-Echo Images (1.5T)**

<table>
<thead>
<tr>
<th></th>
<th>T1-w</th>
<th>T2-w</th>
<th>PD-w</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TR (ms)</strong></td>
<td>mid- (~$T_{1\text{av}}$) 300 – 700</td>
<td>long 1,500 – 3,500</td>
<td>long 1,500 – 3,500</td>
</tr>
<tr>
<td><strong>TE (ms)</strong></td>
<td>short 0 – 25</td>
<td>mid- (~$T_{2\text{av}}$) 60 – 150</td>
<td>short 10 – 25</td>
</tr>
<tr>
<td><strong>Bright</strong></td>
<td>short T1</td>
<td>long T2</td>
<td>high PD</td>
</tr>
<tr>
<td><strong>SNR</strong></td>
<td>good</td>
<td>lower</td>
<td>best</td>
</tr>
</tbody>
</table>
Which of the following would appear bright on a T2-weighted image of the brain?

- a. CSF
- b. Fat
- c. Bone
- d. White Matter
- e. Air
Which of the following would appear bright on a T2-weighted image of the brain?

(a) CSF
(b) Fat
(c) Bone
(d) White matter
(e) Air

Answer: (a) cerebral spinal fluid.

One of the earliest 2D imaging methods was Sensitive Point Reconstruction.

This method used oscillating gradient fields to produce a net field stable in only one voxel at a time. That “sensitive point” was then scanned in space.
In 2-D MRI, phase is as important as frequency and is used to add a second spatial dimension.

**3 components**

- \( \sin x \)
- \( \frac{1}{3} \sin (3x + \pi/2) \)
- \( \frac{1}{3} \sin 3x \)

**2 different superimpositions**

- \( \sin x + \frac{1}{3} \sin 3x \)
- \( \sin x + \frac{1}{3} \sin (3x + \pi/2) \)
2D Spin-Warp 2X2 Matrix Illustration

Assume: PD Map of Thin-Slice 2D, 4-Voxel Patient after 90° pulse drives $M(t)$ into $x$-$y$ plane…

viewed from feet to head
Spin-Echo, Spin-Warp Sequence for 2x2 Matrix

involves 2 spin-echo pulse sequences

180° phase difference between rows

0° phase difference between rows

TR slice selection

RF

G<sub>z</sub>

G<sub>y</sub>

G<sub>x</sub>

TE/2 TE TE/2 TE

for S<sub>0</sub> for S<sub>π</sub>
e.g., No x- or y-Gradients: $G_y = G_x = 0$

After ($G_y=0$) phase encoding

$$S_0(1,2,3,4) = 7 \quad (e.g.)$$

During ($G_x=0$) readout

$$S_0(1,2,3,4)$$

$$\mathcal{F}[S_0(1,2,3,4)] \rightarrow 7 = m(1) + m(2) + m(3) + m(4)$$
$1^{st}$ S-E Sequence: $x$-Gradient Only, On During Readout

$G_y = 0$

(Same phase)

$S_0(1,2) = 5$

$S_0(3,4) = 2$

$\mathcal{F} [S_0(1,2) + S_0(3,4)]$

$S_0(1,2) = 5 = m(1) + m(2)$

$S_0(3,4) = 2 = m(3) + m(4)$
2nd S-E Sequence: y-Gradient, then x-Gradient

\[ G_y \rightarrow 180^{\circ} \Delta . \ G_x \text{ On During Readout} \]

First, \( G_y \neq 0 \)

\[ \varphi + \pi \]

\[ \varphi \]

(Different phases)

Then, \( G_x \neq 0 \)

\[ \begin{array}{c|c}
1 & 3 \\
\hline
2 & 4
\end{array} \]

(Different frequencies)

\[ S_{\pi}(3,4) = 2 \]

\[ S_{\pi}(1,2) = -3 \]

\[ S_{\pi}(1,2) + S_{\pi}(3,4) \]

\[ \mathcal{F} \]

\[ S_{\pi}(1,2) = -3 = m(1) - m(2) \]

\[ S_{\pi}(3,4) = 2 = m(3) - m(4) \]
4 Equations in 4 Unknowns

\[ S_0(1,2) + S_0(3,4) \]
\[ S_0(3,4) = 2 = m(3) + m(4) \]
\[ S_0(1,2) = 5 = m(1) + m(2) \]

\[ S_\pi(1,2) + S_\pi(3,4) \]
\[ S_\pi(3,4) = 2 = m(3) - m(4) \]
\[ S_\pi(1,2) = -3 = m(1) - m(2) \]

180° out of phase: subtract

Solution:
\[ m(1) = 1 \]
\[ m(2) = 4 \]
\[ m(3) = 2 \]
\[ m(4) = 0 \]

Check:
\[ S_0(1,2,3,4) \]
\[ 7 = m(1) + m(2) + m(3) + m(4) \]
Spin-Echo, Spin-Warp…. (256×192 matrix)

90° pulse with $G_z$ on selects $z$-plane and initiates spin-echo

Free precession with $G_y$ on phase-encodes $y$-position
$G_y(n) = n \times G_y(1), \quad n = 1, 2, \ldots, 192$

Reading NMR signal with $G_x$ on frequency-encodes $x$-position

Repeat, but with $G_y(n+1)$
$n = 1, 2, \ldots, 192$
Creation of an MRI ‘Image’ in 2D $k$-Space

Each line in $k$-space from data obtained during one $M_{xy}(TE)$ readout; different lines for different phase-encode gradient strength.
2D, 3D, 4D Imaging
Inversion Recovery (STIR and FLAIR)
Fast S-E; Gradient Recovery Imaging (GRE, e.g., EPI)
Dynamic Contrast-Agent Enhancement (DCE)
Magnetization Transfer
CNR and Other Quantitative Measures of Image Quality
Parallel-Coil Receive, Transmit; Shim Coils
Magnetic Resonance Angiography (MRA)
Perfusion Imaging
Diffusion Tensor Imaging (DTI)
Functional MRI (fMRI)
Image QA, and ACR Accreditation
MRI/PET, MRI-Elastography, MRI/DSA, MRI/US
Highly Mobile MRI (e.g., for strokes)
Zero-Quantum Imaging
Artificial Intelligence Diagnosis

**MRI-Guided Radiation Therapy**
A Brief Introduction to Magnetic Resonance Imaging

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