Review of PET Physics

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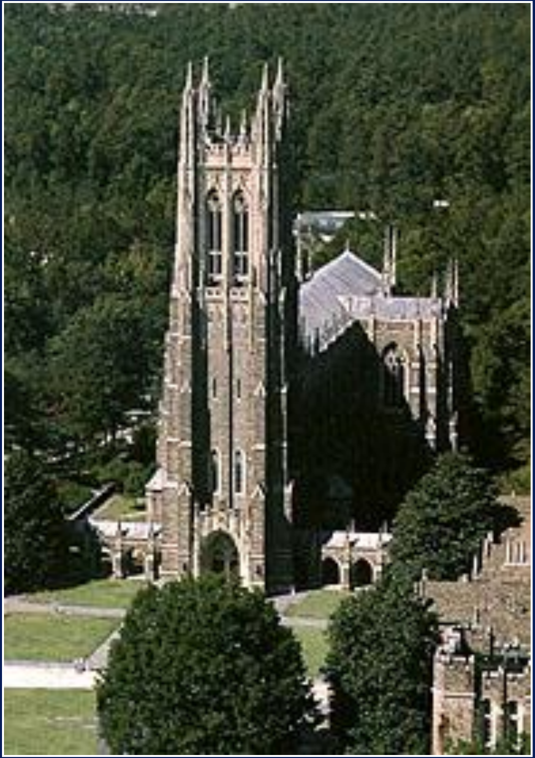
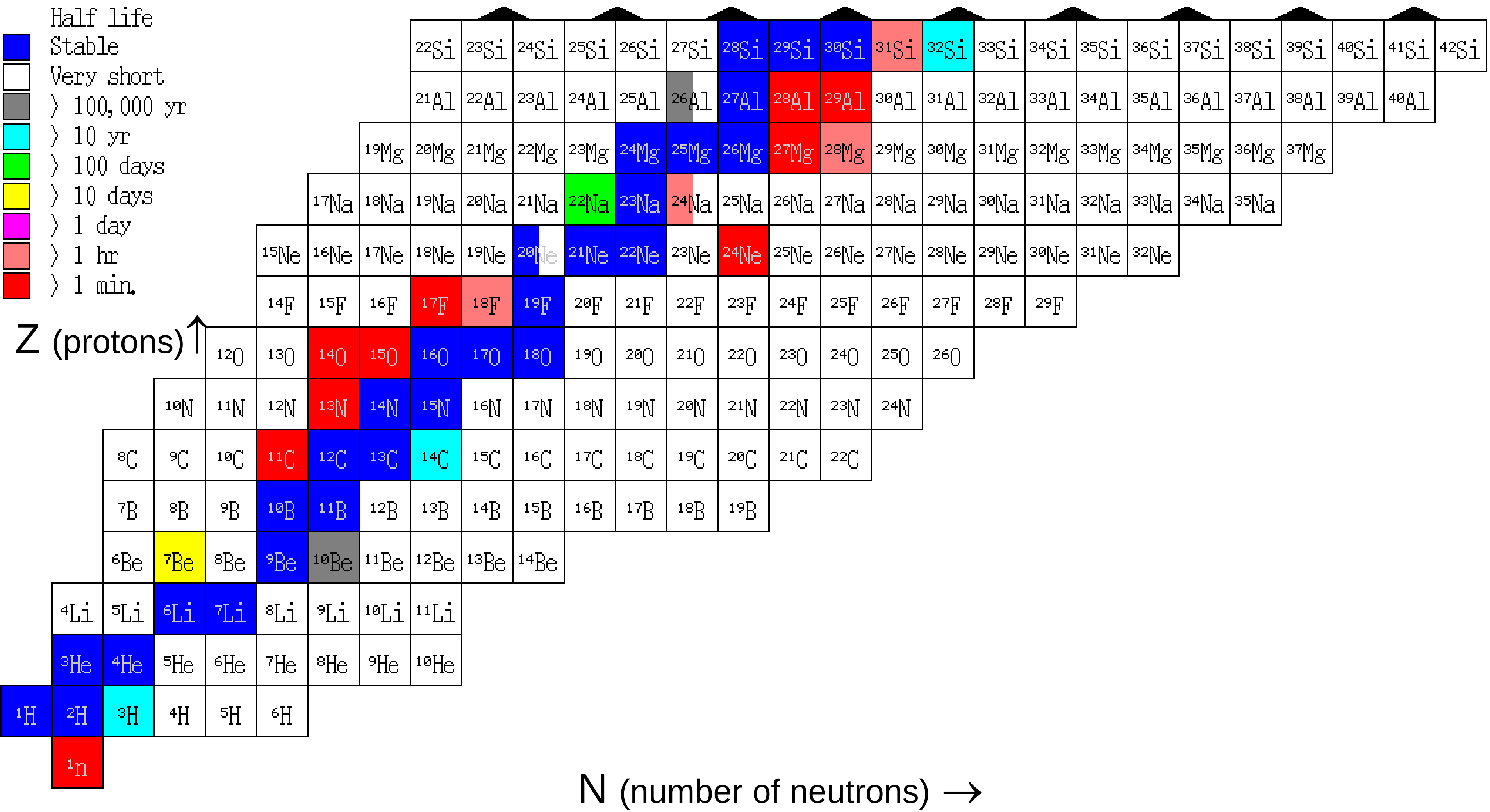


Chart of Nuclides



Nuclear Data Evaluation Lab.

Korea Atomic Energy Research Institute

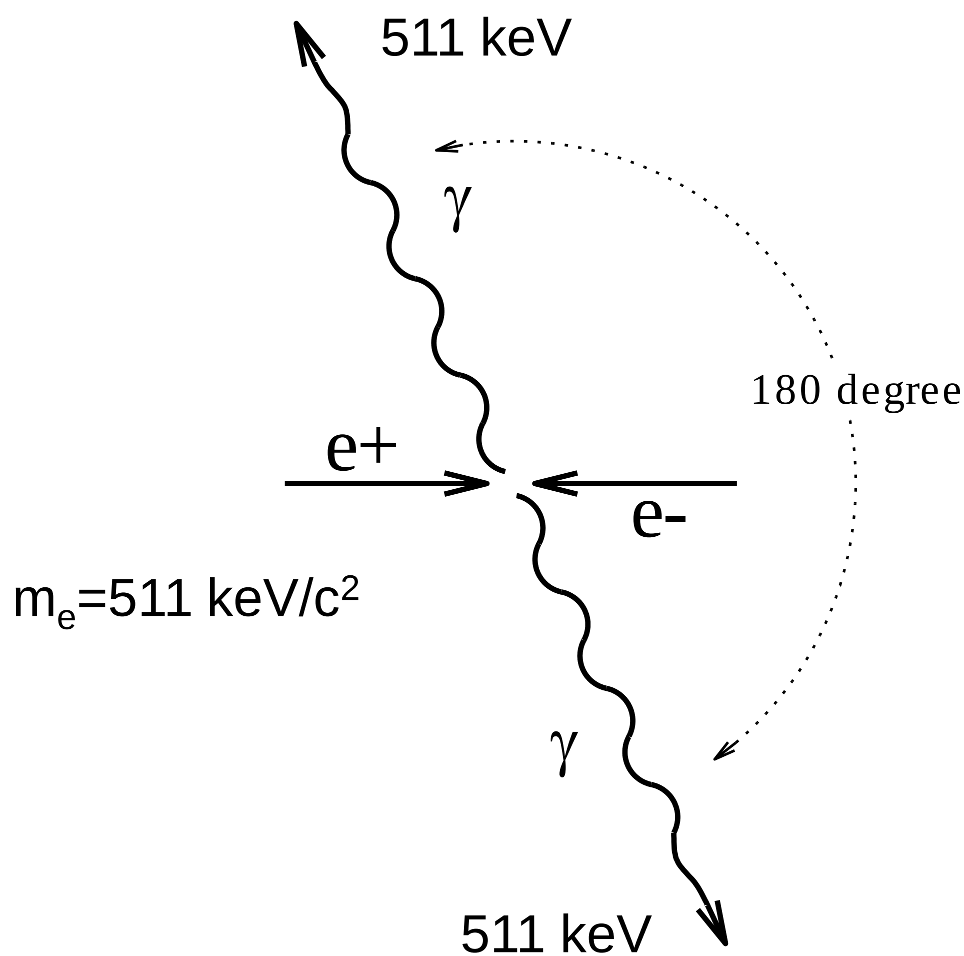
Positron Decay

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z-1}Y_{N+1} + e^{+} + \upsilon $$

Nuclide	half-life
C-11	20.3 min
N-13	10 min
O-15	124 sec
F-18	110 min
Rb-82	*75 sec*

e.g., $^{18}F \rightarrow {}^{18}O + e^{+} + \nu$

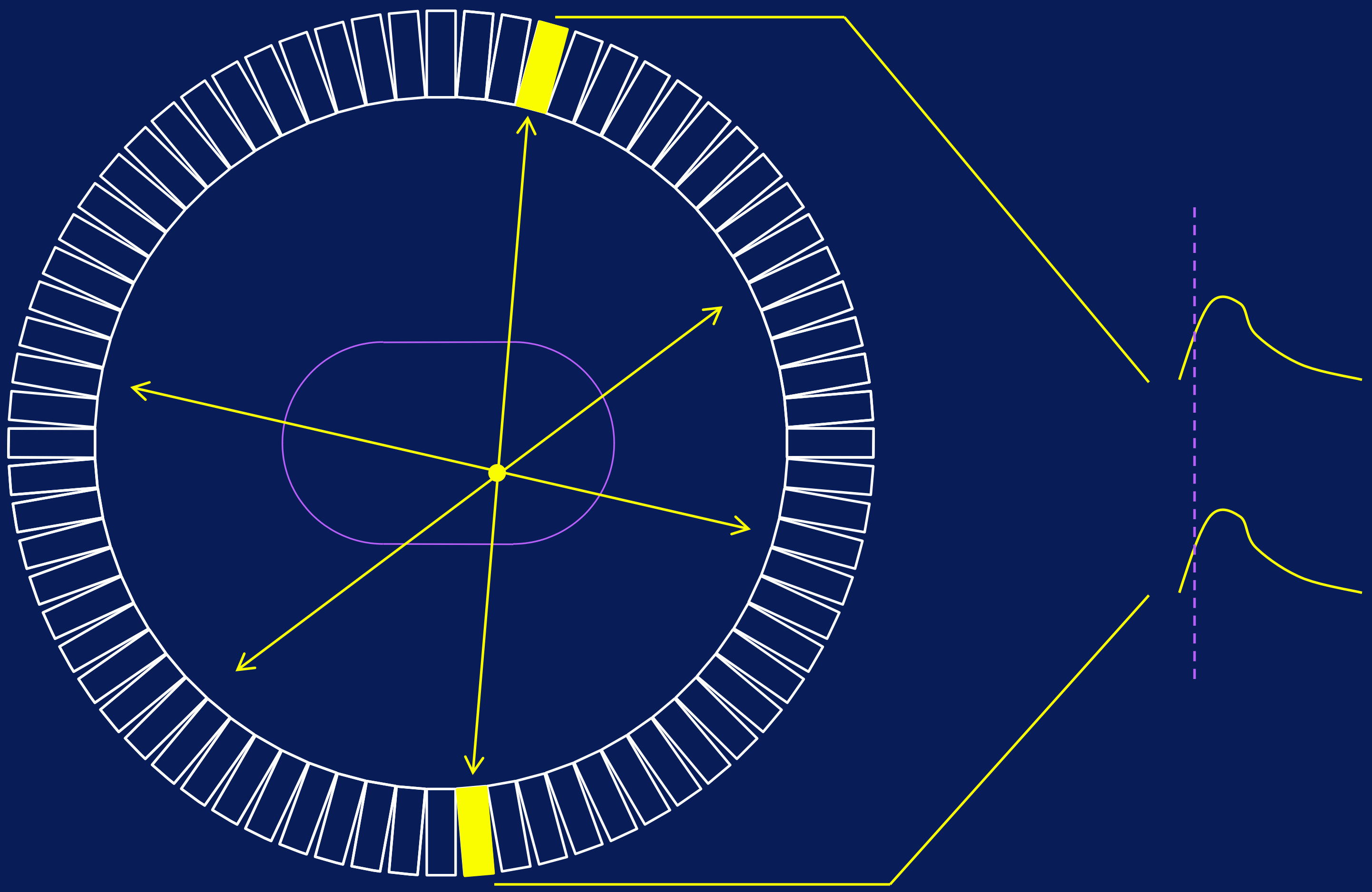
Positron Annihilation



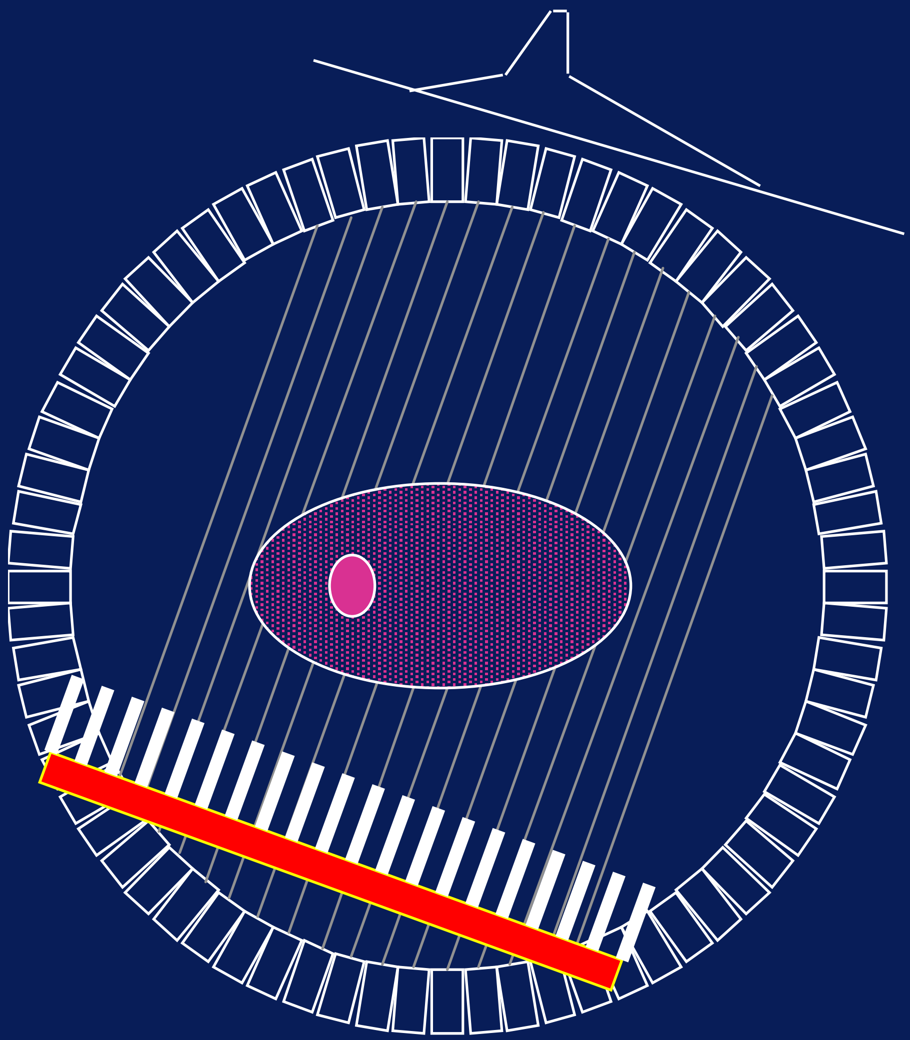
Linear attenuation values for lead and water

Material (Eγ)	μ (cm^{-1})	HVT (cm)	TVT (cm)
lead (140 keV)	22.7	0.031	0.10
lead (511 keV)	1.7	0.41	1.35
water (140 keV)	0.15	4.6	15.4
water (511 keV)	0.096	7.2	24.0

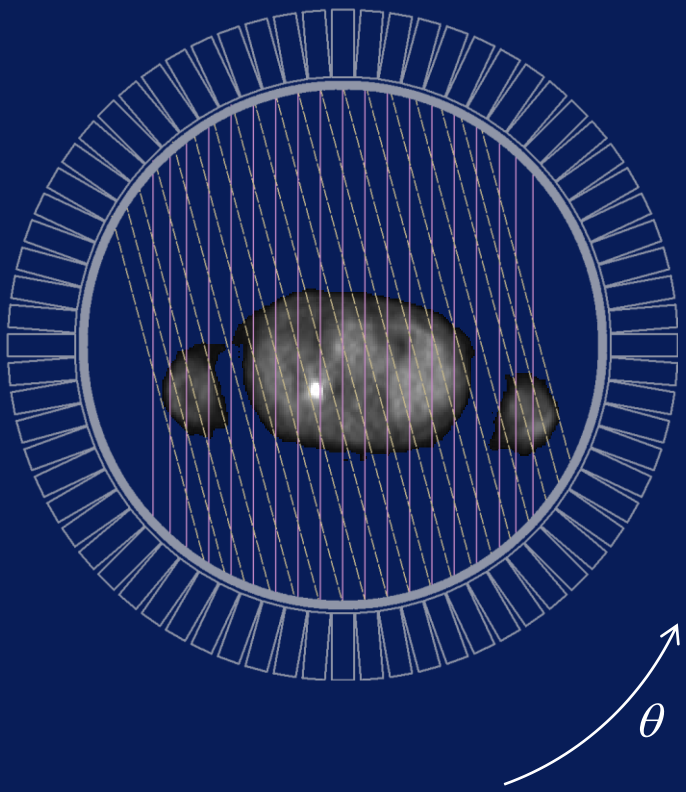
Coincidence Event



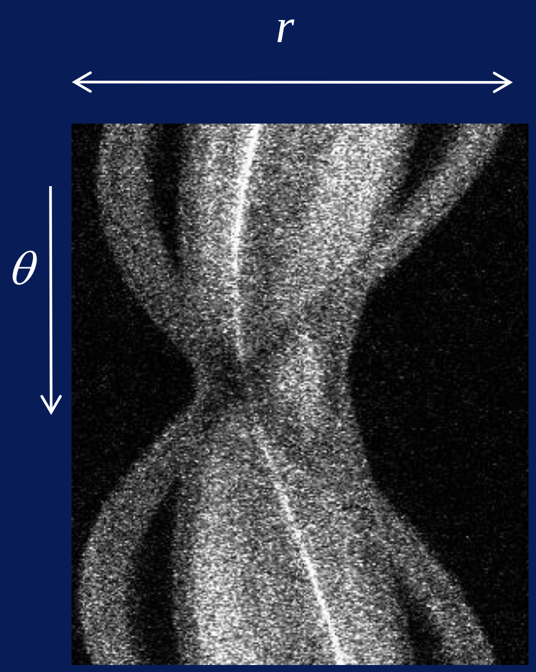
Projections



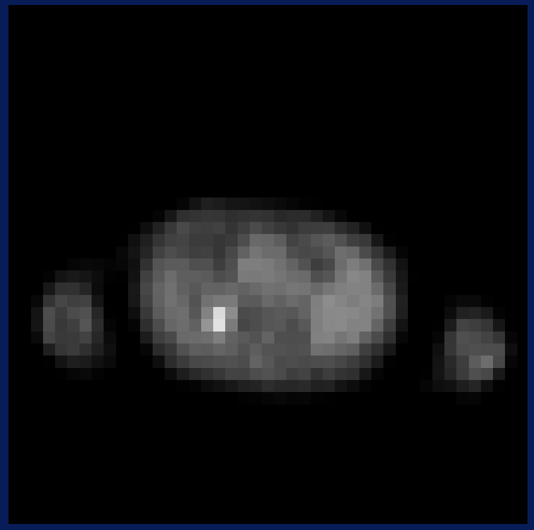
Acquisition and Reconstruction



Data Acquisition



Sinogram
(raw data)

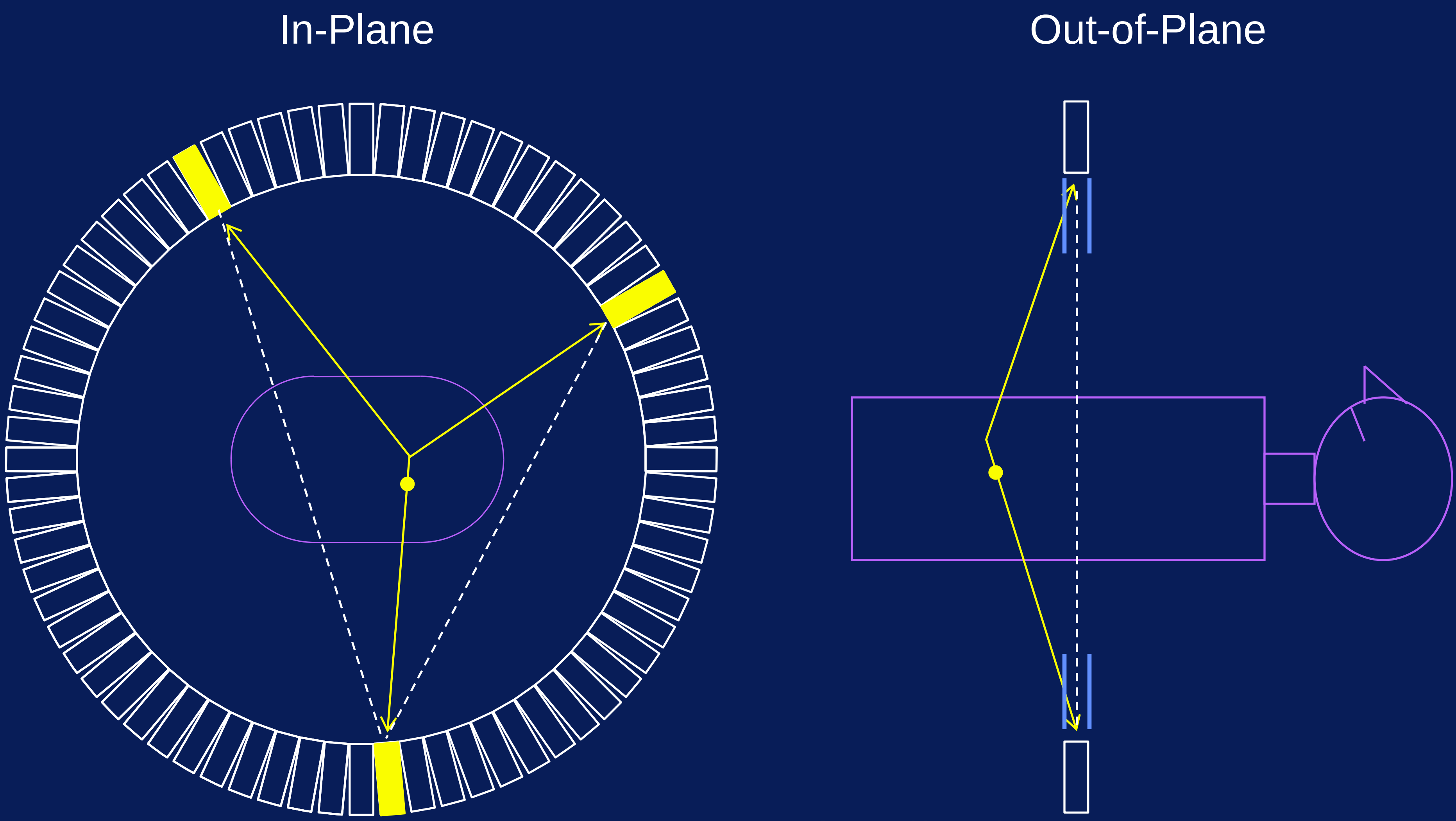


Reconstructed Image

PET Background Events

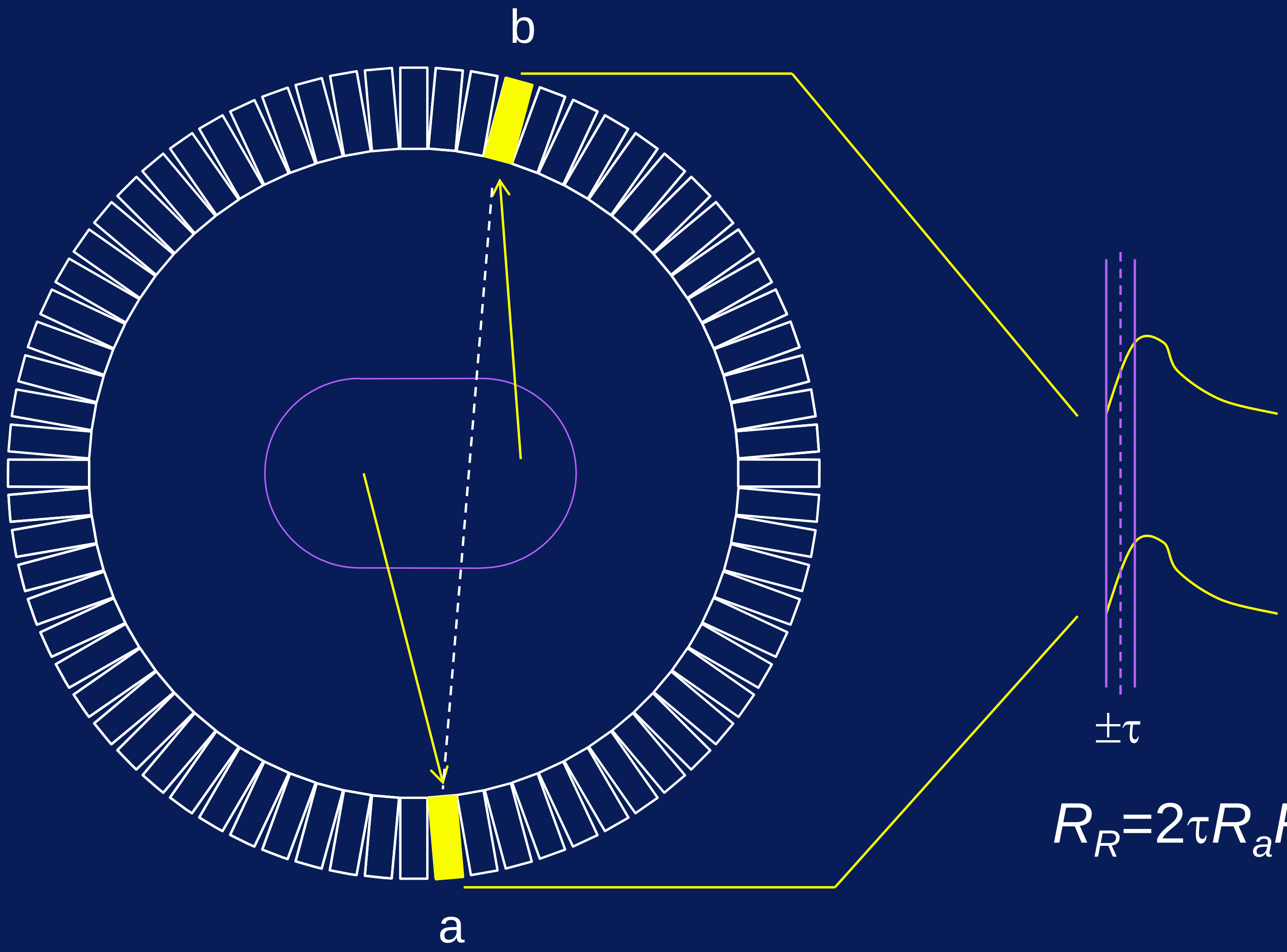
- Scatter
- Randoms

Scattered Coincidence Event



<u>Scatter Fraction S/(S+T)</u>
With septa ~10-20%
w/o septa   ~30-80%

Random Coincidence Event



Correcting Background; Noise Equivalent Counts

$$P_{prompts} = T_{trues} + S_{scatter} + R_{randoms}$$

$$T' = P - S' - R'$$ *(Estimation of true events by subtracting S and R estimates)*

$$\langle T' \rangle = \langle P \rangle + \langle S' \rangle + \langle R' \rangle = P + \begin{vmatrix} 0 \\ R \end{vmatrix} \geq P \geq T$$

(Variance propagation and Poisson properties. 0 vs. R depends on randoms correction method.)

$$SNR \equiv \frac{T'}{\sqrt{\langle T' \rangle}} \approx \frac{T}{\sqrt{P + \begin{vmatrix} 0 \\ R \end{vmatrix}}}; \; NEC = \frac{T^2}{P + \begin{vmatrix} 0 \\ R \end{vmatrix}} = \frac{T}{(1 + S/T + (2\,?)R/T)}$$

S and R refer to scattered and random events on LORs that subtend the imaged object.

More background $\rightarrow$ more statistical image noise.

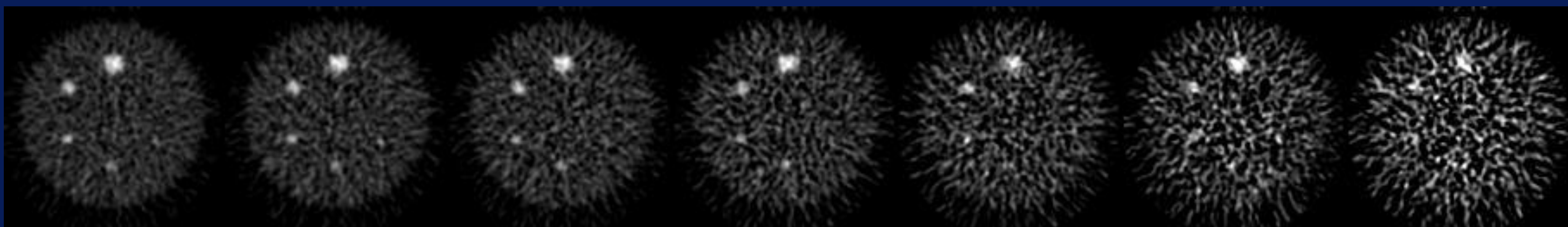
NEC Examples

$$NEC = \frac{T}{\left(1 + S/T + R/T\right)}$$

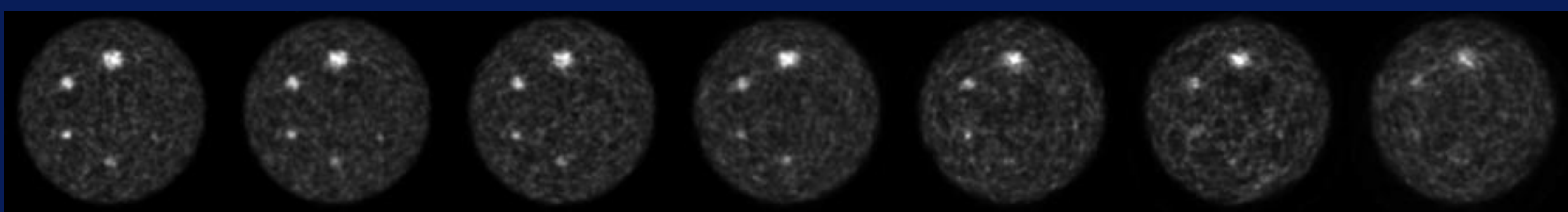
Prompts	Trues	Scatter	SF	NEC
100	100	0	0	100
200	100	100	0.5	50
400	200	200	0.5	100

Image Noise and Lesion Detection

FBP



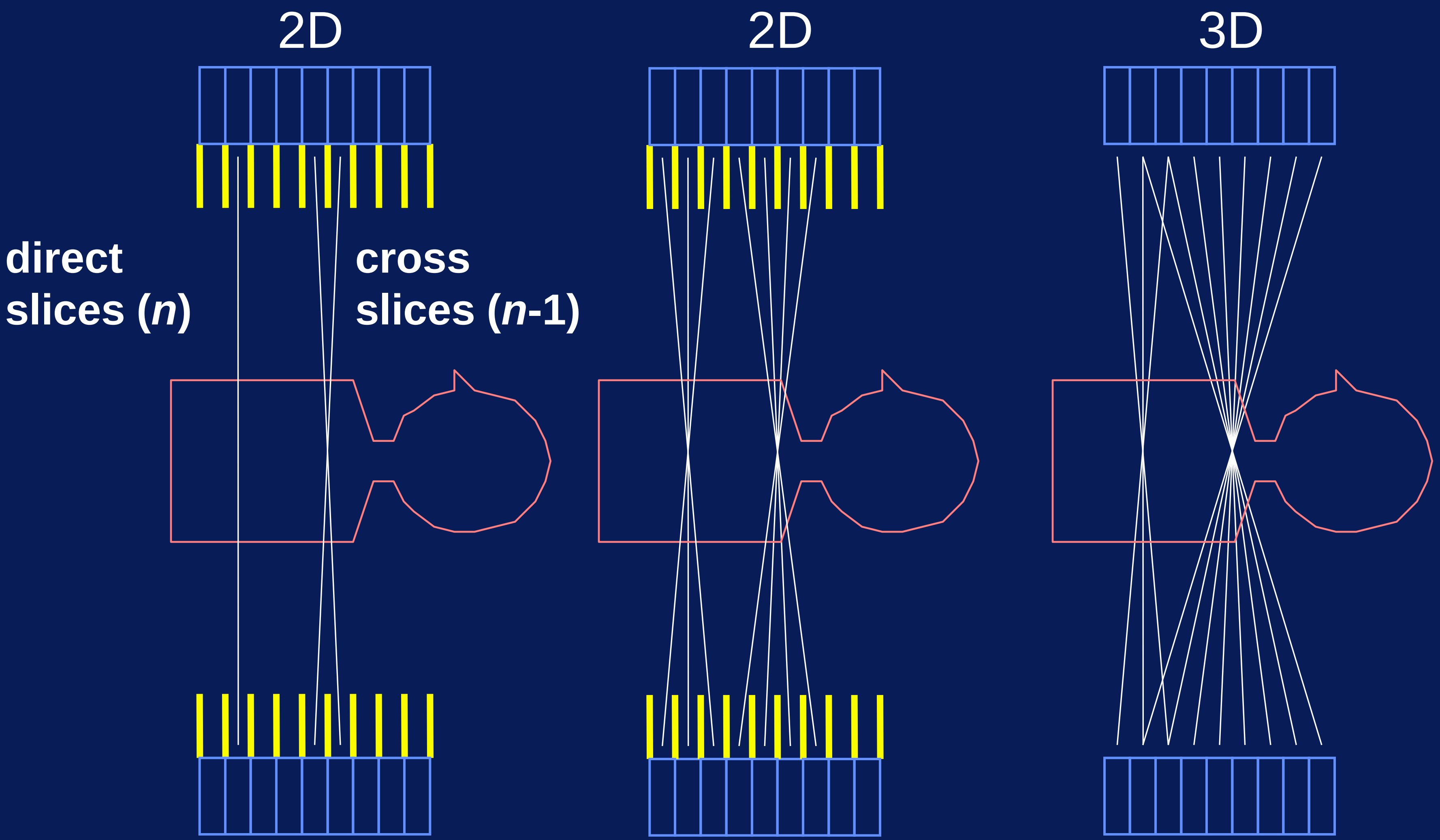
OS-EM



600s 300s 150s 75s 38s 19s 10s

Multiple Rings, 2D – 3D

For *n* detector rings:



total slices = 2*n*-1

Higher Sensitivity
(degraded axial resolution)

Time Of Flight PET: The influence of background

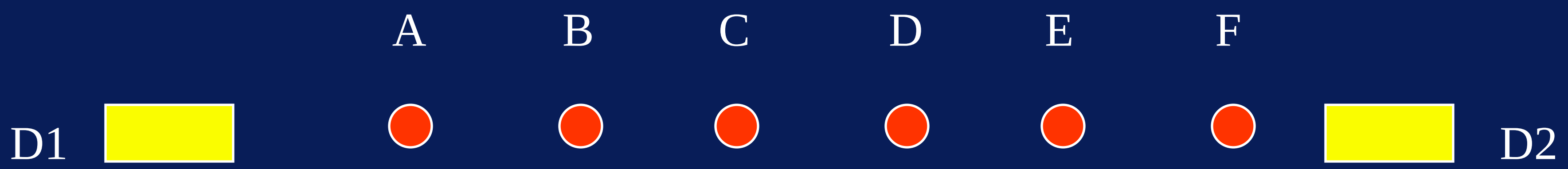
How strong is source A?

Detectors measure counts C_A+C_B.

S.D. of measurement is $\sqrt{C_A + C_B}$

SNR = $C_A / \sqrt{C_A + C_B}$

The influence of even more background



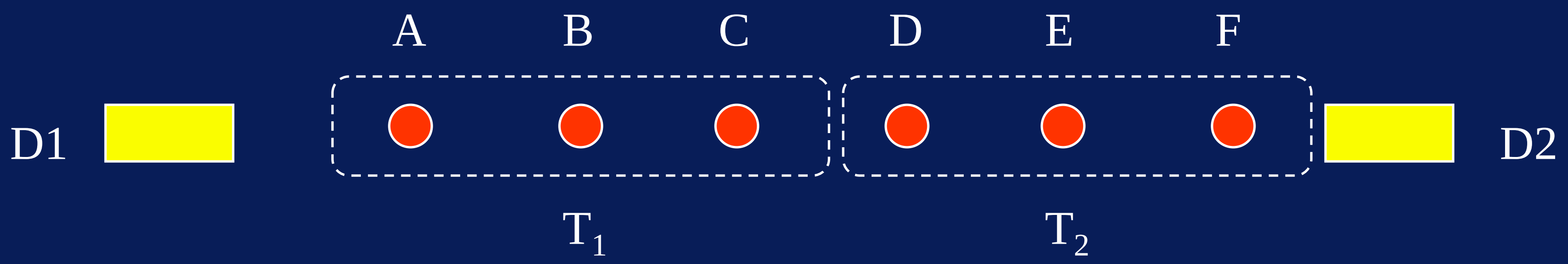
SNR of measurement of A?

$$\text{SNR} = C_A / \sqrt{C_A + C_B + C_C + C_D + \ldots}$$

If all N sources are equal,

$$\text{SNR} = C_A / \sqrt{N C_A}$$

Magical new detectors



Can distinguish between counts originating in segment T_1 and counts originating in segment T_2.

If all N sources are equal,

SNR = $C_A / \sqrt{nC_A}$

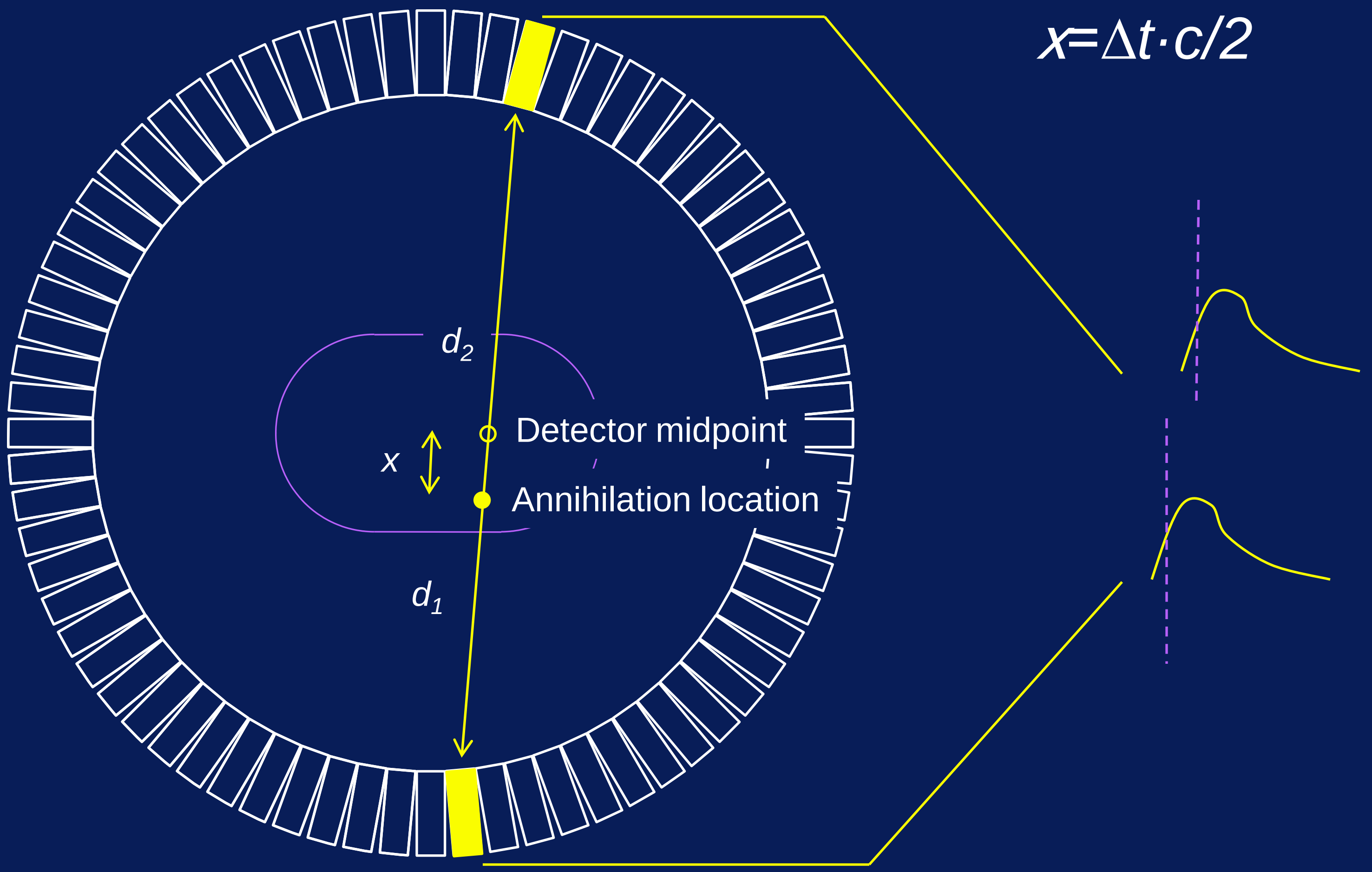
where n is the number of sources within zone T_1.

SNR improvement of sqrt(N/n); similar to counting longer by N/n.

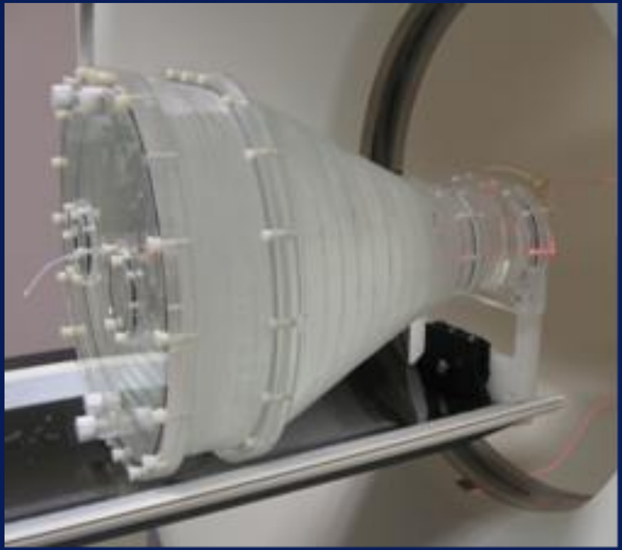
Time-of-Flight PET

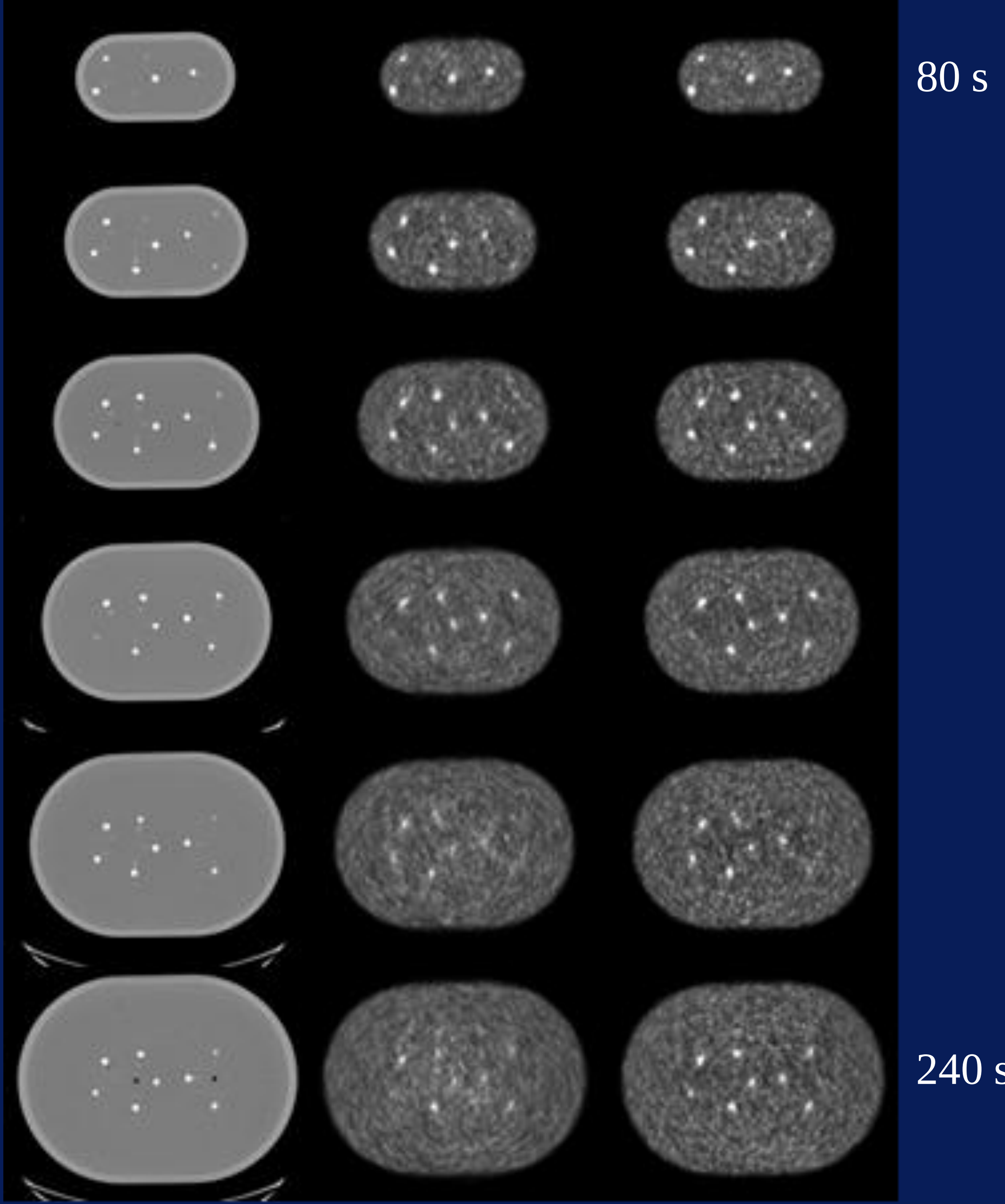
$$\Delta t=(d_2-d_1)/c=2x/c$$

$$x=\Delta t \cdot c/2$$

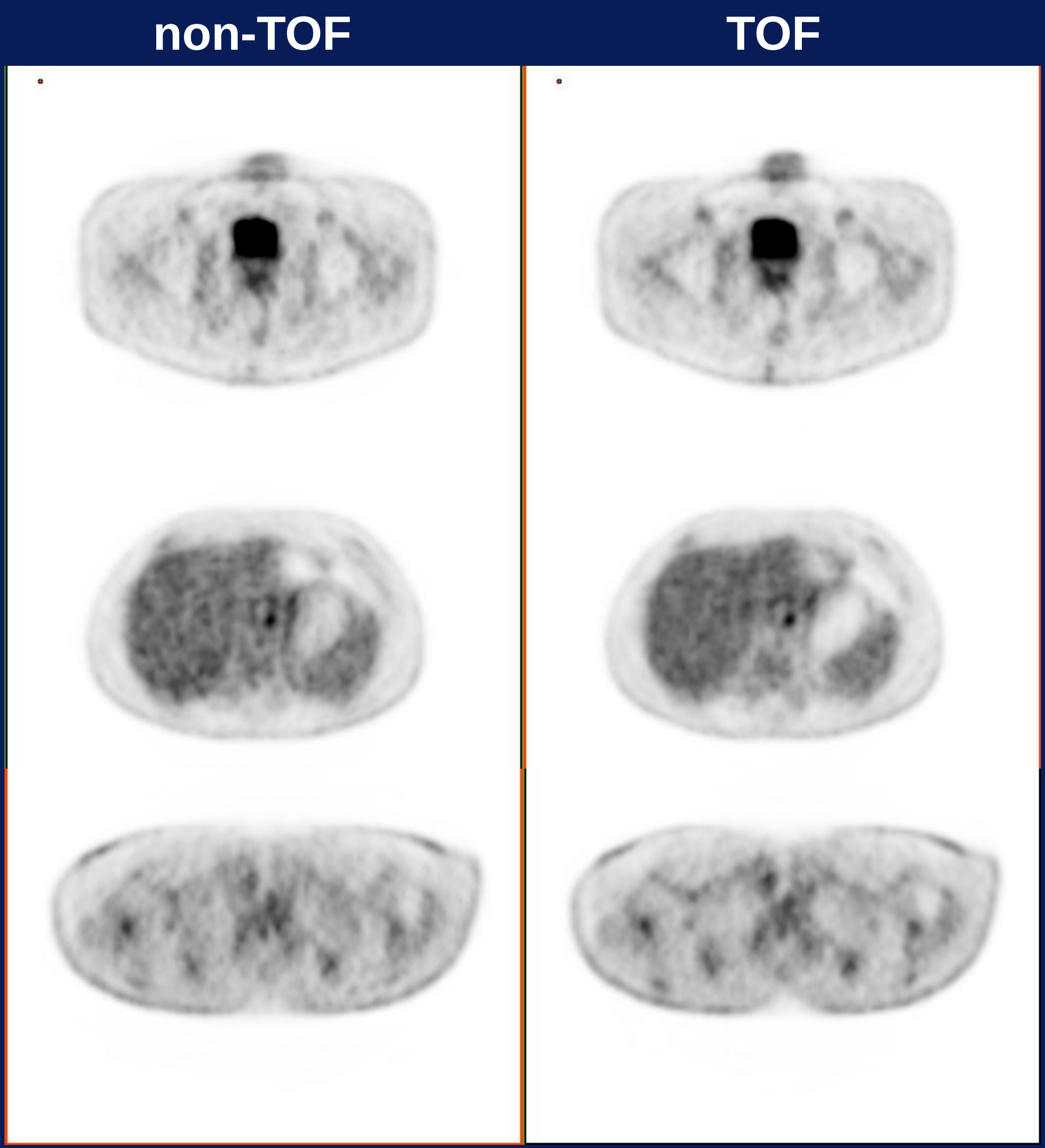


Fillable, Tapering Phantom with 1 cm lesions



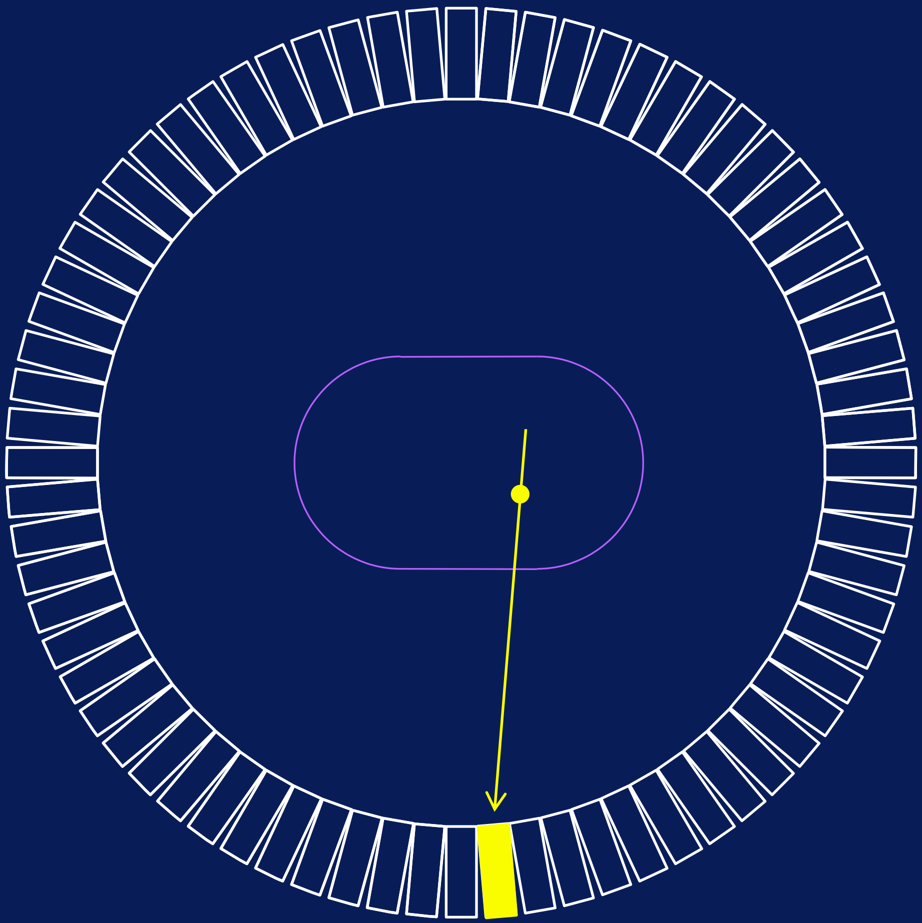


Clinical Example 2

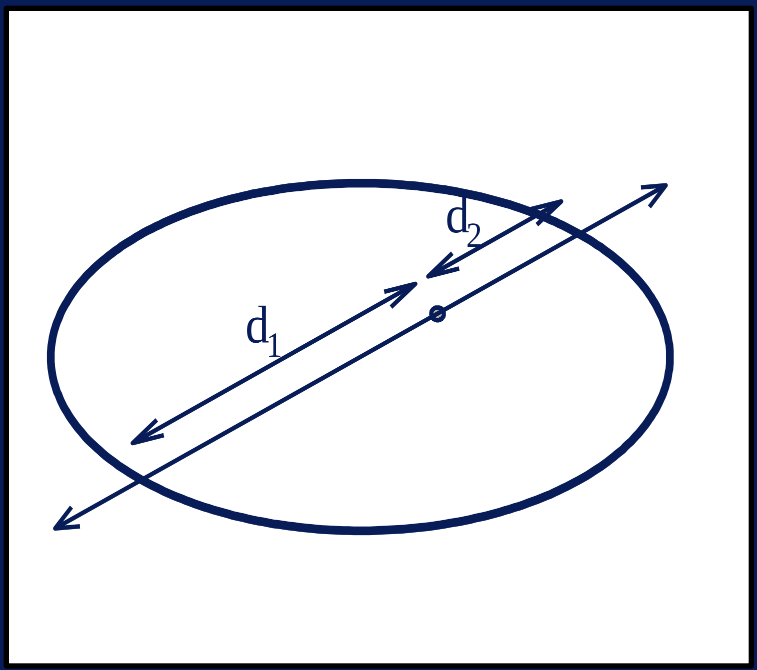


68 kg male

Attenuated Event



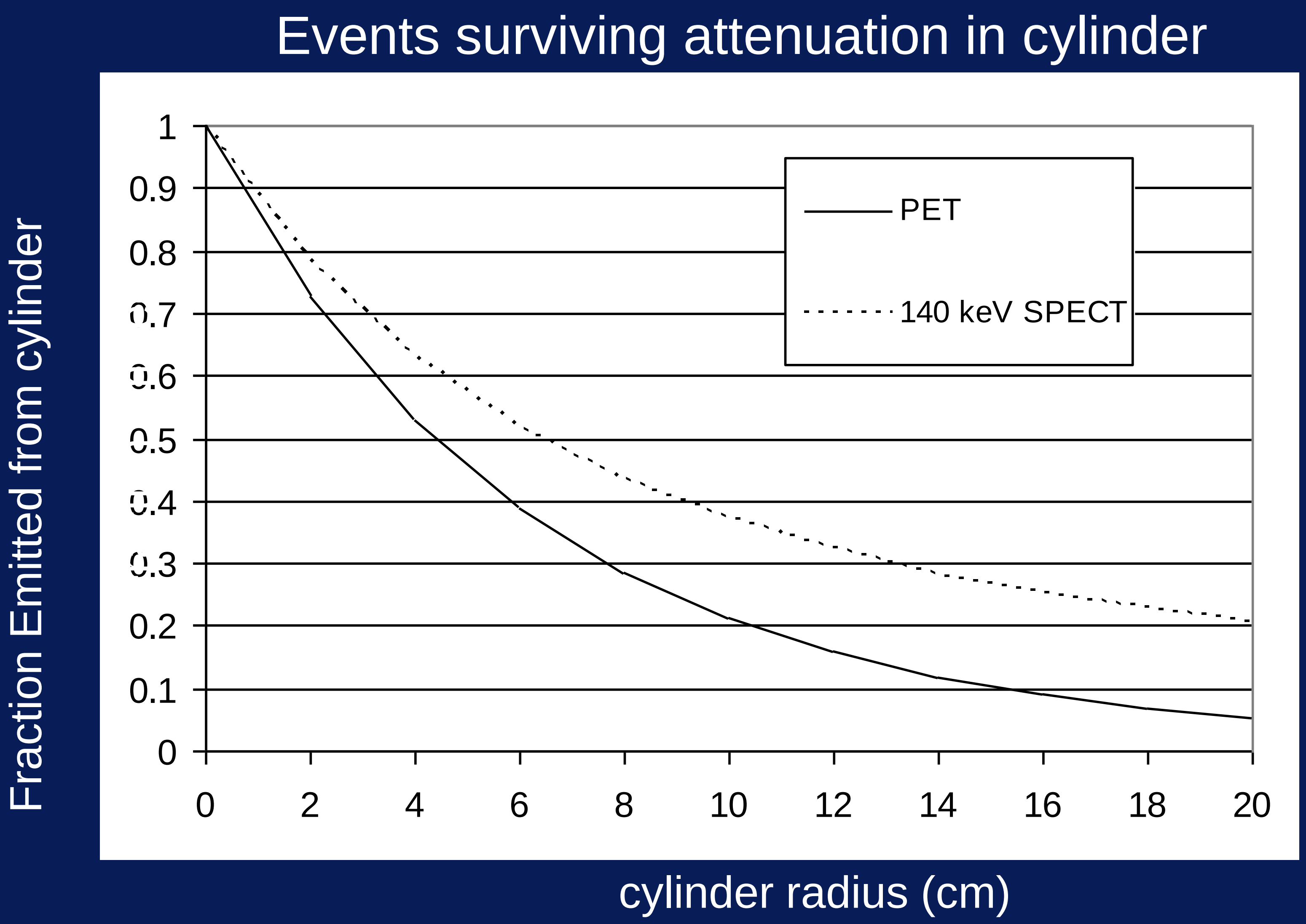
Coincidence Attenuation



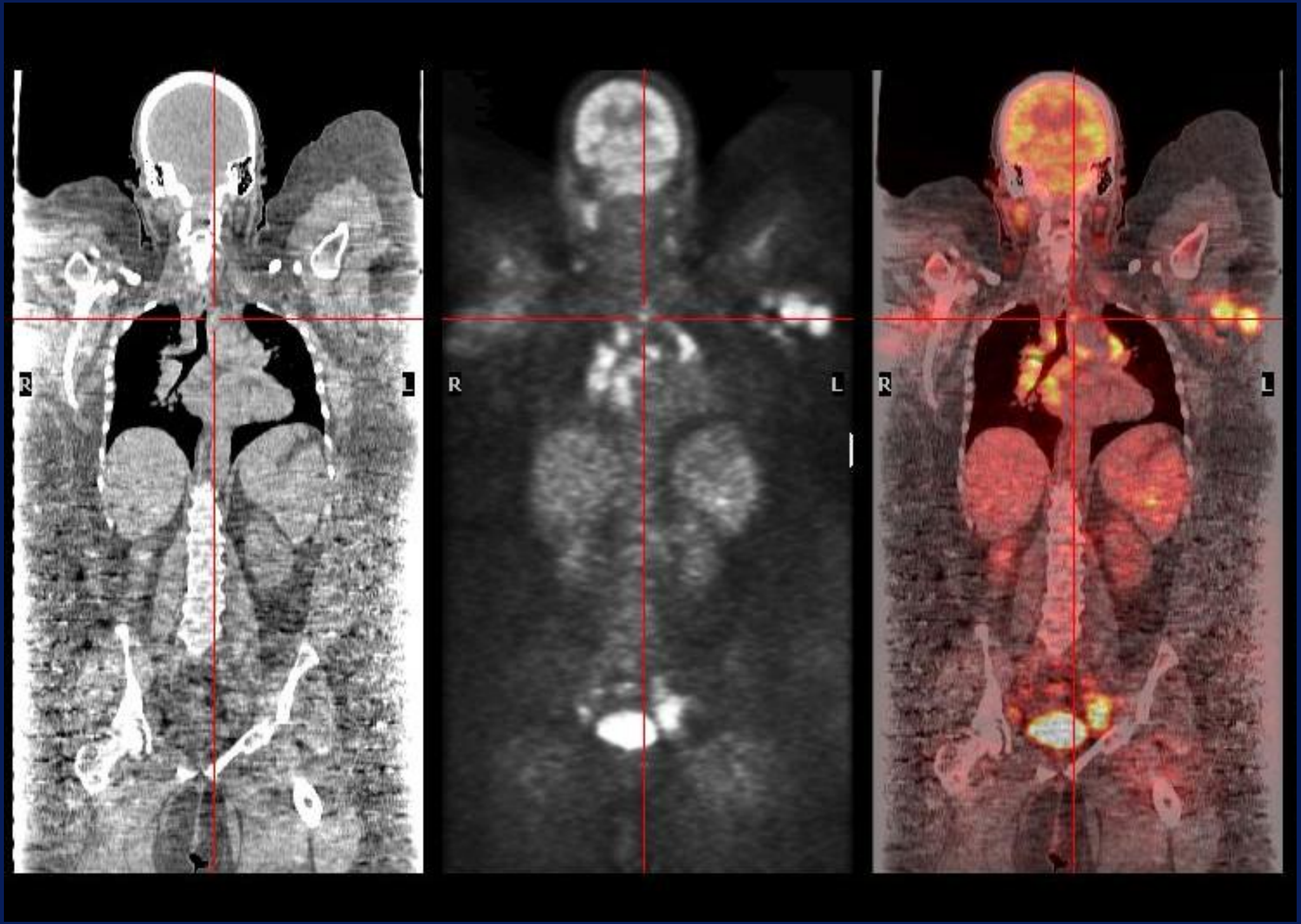
$$
\begin{aligned}
P_C &= P_1 P_2 \\
&= e^{-\mu \cdot d_1} e^{-\mu \cdot d_2} \\
&= e^{-\mu \cdot (d_1 + d_2)}
\end{aligned}
$$

Annihilation radiation emitted along a particular line of response has the same attenuation probability, regardless of where it originated on the line.

Attenuation losses - PET and SPECT



Obese Patient

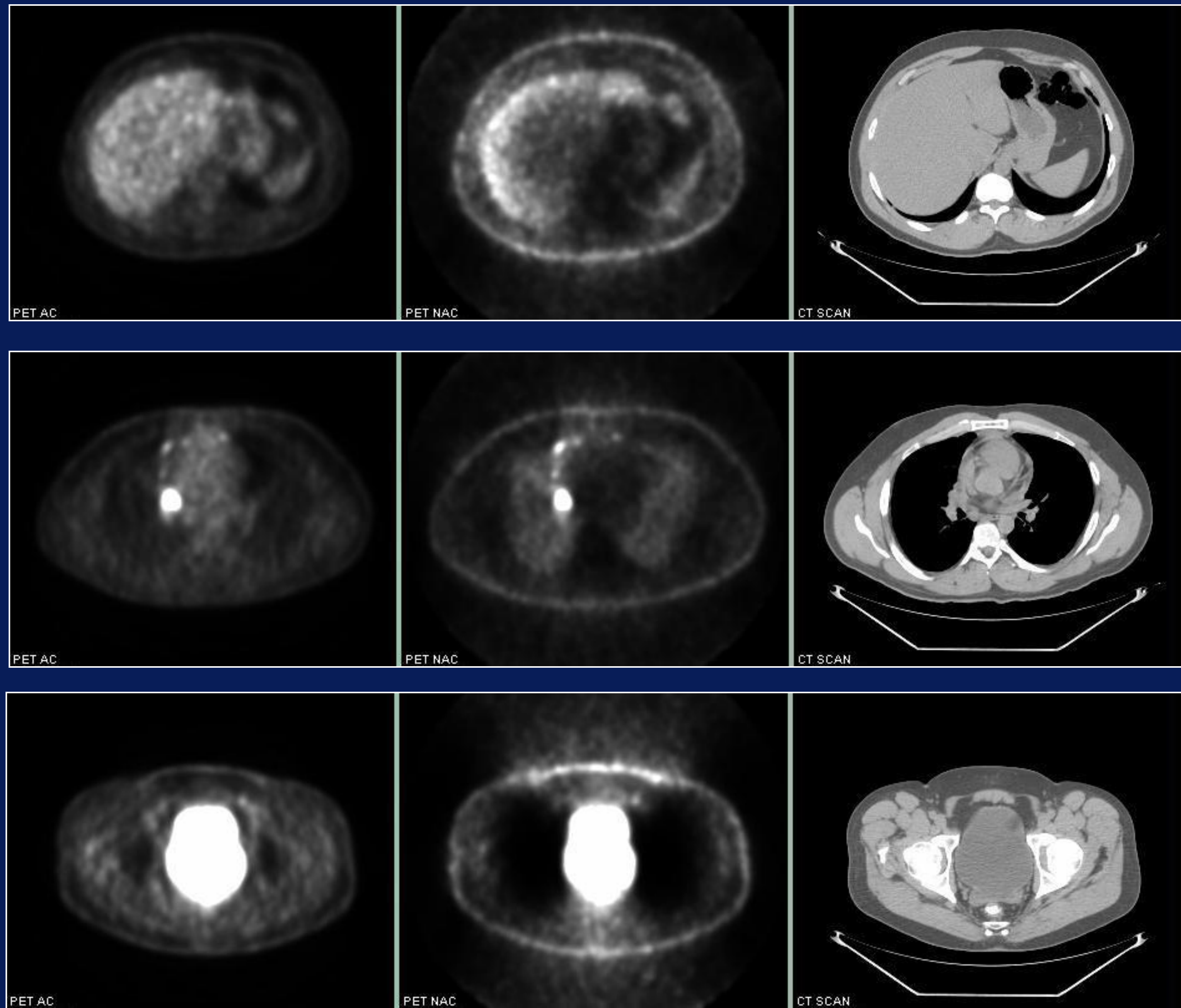


Technologist Size Variations

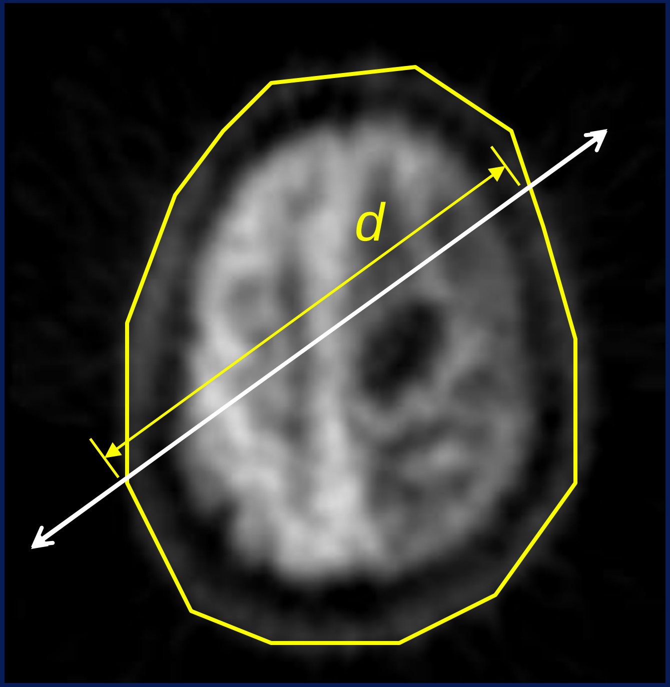
$m_b \sim 2.5 m_a$

Attenuation Effects

AC NAC x-ray CT

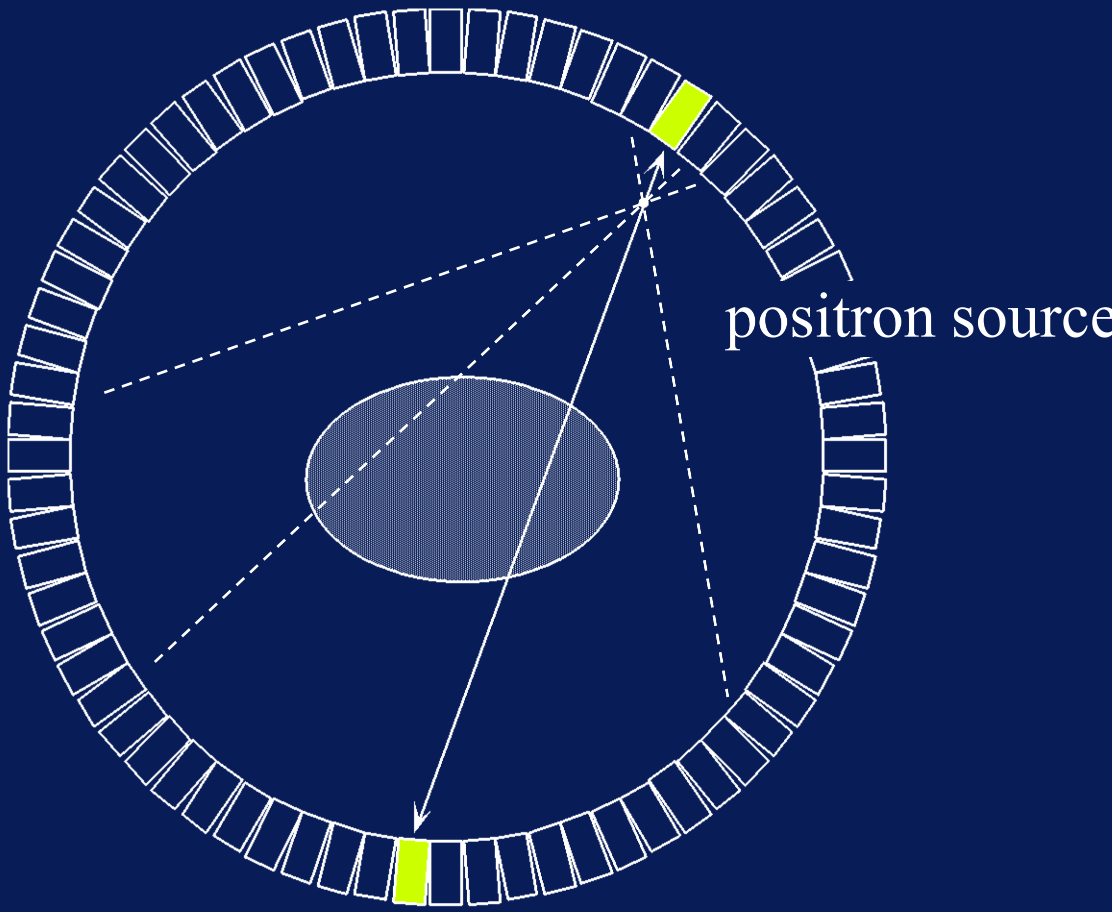


Calculated Attenuation Correction

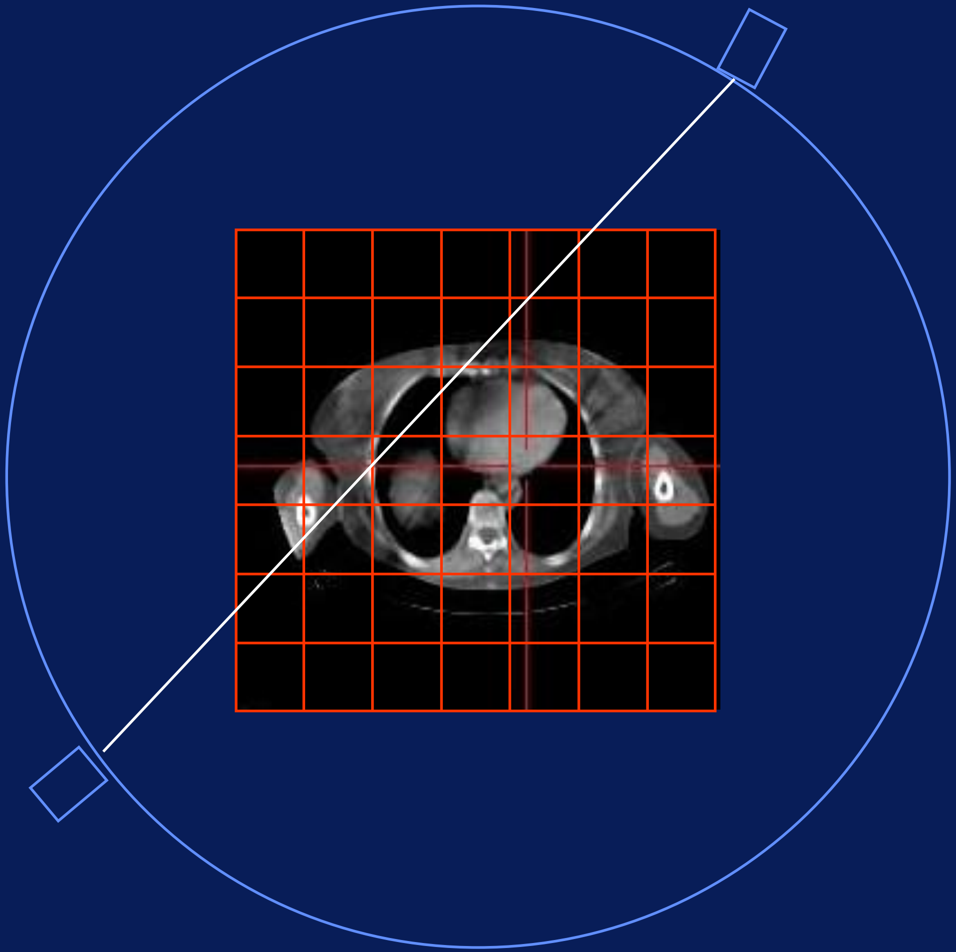


$$I = I_0 e^{-\mu d}$$

Transmission Attenuation Measurement



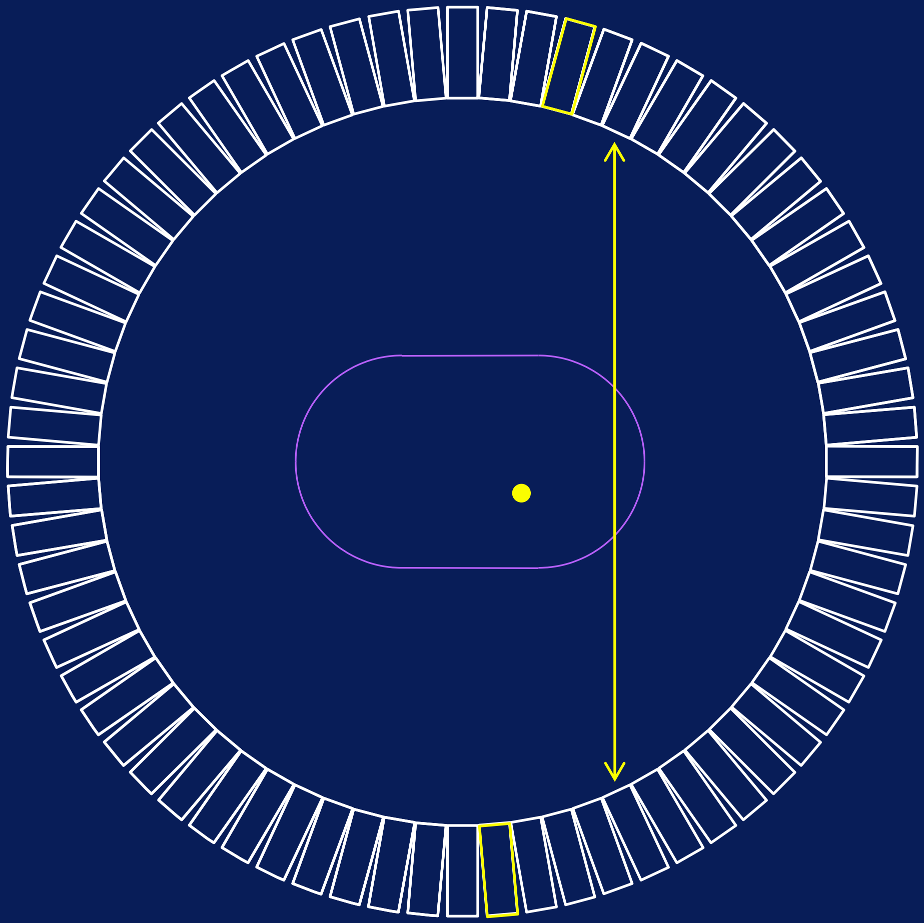
Using CT image for AC



Attenuation Correction Accuracy

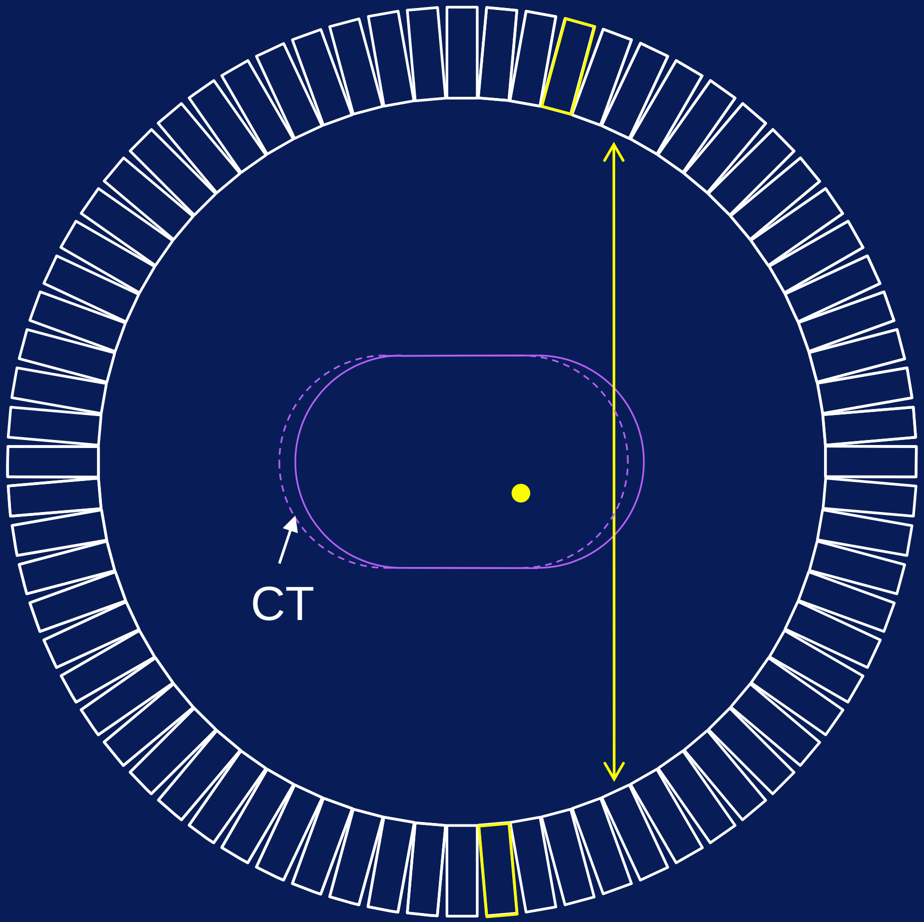
- CT-based attenuation correction is performed on almost all PET studies.
- Is it being done well?
  - Is the CT accurate (e.g., water = 0)?
  - Is the CT accurate under all relevant conditions?
  - Is the translation between CT# and 511 keV μ appropriate?
  - Patient motion between CT and PET?
  - …

Attenuation Correction



Photons emitted along this line will be attenuated by a factor that can be determined from the corresponding CT scan.

Attenuation Correction



The biggest source of error in PET AC is patient motion between the CT and the PET scans.
This particular PET photon trajectory will be undercorrected. The intensity on this side of the body will be artificially low.

Spatial Resolution

$$R_{sys} = \sqrt{R_{det}^2 + R_{acol}^2 + R_{range}^2 + b^2}$$

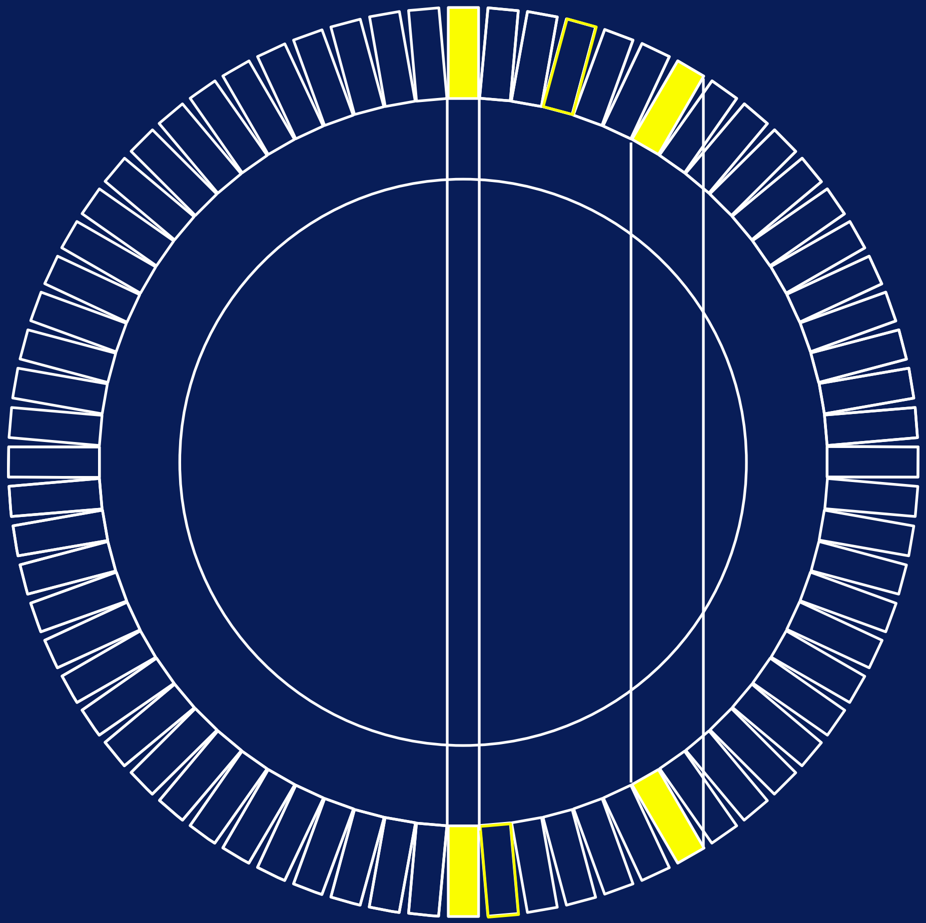
R_{det} = resolution of detectors ($\leq d$)

R_{acol} = resolution from photon acollinearity ($=0.0022D$)

R_{range} = resolution from positron range

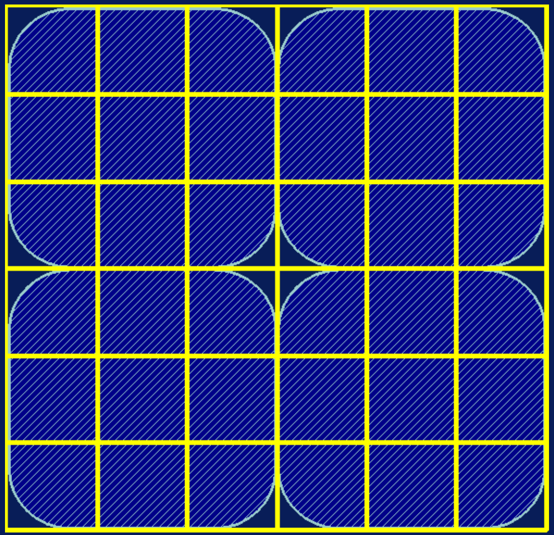
b = block effect

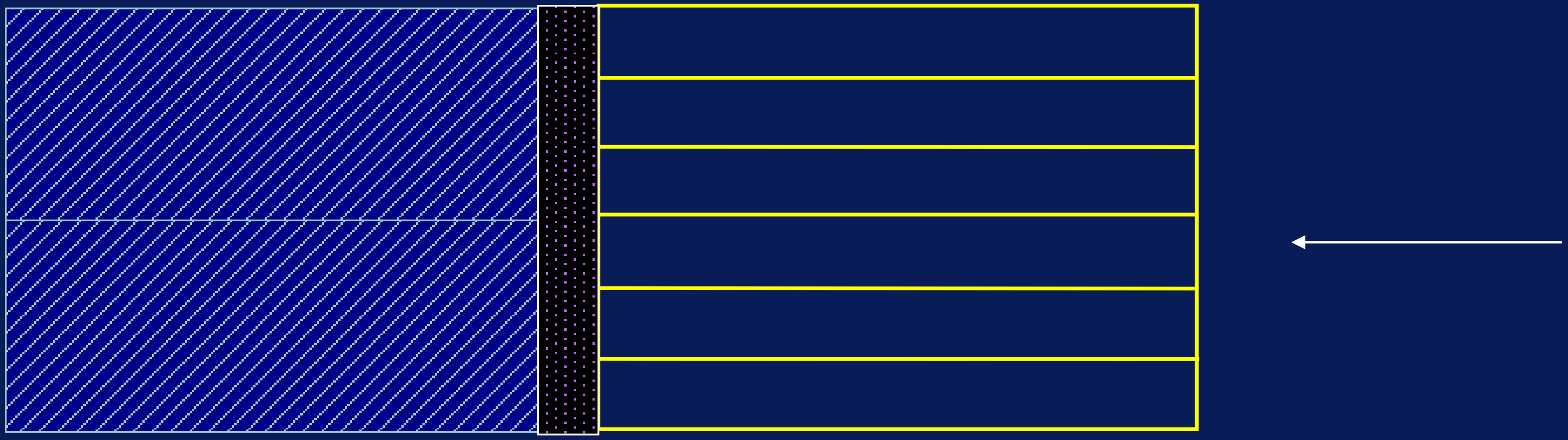
Depth of Interaction Uncertainty



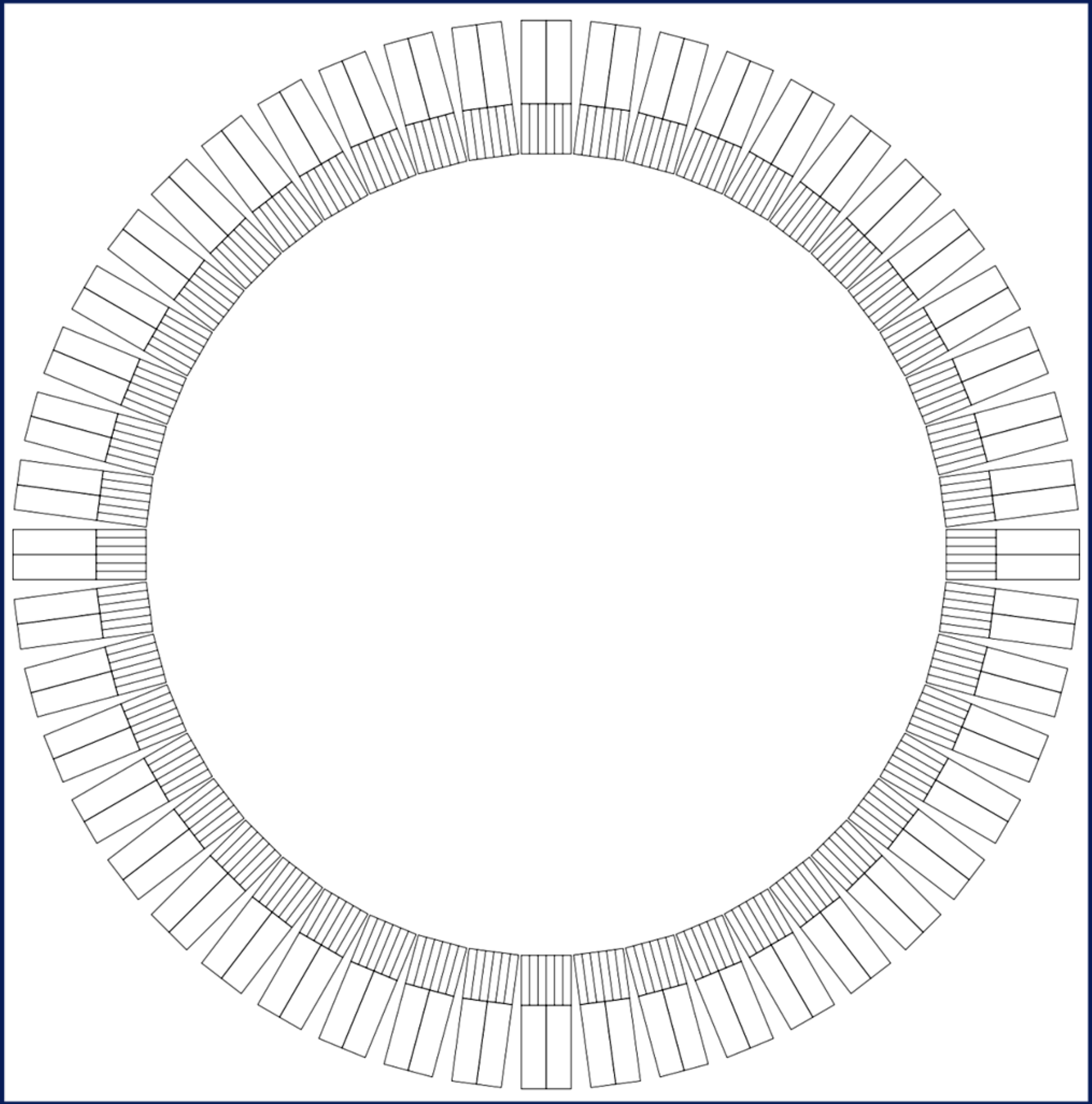
- Uncertainty in the origin of radiation when measured obliquely in a detector.
- Resolution in radial direction worsens with increasing radius.
- High stopping power helps: Interactions more likely in front of detector
- Some high resolution systems sacrifice sensitivity by shortening detectors (to mitigate DOI effects.)
- Some systems (HRRT) use two layers of detectors to lessen effect. Others propose a measurement of the DOI.

Block Detector (GE, Siemens)





PET Ring with Block Detectors



Curved Plate Pixelated Camera (Philips)

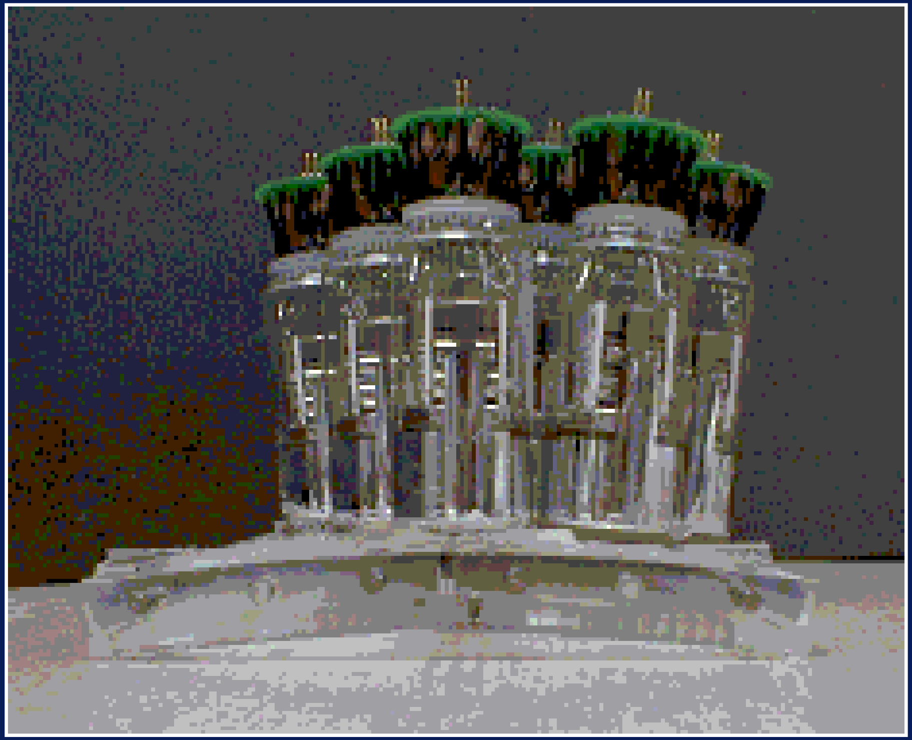
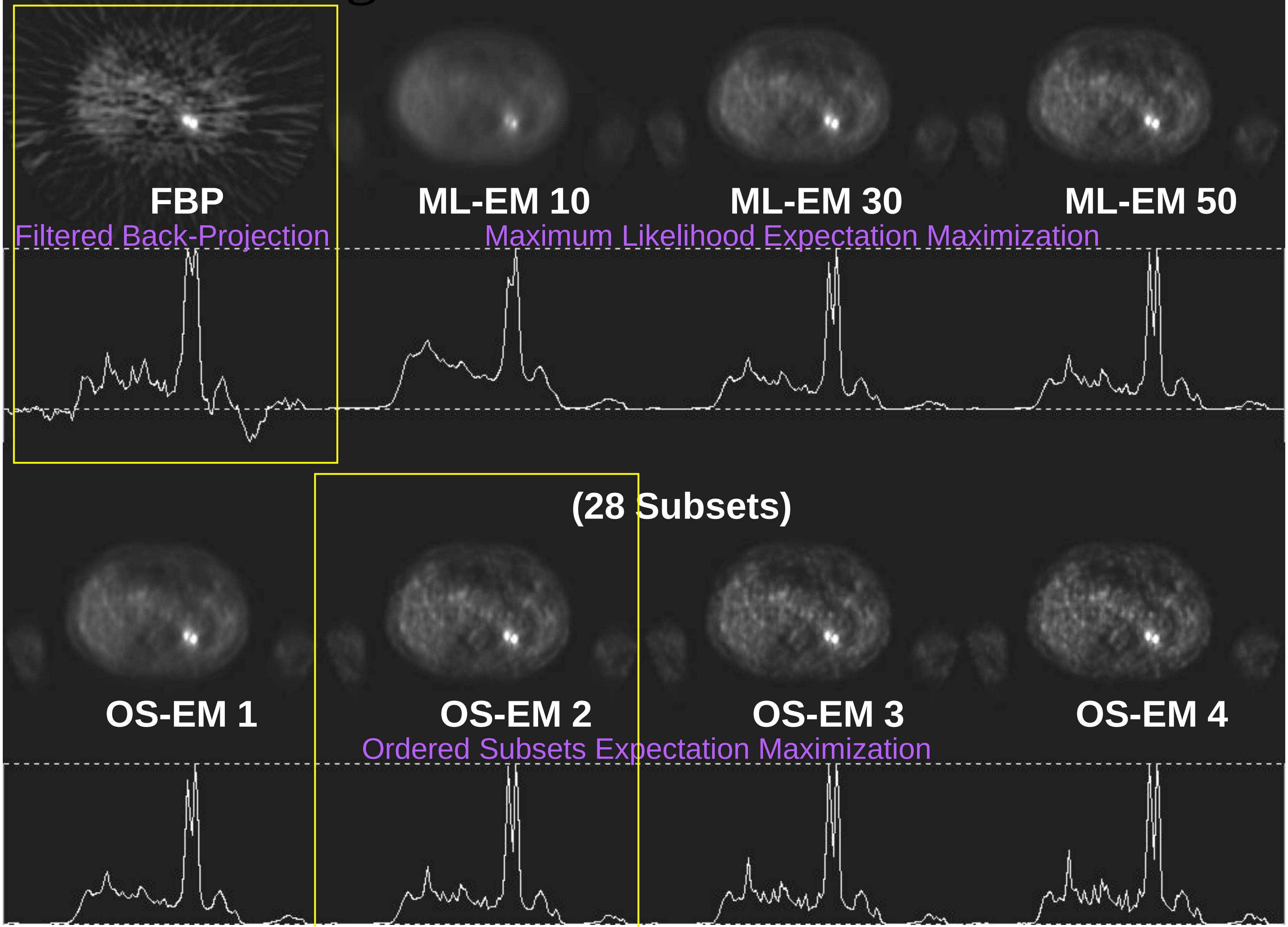


Image Reconstruction

Image Reconstruction Methods



Detection Process

$$m_i = b_i + \sum_{j=1}^{npix} p_{ij} \lambda_j$$

i = projection line (line of response)
j = source pixel
λ_j= radioactivity at voxel j
p_{ij} = probability of emission from j
being measure in i
b_i = background contributing to i
m_i = expected counts on projection i

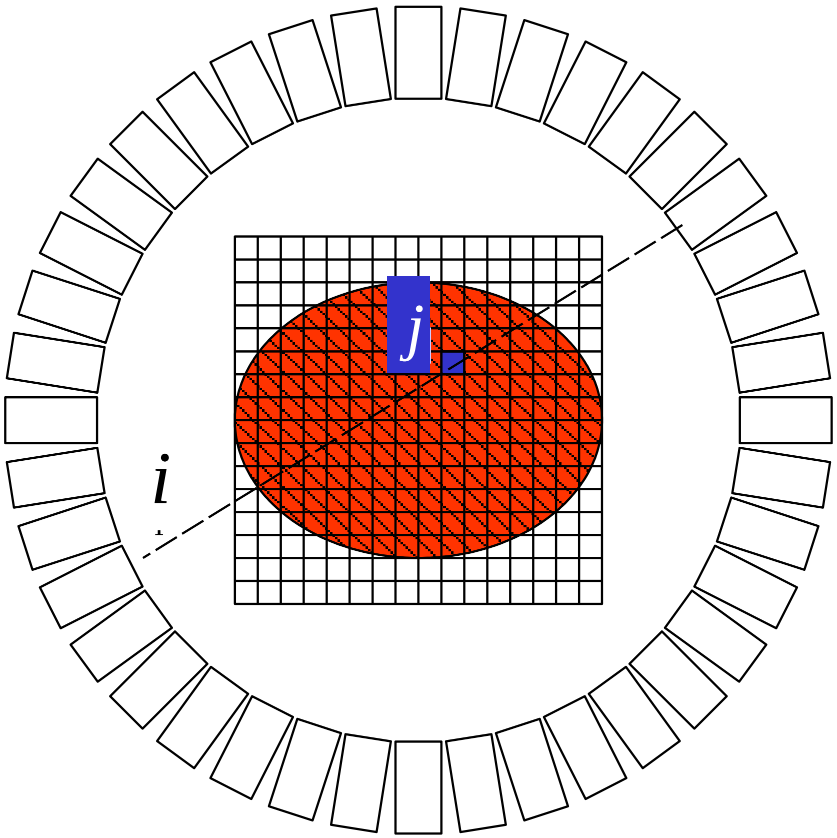
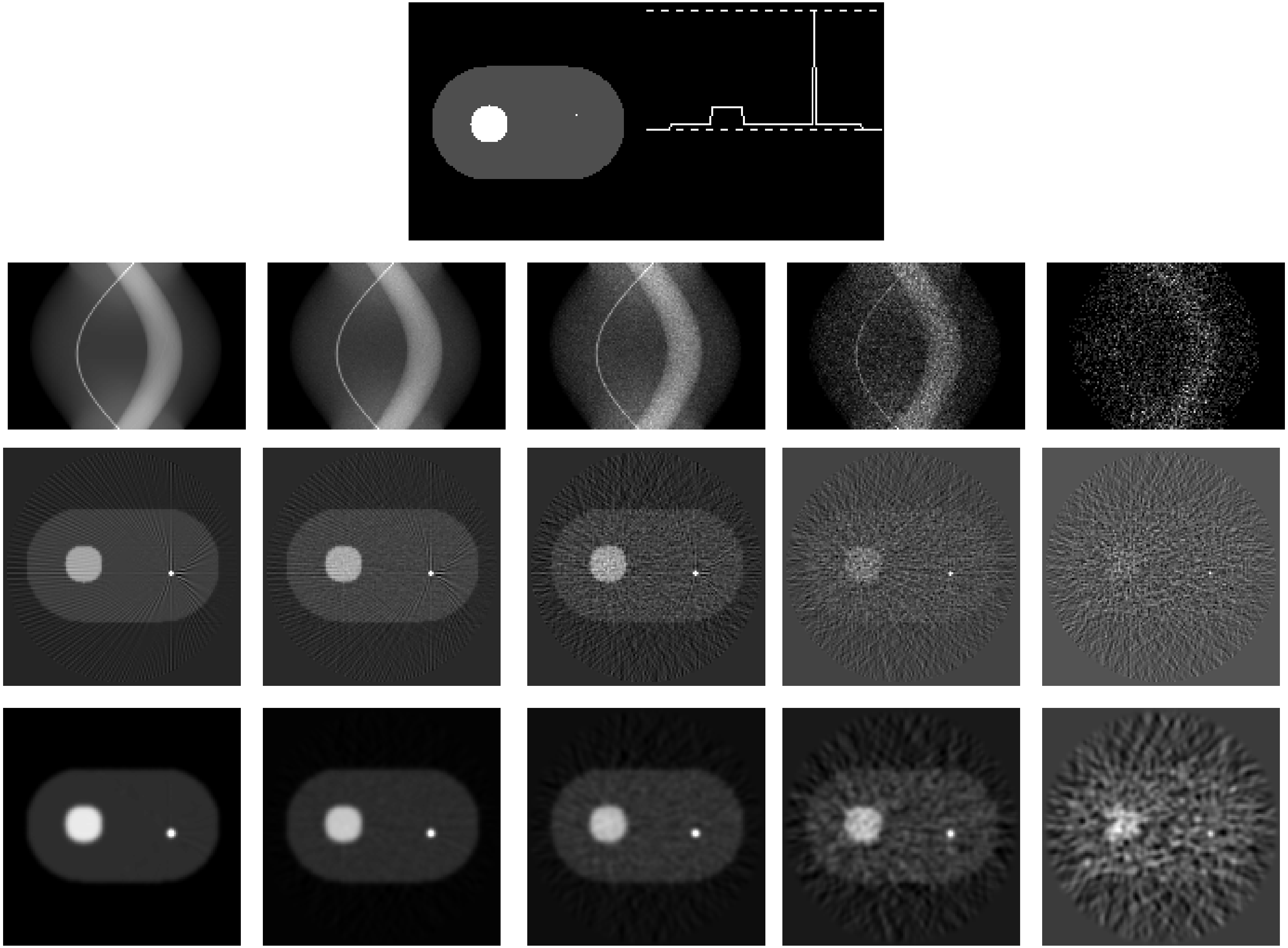


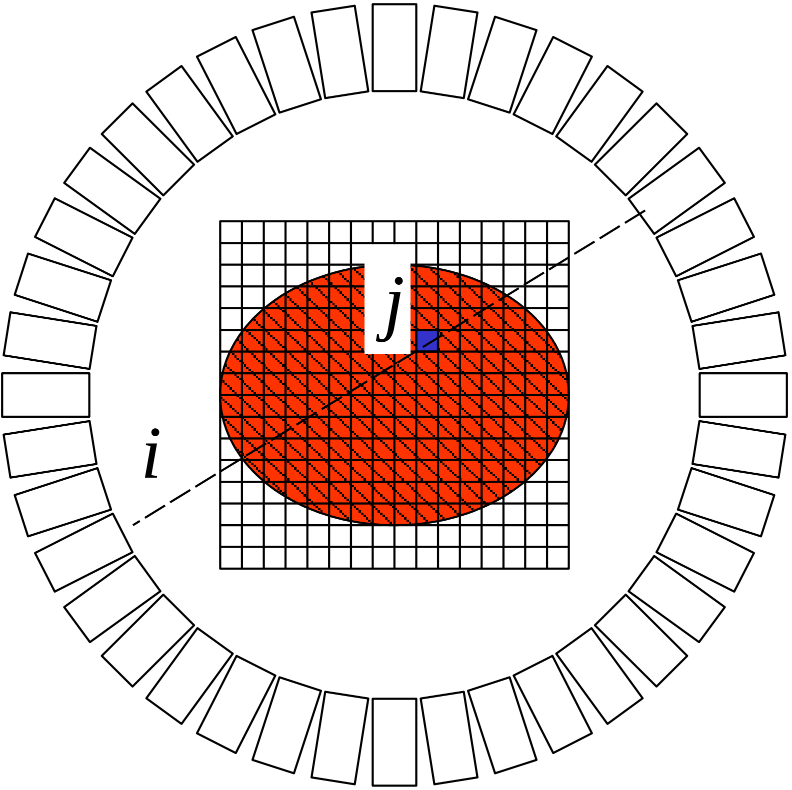
Image reconstruction: $\lambda = \boldsymbol{p}^{-1}(m_i - b_i)$ What is $\boldsymbol{p}^{-1}$?

Extended Distribution Example



smoothed

Maximum Likelihood Expectation Maximization (ML-EM)



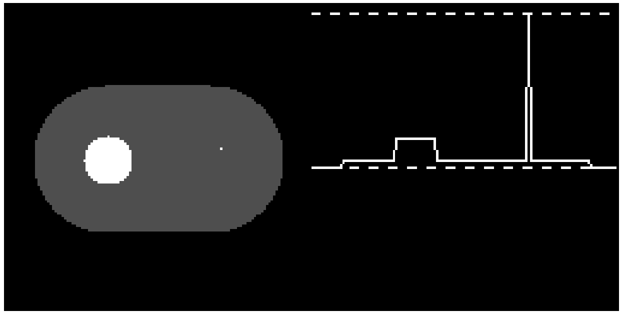
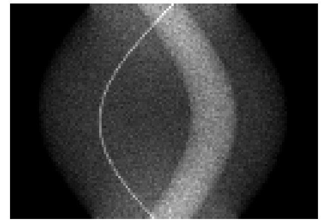
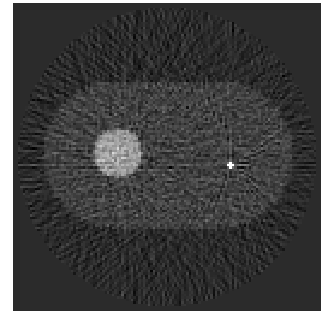
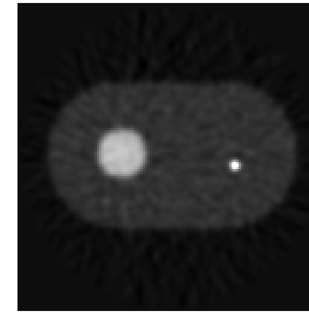
Shepp LA, Vardi Y., *IEEE Trans Med Imag* 1:113-121, 1982.
Lange K, Carson R., *J Comput Assist Tomo* 8:306-316, 1984.

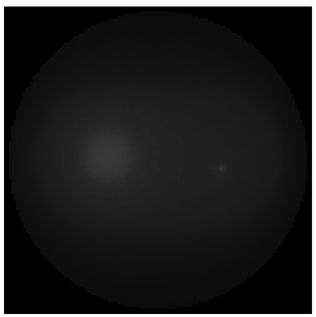
$$\lambda_j^{(n+1)} = \frac{1}{\sum_{i=1}^{nbin} p_{ij}} \sum_{i=1}^{nbin} \frac{p_{ij}\lambda_j^{(n)}}{b_i + \sum_{k=1}^{nvox} p_{ik}\lambda_k^{(n)}} m_i$$

$$m_i = b_i + \sum_{j=1}^{npix} p_{ij}\lambda_j$$

$\lambda_j^{(n)}$ is the estimated activity in voxel *j* at iteration *n*.

Extended Distribution Example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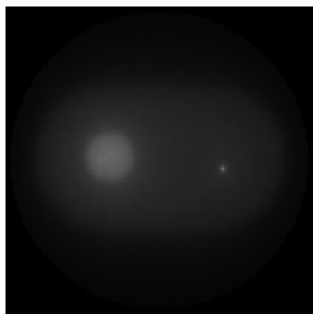
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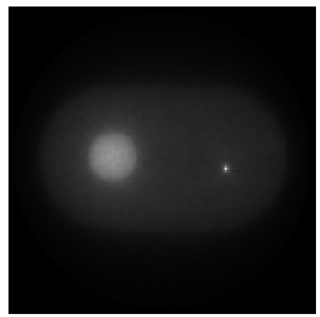
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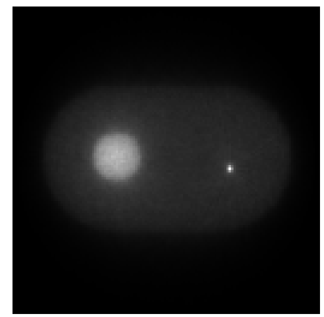
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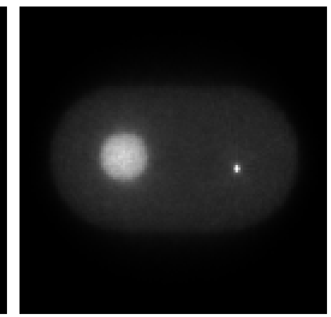
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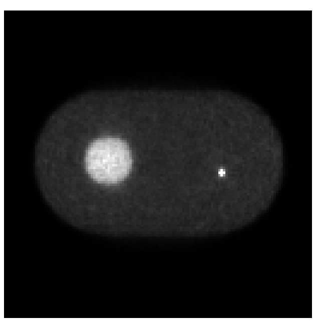
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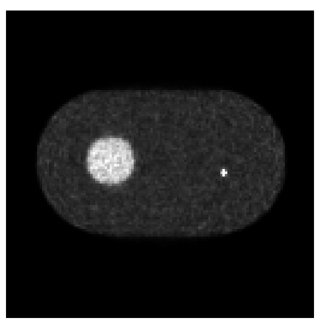
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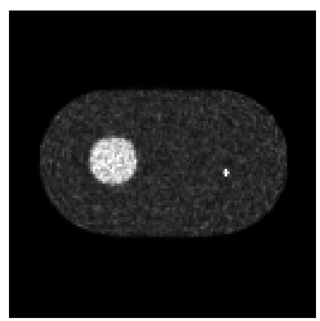
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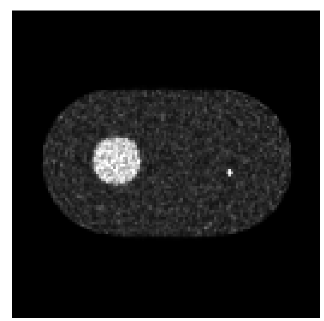
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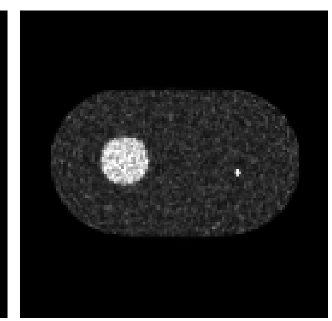
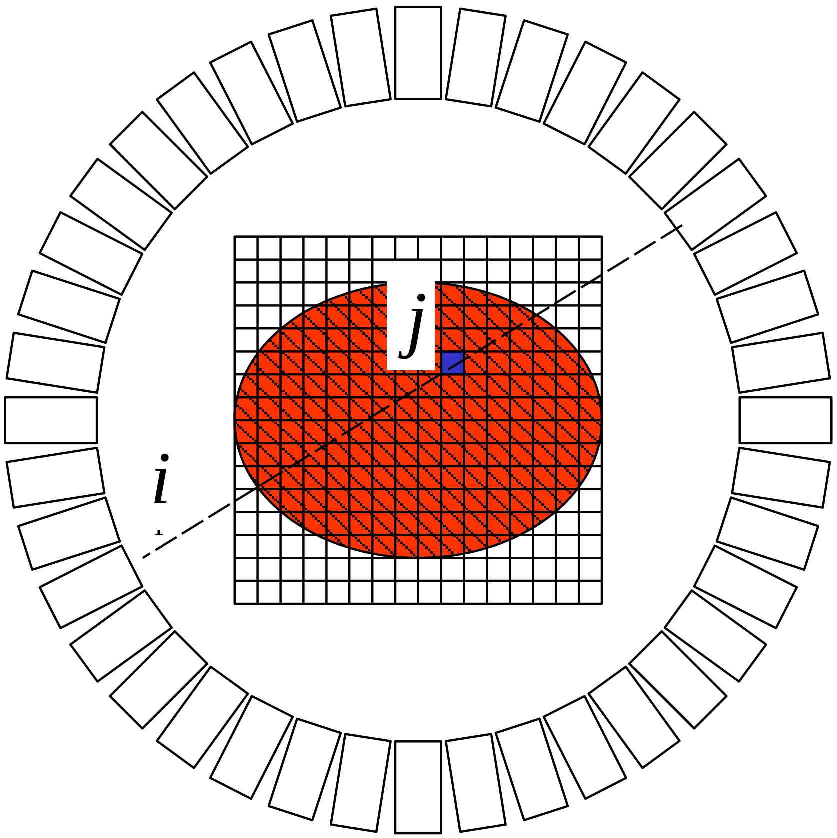

50

What is **p**?

$$m_i = b_i + \sum_{j=1}^{npix} p_{ij} \lambda_j$$



Increasing levels of complexity:

1) **p** is 1's and 0's - very sparse
2) **p** is fractional values, depending on how column intersects with voxel - still pretty sparse
3) **p** includes attenuation (lower values) - still pretty sparse
4) **p** includes resolution effects (collimator blurring, etc) - somewhat sparse
5) **p** includes background - not sparse at all

Compensations/Corrections

Putting physical effects into **p** allows the reconstruction to compensate for those effects.

For example, including attenuation in the model leads to attenuation correction in the final image. Putting system blurring (imperfect spatial resolution) can improve the final image resolution (and/or noise).

ML-EM Characteristics

- Not Fast
- Non-negative pixel values - can lead to biases
- Noise is greatest in areas of high activity
- Can compensate for physical effects
- Allows image reconstruction from limited projections.

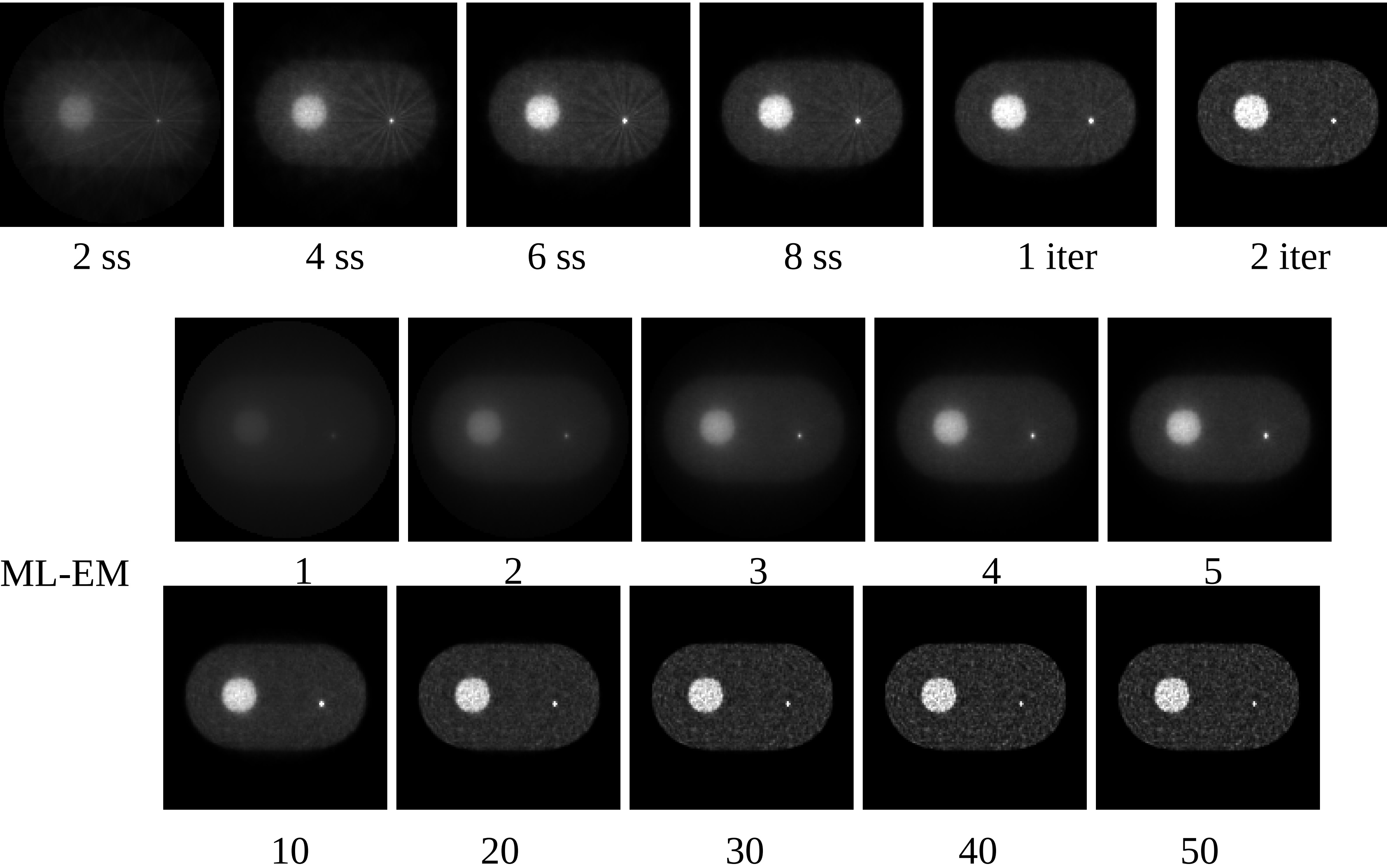
Ordered Subsets Expectation Maximization (OS-EM)

Hudson HM, Larkin RS . *IEEE Trans Med Imag* 13:601-609, 1994.

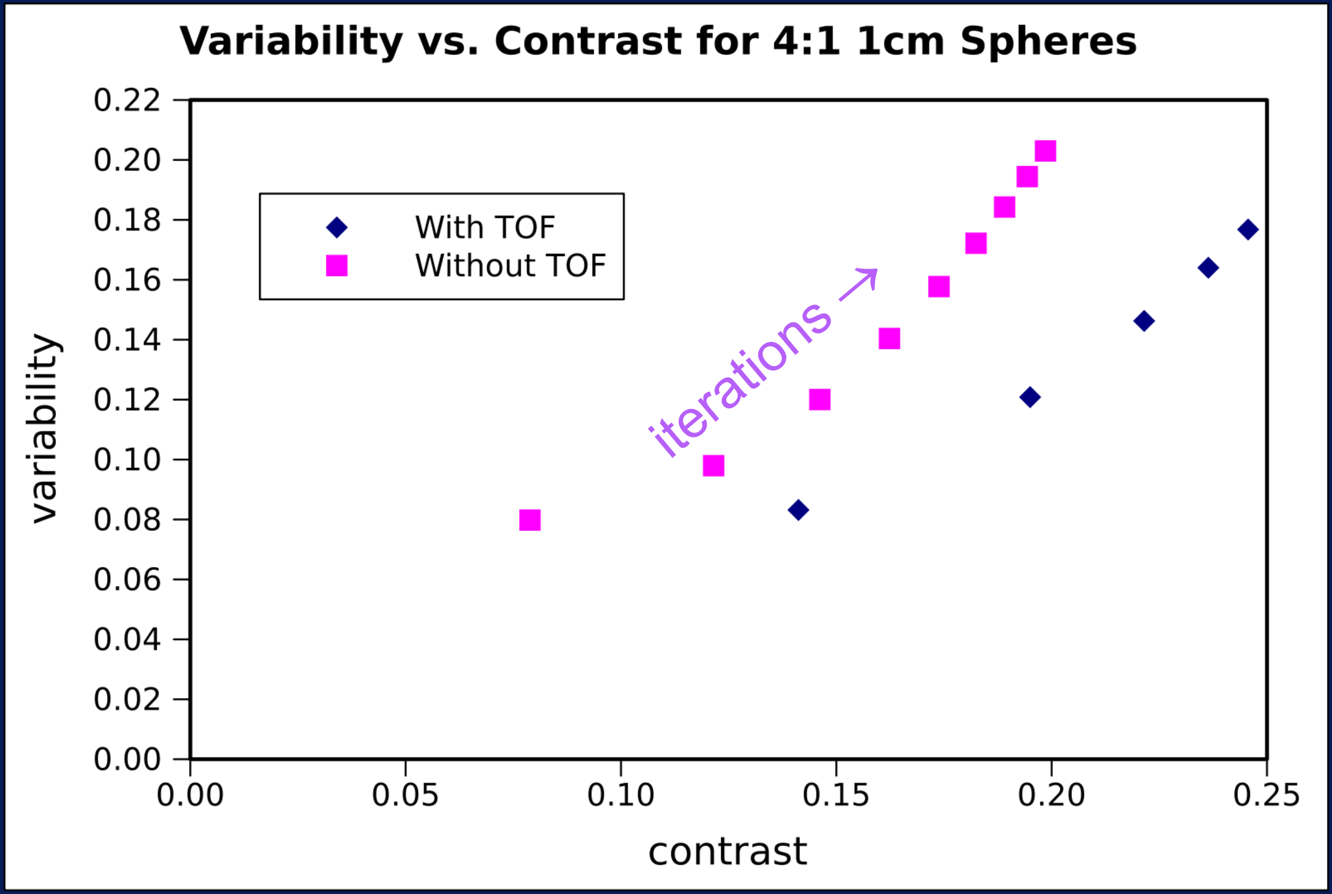
$$\lambda_j^{(n+1)} = \frac{1}{\sum_{i=1}^{nbin} p_{ij}} \sum_{i=1}^{nbin} \frac{p_{ij}\lambda_j^{(n)}}{b_i + \sum_{k=1}^{nvox} p_{ik}\lambda_k^{(n)}} m_i$$

Instead of processing all projection for each image update, projections are divided into subsets, and the image is updated after processing each subset.

OS-EM   10 subsets



Results – 4:1 Hot Spheres



Regularized Reon (Q.Clear)

$$\sum_{j=1}^{n} \sum\nolimits_{k \in N_j} w_j \, w_k \frac{\left(x_j - x_k\right)^2}{\left(x_j - x_k\right) + \gamma \left|x_j - x_k\right|}$$

Regularized Recon (“Q.Clear”)

4min 2min 1min 30s

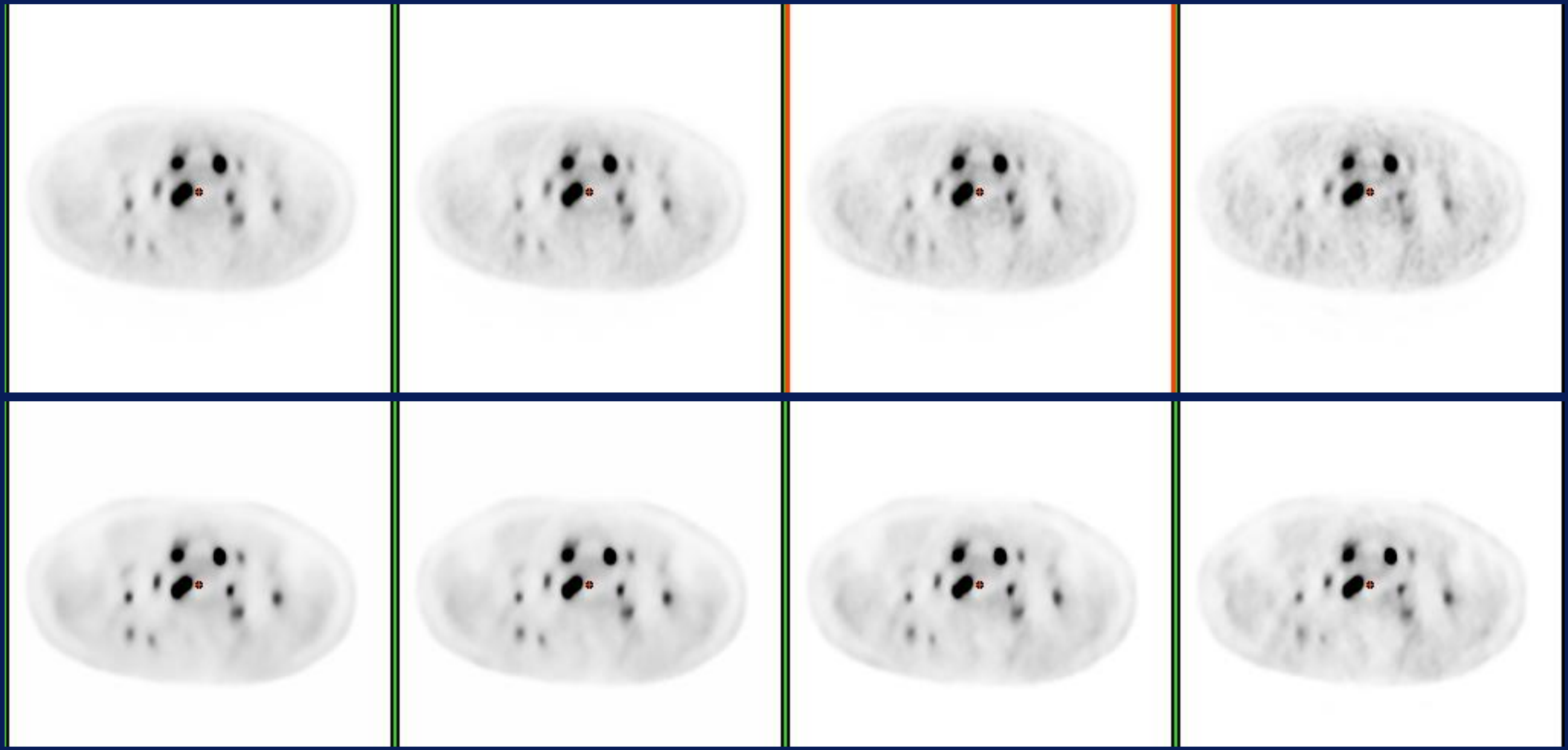


Image Quantitation

What is Image Quantitation?

- Generally - Deriving numbers from images
- Volume measurement
- Motion measurements (e.g., ejection fraction)
- Distributions of radiotracers, and, under the right circumstances, the underlying physiological processes.

What Factors Affect Quantitation of Radionuclide Distributions?

- Successful calibration of scanning system (counts/s to activity)
- Accurate corrections for
  - attenuation
  - scatter
  - randoms
  - dead time
    (the bigger the effect, the more accurate the correction must be)
- Quantitative reconstruction algorithm
- Resolution effects (degradation of small structures)
- ROI (Region Of Interest) Analysis

Standardized Uptake Value (SUV)

- The SUV radioactivity concentration (what the scanner measures) normalized to injected dose and body mass:

$$SUV = \frac{radioactivity\ concentration}{injected\ dose / body\ mass}$$

- This can also be thought of as the local concentration divided by the body mean concentration
- Dimensions are mass/volume (e.g. g/ml). Since this is the body (mostly water; 1ml=1g), it is *almost* dimensionless.
- This is sometimes referred to as a semi-quantitative measure (compared to kinetic modeling.)

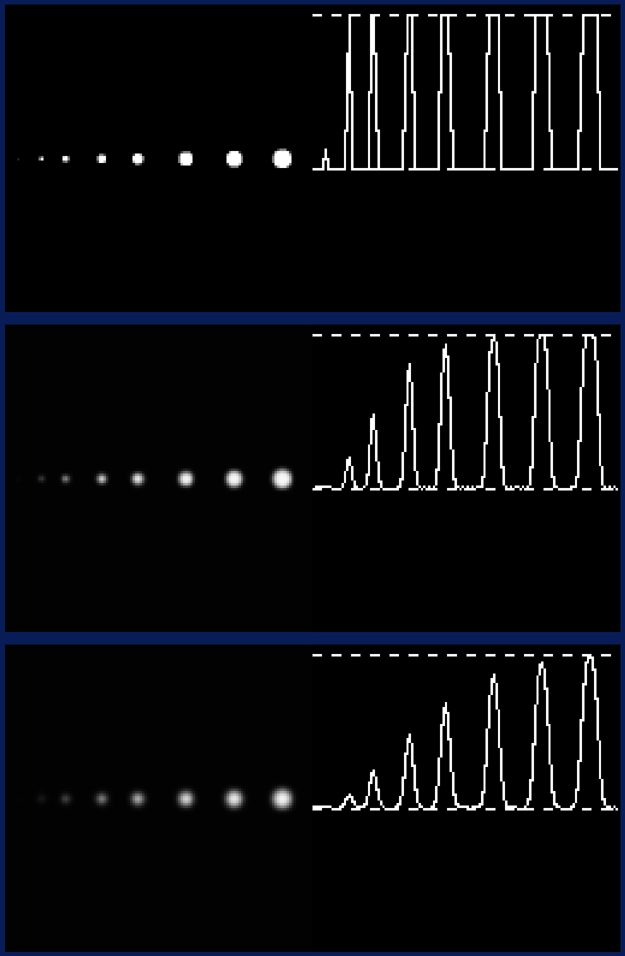
Sphere Size vs. Spatial Resolution

"Full Recovery":
When at least some pixels in a region retain full intensity

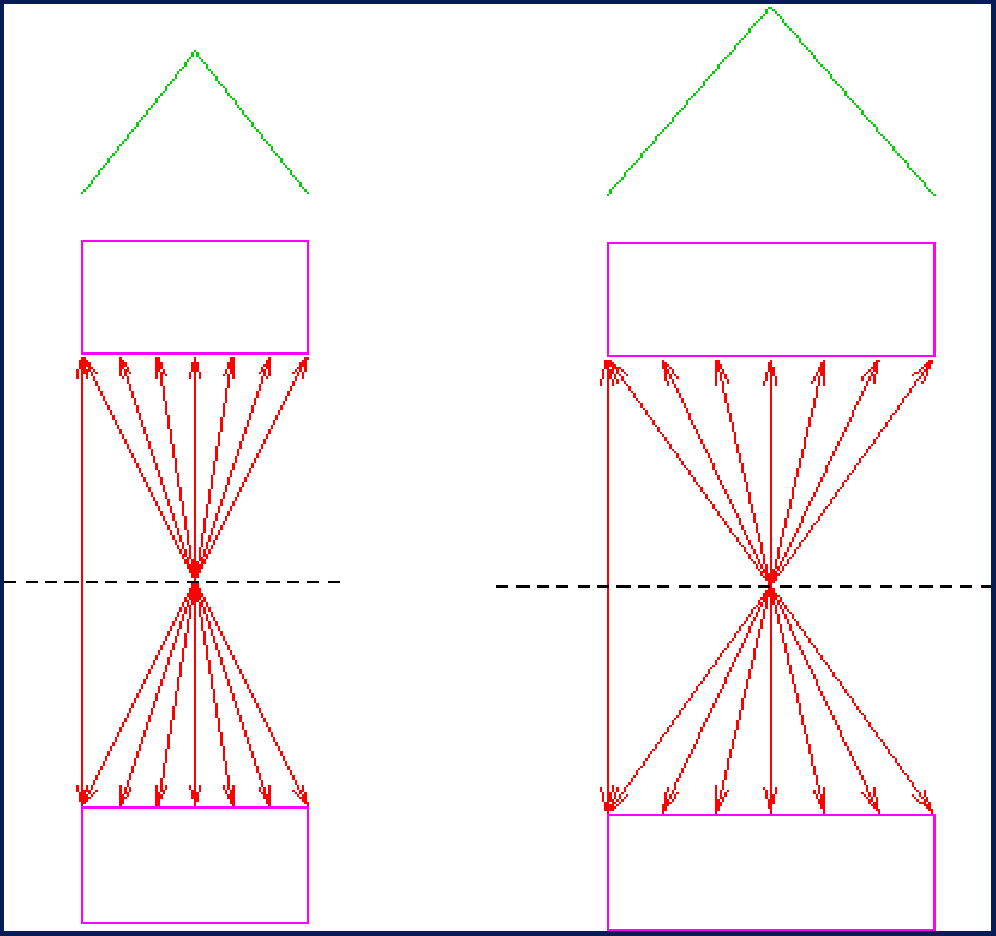
"Recovery coefficient":
Ratio between measured and actual values (due to resolution effects only)
RC ≤ 1

Sphere diameter ≥ 3X FWHM resolution → full recovery (for 3D blurring)
(Not so bad if non-spherical, or non-3D blurring

Sphere diameters:
"1", 2, 3, 4, 5, 6, 7, 8 pixels

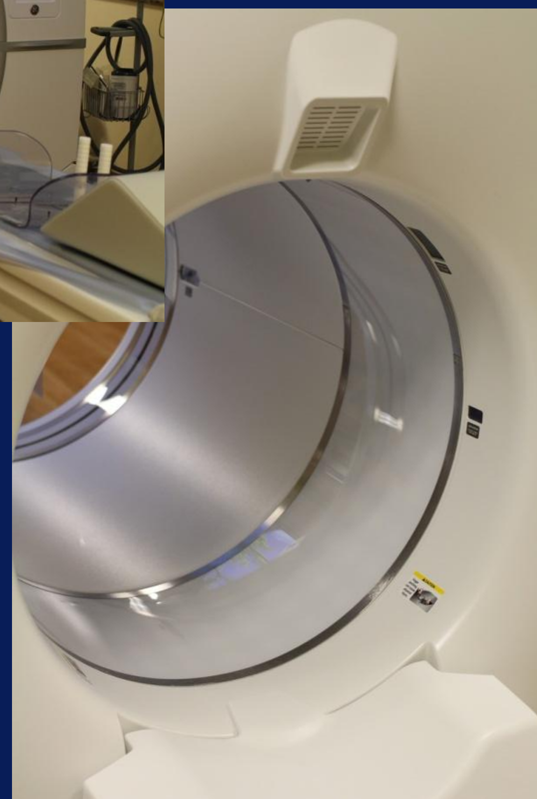


3D Sensitivity vs. axial FOV



Sens $\propto$ (FOV)^2

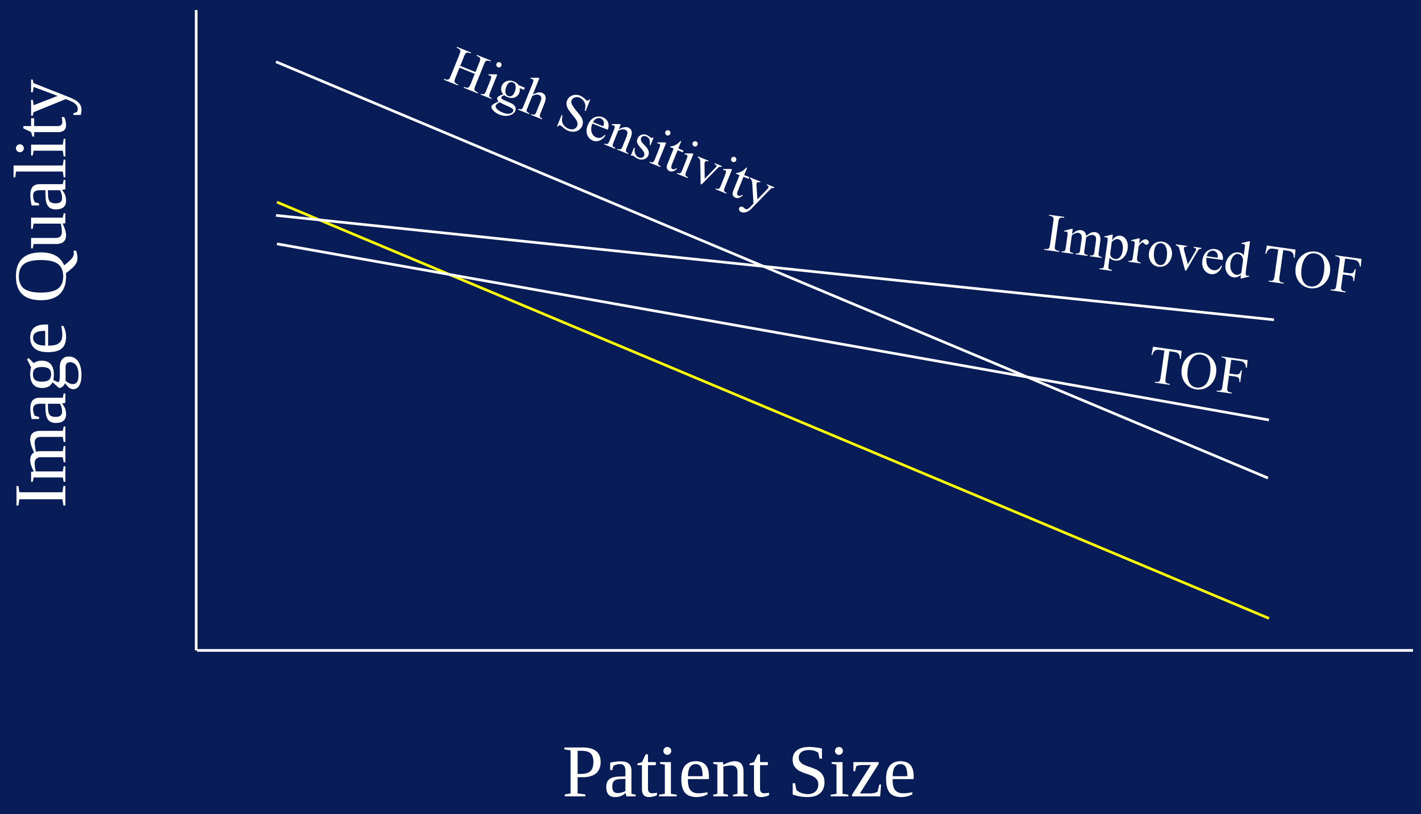
GE Discovery IQ



Solid State Photodetectors

- Avalanche Photodiodes
  - Compact, work in B field
  - Siemens PET/MR
- SSPM, SiPM, etc.
  - Compact, work in B field, excellent timing
  - GE PET/MR, Philips PET/CT (Vereos)

Image Quality vs. Size

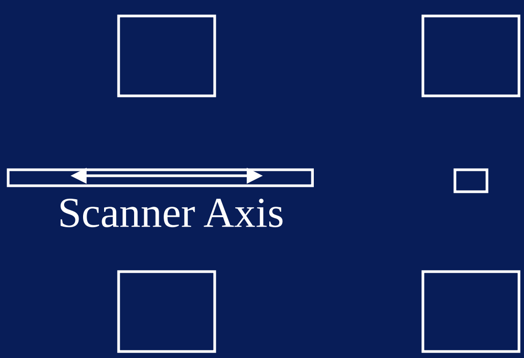


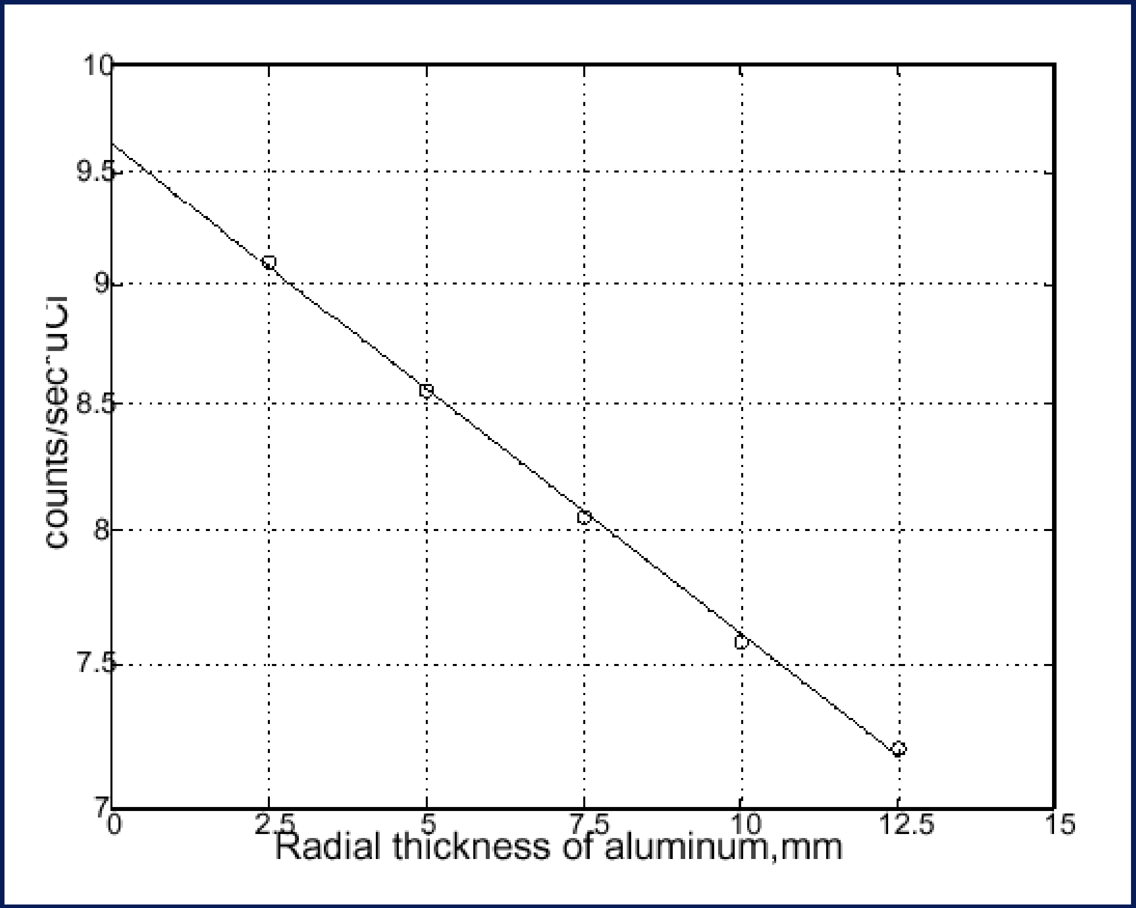
NEMA NU-2 Performance Tests

- Spatial Resolution
- Sensitivity
- Count Rate and Scatter Fraction
- Image Quality

PET Performance - Sensitivity

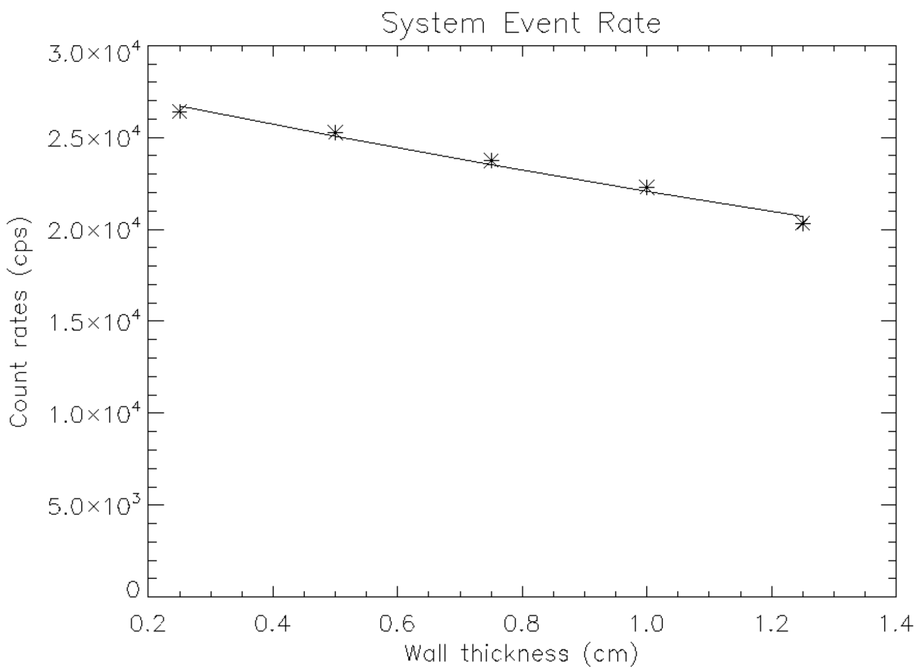
Five Concentric 70 cm long aluminum tubes surrounding source

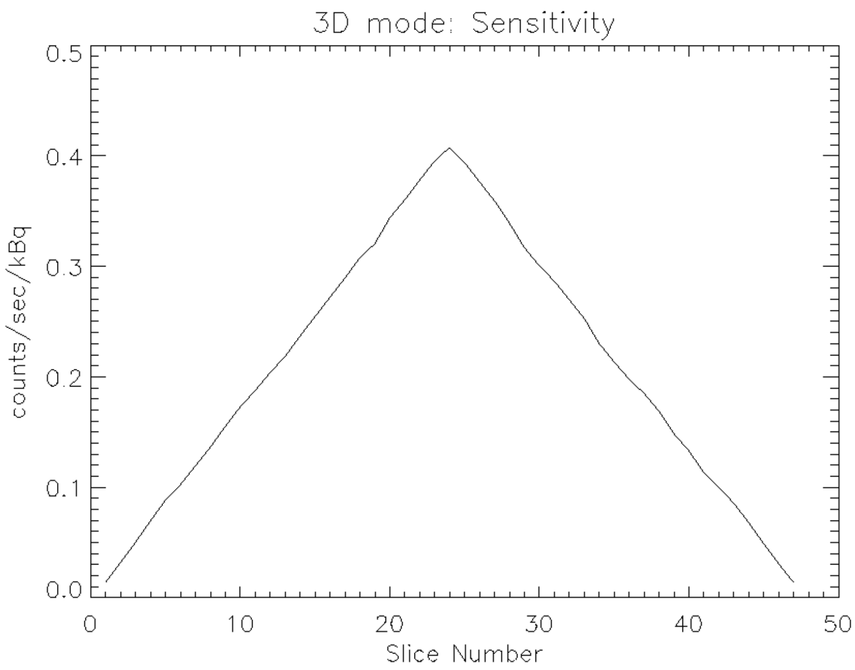




Why long line source? Whole body sensitivity.

Sensitivity - 3D, r=0





System sensitivity: 9.75084 (counts/sec/kBq)

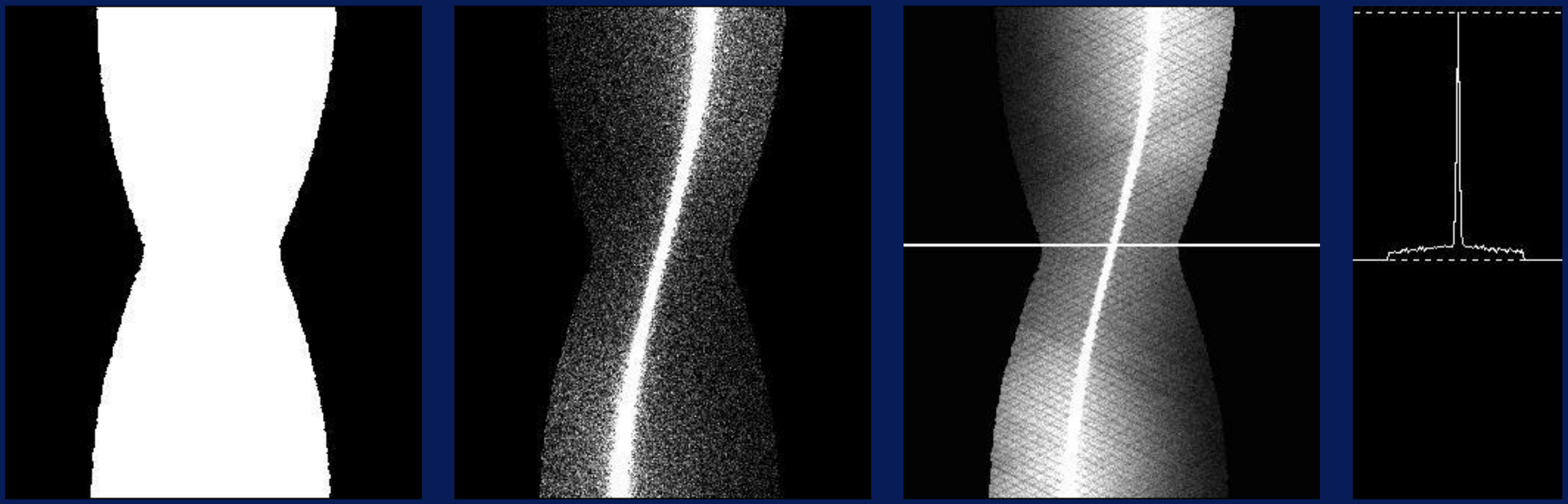
Attenuation Coef. for sleeves: 0.127310 (1/cm)

Results generated from scan with NU2-2001 3D Sens R0 sleeve 5

Scatter Fraction and Count Rate

Scatter Fraction

- S/(S+T) --- low is good
- reflects energy resolution and geometry
- Method:
  - Axial low-intensity line source in phantom
  - Mask out LORs not subtending phantom
  - Line source makes sinusoid in each slice's sinogram
  - Pixels off the sinusoid measure background (scatter and random)
  - At very low rates, no randoms.

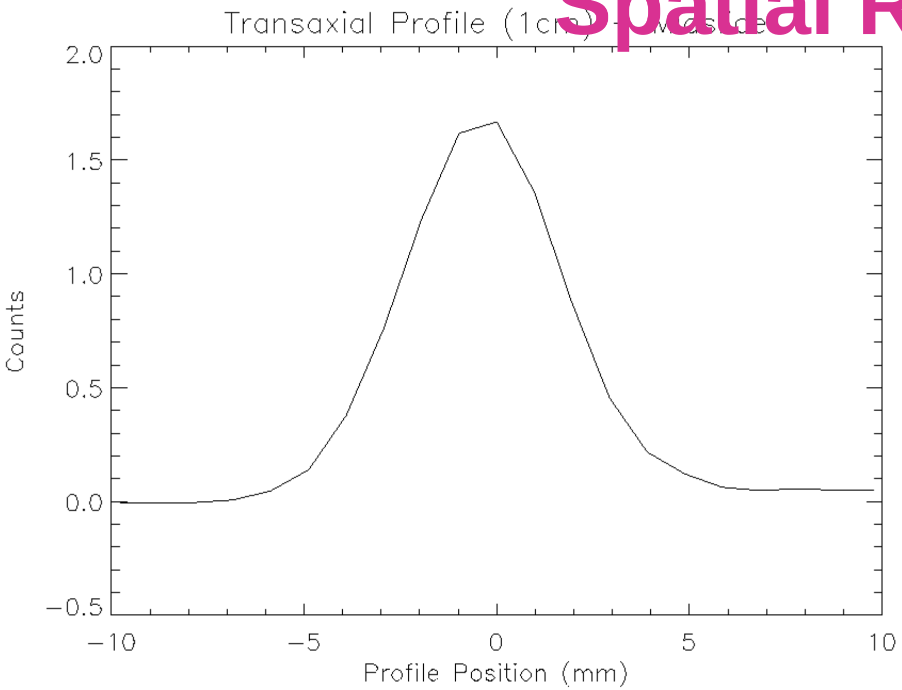


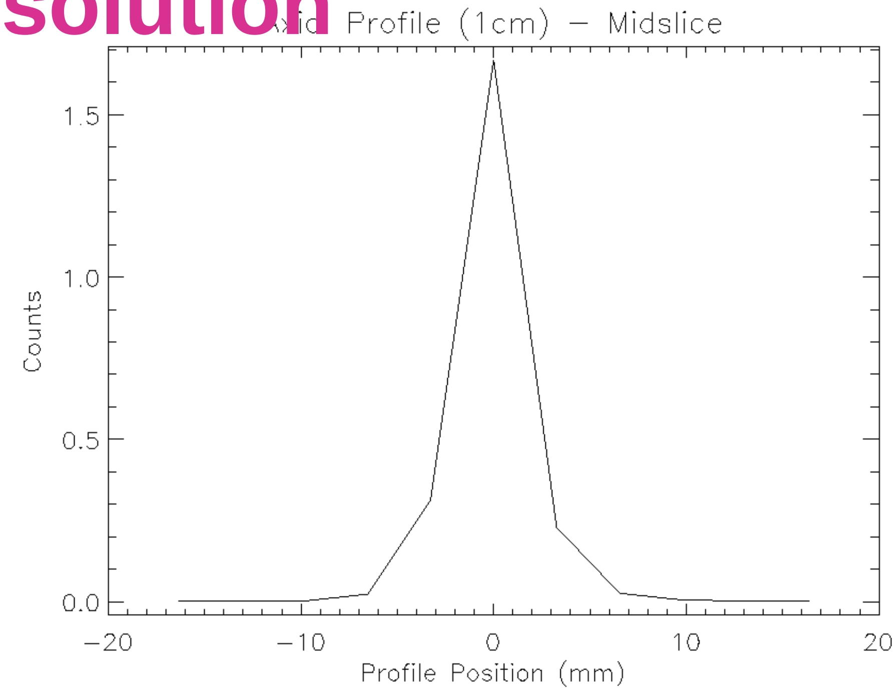
Spatial Resolution

- Small (< 1 mm) point sources placed in several locations within FOV
- Scan, reconstruct with very small (< 1/10 expected FWHM) pixels.
- Measure FWHM of resulting profiles in all three directions

Generally, pixels should be 1/3 the expected FWHM or smaller, for any NM application.   Combination of recon FOV and image matrix.

Spatial Resolution





Results for 1 cm source

Trans FWHM: 5.11617 (mm)

Trans FWTM: 10.0089 (mm)

Axial FWHM: 5.03312 (mm)

Axial FWTM: 10.7149 (mm)

Results for 10 cm source

Radial FWHM: 5.91931 (mm)

Radial FWTM: 12.9862 (mm)

Tang. FWHM: 6.72302 (mm)

Tang. FWTM: 32.1492 (mm)

Axial FWHM: 5.99356 (mm)

Axial FWTM: 11.4418 (mm)

Which count-based PET metric is the best predictor of image quality?

0%	1.	Total counts (prompts)
19%	2.	True counts
0%	3.	Random counts
71%	4.	Noise-equivalent counts (NEC)
10%	5.	Scatter fraction

Which count-based PET metric is the best predictor of image quality?

4. Noise-equivalent counts (NEC)

Cherry, Sorenson, and Phelps “Physics in Nuclear Medicine”

Why is photon attenuation a large effect in PET (compared to SPECT)?

0%	1. There's no way to correct it.
81%	2. Both photons must be detected.
5%	3. The photon energy is high.
0%	4. The photon energy is low.
14%	5. Timing resolution is imperfect.

Why is photon attenuation a large effect in PET (compared to SPECT)??

2. Both photons must be detected

Cherry, Sorenson, and Phelps “Physics in Nuclear Medicine”

What factor is essential to localizing the PET annihilation locations?

14%	1.	The 511 keV γ energy.
0%	2.	The F-18 half-life.
64%	3.	The anti-parallel γ paths.
18%	4.	The scanner sensitivity.
5%	5.	The electron mass.

What factor is essential to localizing the PET annihilation locations?

3. The anti-parallel γ paths

Cherry, Sorenson, and Phelps "Physics in Nuclear Medicine"

What effect does increasing iterations in ML-EM image reconstruction have?

0%	1. Increased patient comfort.
0%	2. Less CPU time.
5%	3. Lower resolution.
76%	4. Increased noise.
19%	5. Shorter scan time.

What effect does increasing iterations in ML-EM image reconstruction have?

4. Increased noise.

Cherry, Sorenson, and Phelps “Physics in Nuclear Medicine”

Which of the following is directly measured in current NEMA NU-2 PET tests?

82%	1.	Spatial resolution.
14%	2.	Timing resolution.
0%	3.	Energy resolution.
5%	4.	PET-CT alignment accuracy.
0%	5.	Image reconstruction speed.

Which of the following is directly measured in current NEMA NU-2 PET tests?

1. Spatial Resolution.

"Performance Measurements of Positron Emission Tomographs (PETs)", NEMA-NU2 2012, National Electrical Manufacturer's Association

Thank you.