Review of PET Physics

Timothy Turkington, Ph.D.
Radiology and Medical Physics
Duke University
Durham, North Carolina, USA
Positron Decay

\[ \frac{AX_N}{Z} \rightarrow \frac{AY_{N+1}}{Z-1} + e^+ + \nu \]

<table>
<thead>
<tr>
<th>Nuclide</th>
<th>half-life</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-11</td>
<td>20.3 min</td>
</tr>
<tr>
<td>N-13</td>
<td>10 min</td>
</tr>
<tr>
<td>O-15</td>
<td>124 sec</td>
</tr>
<tr>
<td>F-18</td>
<td>110 min</td>
</tr>
<tr>
<td>Rb-82</td>
<td>75 sec</td>
</tr>
</tbody>
</table>

e.g., \(^{18}\text{F} \rightarrow ^{18}\text{O} + e^+ + \nu\)
Positron Annihilation

\[ m_e = 511 \text{ keV}/c^2 \]
## Linear attenuation values for lead and water

<table>
<thead>
<tr>
<th>Material (E$_\gamma$)</th>
<th>$\mu$ (cm$^{-1}$)</th>
<th>HVT (cm)</th>
<th>TVT (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>lead (140 keV)</td>
<td>22.7</td>
<td>0.031</td>
<td>0.10</td>
</tr>
<tr>
<td>lead (511 keV)</td>
<td>1.7</td>
<td>0.41</td>
<td>1.35</td>
</tr>
<tr>
<td>water (140 keV)</td>
<td>0.15</td>
<td>4.6</td>
<td>15.4</td>
</tr>
<tr>
<td>water (511 keV)</td>
<td>0.096</td>
<td>7.2</td>
<td>24.0</td>
</tr>
</tbody>
</table>
Coincidence Event
Projections
PET Background Events

- Scatter
- Randoms
Scattered Coincidence Event

Scatter Fraction $S/(S+T)$
- With septa ~10-20%
- w/o septa ~30-80%
Random Coincidence Event

\[ R_R = 2\tau R_a R_b \]
Correcting Background; Noise Equivalent Counts

\[ P_{\text{prompts}} = T_{\text{true}} + S_{\text{scatter}} + R_{\text{randoms}} \]

\[ T' = P - S' - R' \quad (\text{Estimation of true events by subtracting } S \text{ and } R \text{ estimates}) \]

\[ \langle T' \rangle = \langle P \rangle + \langle S' \rangle + \langle R' \rangle = P + \begin{vmatrix} 0 \\ R \end{vmatrix} \geq P \geq T \]

\[ \text{SNR} \equiv \frac{T'}{\sqrt{\langle T' \rangle}} \approx \frac{T}{\sqrt{P + \begin{vmatrix} 0 \\ R \end{vmatrix}}} ; \quad \text{NEC} = \frac{T^2}{P + \begin{vmatrix} 0 \\ R \end{vmatrix}} = \frac{T}{(1 + S/T + (2?)R/T)} \]

S and R refer to scattered and random events on LORs that subtend the imaged object.

More background \( \rightarrow \) more statistical image noise.
NEC Examples

\[ NEC = \frac{T}{(1 + S/T + R/T)} \]

<table>
<thead>
<tr>
<th>Prompts</th>
<th>Trues</th>
<th>Scatter</th>
<th>SF</th>
<th>NEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>200</td>
<td>100</td>
<td>100</td>
<td>0.5</td>
<td>50</td>
</tr>
<tr>
<td>400</td>
<td>200</td>
<td>200</td>
<td>0.5</td>
<td>100</td>
</tr>
</tbody>
</table>
Image Noise and Lesion Detection

600s  300s  150s  75s  38s  19s  10s

FBP

OS-EM
Multiple Rings, 2D – 3D

For $n$ detector rings:

- **2D**
  - Direct slices ($n$)
  - Cross slices ($n-1$)

- Total slices = $2n-1$

- **3D**

Higher Sensitivity (degraded axial resolution)
Time Of Flight PET: The influence of background

How strong is source A?

Detectors measure counts $C_A + C_B$.

S.D. of measurement is $\sqrt{C_A + C_B}$

SNR = $\frac{C_A}{\sqrt{C_A + C_B}}$
The influence of even more background

SNR of measurement of A?

\[ \text{SNR} = \frac{C_A}{\sqrt{C_A + C_B + C_C + C_D + L}} \]

If all \( N \) sources are equal,

\[ \text{SNR} = \frac{C_A}{\sqrt{NC_A}} \]
If all $N$ sources are equal,

$$\text{SNR} = \frac{C_A}{\sqrt{nC_A}}$$

where $n$ is the number of sources within zone $T_1$.

SNR improvement of $\sqrt{\frac{N}{n}}$; similar to counting longer by $\frac{N}{n}$.

Magical new detectors

Can distinguish between counts originating in segment $T_1$ and counts originating in segment $T_2$. 
Time-of-Flight PET

\[ \Delta t = \frac{(d_2 - d_1)}{c} = \frac{2x}{c} \]

\[ x = \Delta t \cdot \frac{c}{2} \]
Fillable, Tapering Phantom with 1 cm lesions
Clinical Example 2

68 kg male

non-TOF

TOF
Attenuated Event
Annihilation radiation emitted along a particular line of response has the same attenuation probability, regardless of where it originated on the line.

\[ P_C = P_1 P_2 \]

\[ \begin{align*}
&= e^{-\mu \cdot d_1} e^{-\mu \cdot d_2} \\
&= e^{-\mu \cdot (d_1 + d_2)}
\end{align*} \]
Attenuation losses - PET and SPECT

Events surviving attenuation in cylinder

Fraction emitted from cylinder

Fraction emitted

PET

140 keV SPECT

cylinder radius (cm)
Obese Patient
Technologist Size Variations

$m_b \sim 2.5m_a$
Attenuation Effects

<table>
<thead>
<tr>
<th>AC</th>
<th>NAC</th>
<th>x-ray CT</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="PET AC" /></td>
<td><img src="image2.png" alt="PET NAC" /></td>
<td><img src="image3.png" alt="CT SCAN" /></td>
</tr>
<tr>
<td><img src="image4.png" alt="PET AC" /></td>
<td><img src="image5.png" alt="PET NAC" /></td>
<td><img src="image6.png" alt="CT SCAN" /></td>
</tr>
<tr>
<td><img src="image7.png" alt="PET AC" /></td>
<td><img src="image8.png" alt="PET NAC" /></td>
<td><img src="image9.png" alt="CT SCAN" /></td>
</tr>
<tr>
<td><img src="image10.png" alt="PET AC" /></td>
<td><img src="image11.png" alt="PET NAC" /></td>
<td><img src="image12.png" alt="CT SCAN" /></td>
</tr>
</tbody>
</table>
Calculated Attenuation Correction

\[ I = I_0 e^{-\mu d} \]
Transmission Attenuation Measurement

positron source
Using CT image for AC
Attenuation Correction Accuracy

- CT-based attenuation correction is performed on almost all PET studies.

- Is it being done well?
  - Is the CT accurate (e.g., water = 0)?
  - Is the CT accurate under all relevant conditions?
  - Is the translation between CT# and 511 keV $\mu$ appropriate?
  - Patient motion between CT and PET?
  - …
Attenuation Correction

Photons emitted along this line will be attenuated by a factor that can be determined from the corresponding CT scan.
Attenuation Correction

The biggest source of error in PET AC is patient motion between the CT and the PET scans. This particular PET photon trajectory will be undercorrected. The intensity on this side of the body will be artificially low.
Spatial Resolution

\[ R_{\text{sys}} = \sqrt{R_{\text{det}}^2 + R_{\text{acol}}^2 + R_{\text{range}}^2 + b^2} \]

- \( R_{\text{det}} = \) resolution of detectors (\( \leq d \))
- \( R_{\text{acol}} = \) resolution from photon acollinearity (\( = 0.0022D \))
- \( R_{\text{range}} = \) resolution from positron range
- \( b = \) block effect
Depth of Interaction Uncertainty

- Uncertainty in the origin of radiation when measured obliquely in a detector.
- Resolution in radial direction worsens with increasing radius.
- High stopping power helps: Interactions more likely in front of detector.
- Some high resolution systems sacrifice sensitivity by shortening detectors (to mitigate DOI effects.)
- Some systems (HRRT) use two layers of detectors to lessen effect. Others propose a measurement of the DOI.
Block Detector (GE, Siemens)

Photomultiplier(s)  Scintillation Crystals
PET Ring with Block Detectors
Curved Plate Pixelated Camera (Philips)
Image Reconstruction
Image Reconstruction Methods

- **FBP** (Filtered Back-Projection)
- **ML-EM 10** (Maximum Likelihood Expectation Maximization)
- **ML-EM 30**
- **ML-EM 50**
- **OS-EM 1** (Ordered Subsets Expectation Maximization)
- **OS-EM 2**
- **OS-EM 3**
- **OS-EM 4**

(28 Subsets)
Detection Process

\[ m_i = b_i + \sum_{j=1}^{npix} p_{ij} \lambda_j \]

- \( i = \) projection line (line of response)
- \( j = \) source pixel
- \( \lambda_j = \) radioactivity at voxel \( j \)
- \( p_{ij} = \) probability of emission from \( j \)
  being measured in \( i \)
- \( b_i = \) background contributing to \( i \)
- \( m_i = \) expected counts on projection \( i \)

Image reconstruction: \( \lambda = p^{-1}(m_i - b_i) \)

What is \( p^{-1} \)?
Extended Distribution Example

smoothed
Maximum Likelihood Expectation Maximization (ML-EM)

\[
\lambda^{(n+1)}_j = \frac{1}{n_{\text{bin}}} \sum_{i=1}^{n_{\text{bin}}} p_{ij} \lambda^{(n)}_j \\
+ \frac{1}{n_{\text{vox}}} \sum_{k=1}^{n_{\text{vox}}} p_{ik} \lambda^{(n)}_k \\
\]

\[
m_i = b_i + \sum_{j=1}^{n_{\text{pix}}} p_{ij} \lambda_j
\]

\(\lambda_j^{(n)}\) is the estimated activity in voxel \(j\) at iteration \(n\).
Extended Distribution Example
What is $p$?

$$m_i = b_i + \sum_{j=1}^{npix} p_{ij} \lambda_j$$

Increasing levels of complexity:

1) $p$ is 1’s and 0’s - very sparse
2) $p$ is fractional values, depending on how column intersects with voxel - still pretty sparse
3) $p$ includes attenuation (lower values) - still pretty sparse
4) $p$ includes resolution effects (collimator blurring, etc) - somewhat sparse
5) $p$ includes background - not sparse at all
Compensations/Corrections

Putting physical effects into $p$ allows the reconstruction to compensate for those effects.

For example, including attenuation in the model leads to attenuation correction in the final image. Putting system blurring (imperfect spatial resolution) can improve the final image resolution (and/or noise).
ML-EM Characteristics

• Not Fast
• Non-negative pixel values - can lead to biases
• Noise is greatest in areas of high activity
• Can compensate for physical effects
• Allows image reconstruction from limited projections.
Ordered Subsets Expectation Maximization (OS-EM)


\[
\lambda^{(n+1)}_j = \frac{1}{\text{nbin}} \sum_{i=1}^\text{nbin} p_{ij} \frac{\lambda^{(n)}_j}{\text{m}_i} \left[ b_i + \sum_{k=1}^\text{nvox} p_{ik} \lambda^{(n)}_k \right]
\]

Instead of processing all projection for each image update, projections are divided into subsets, and the image is updated after processing each subset.
OS-EM  10 subsets

ML-EM
Results – 4:1 Hot Spheres

Variability vs. Contrast for 4:1 1cm Spheres

- With TOF
- Without TOF

Iterations →
Regularized Reon (Q.Clear)

\[ \sum_{j=1}^{n} \sum_{k \in N_j} w_j w_k \frac{(x_j - x_k)^2}{(x_j - x_k) + \gamma |x_j - x_k|} \]
Regularized Recon ("Q.Clear")

4min  2min  1min  30s
Image Quantitation
What is Image Quantitation?

- Generally - Deriving numbers from images
- Volume measurement
- Motion measurements (e.g., ejection fraction)
- Distributions of radiotracers, and, under the right circumstances, the underlying physiological processes.
What Factors Affect Quantitation of Radionuclide Distributions?

• Successful calibration of scanning system (counts/s to activity)

• Accurate corrections for
  – attenuation
  – scatter
  – randoms
  – dead time
    (the bigger the effect, the more accurate the correction must be)

• Quantitative reconstruction algorithm

• Resolution effects (degradation of small structures)

• ROI (Region Of Interest) Analysis
Standardized Uptake Value (SUV)

• The SUV radioactivity concentration (what the scanner measures) normalized to injected dose and body mass:

\[ SUV = \frac{\text{radioactivity concentration}}{\text{injected dose/ body mass}} \]

• This can also be thought of as the local concentration divided by the body mean concentration.

• Dimensions are mass/volume (e.g. g/ml). Since this is the body (mostly water; 1ml=1g), it is *almost* dimensionless.

• This is sometimes referred to as a semi-quantitative measure (compared to kinetic modeling.)
Sphere Size vs. Spatial Resolution

“Full Recovery”: When at least some pixels in a region retain full intensity.

“Recovery coefficient”: Ratio between measured and actual values (due to resolution effects only). $\text{RC} \leq 1$

Sphere diameter $\geq 3 \times \text{FWHM}$ resolution $\rightarrow$ full recovery (for 3D blurring) (Not so bad if non-spherical, or non-3D blurring)

Sphere diameters: “1”, 2, 3, 4, 5, 6, 7, 8 pixels

- No smoothing
- 2 pixel FWHM 3D smoothing
- 3 pixel FWHM 3D smoothing
3D Sensitivity vs. axial FOV

\[ \text{Sens} \propto (\text{FOV})^2 \]
GE Discovery IQ
Solid State Photodetectors

• Avalanche Photodiodes
  – Compact, work in B field
  – Siemens PET/MR

• SSPM, SiPM, etc.
  – Compact, work in B field, excellent timing
  – GE PET/MR, Philips PET/CT (Vereos)
Image Quality vs. Size

- High Sensitivity
- Improved TOF
- TOF

Image Quality vs. Patient Size
NEMA NU-2 Performance Tests

- Spatial Resolution
- Sensitivity
- Count Rate and Scatter Fraction
- Image Quality
PET Performance - Sensitivity

Five Concentric 70 cm long aluminum tubes surrounding source

Why long line source? Whole body sensitivity.
Sensitivity - 3D, r=0

System Event Rate

3D mode: Sensitivity

System sensitivity: 9.75084 (counts/sec/kBq)

Attenuation Coef. for sleeves: 0.127310 (1/cm)

Results generated from scan with NU2-2001 3D Sens R0 sleeve 5
Scatter Fraction and Count Rate
Scatter Fraction

- \( S/(S+T) \) --- low is good
- reflects energy resolution and geometry
- Method:
  - Axial low-intensity line source in phantom
  - Mask out LORs not subtending phantom
  - Line source makes sinusoid in each slice’s sinogram
  - Pixels off the sinusoid measure background (scatter and random)
  - At very low rates, no randoms.
Spatial Resolution

- Small (< 1 mm) point sources placed in several locations within FOV
- Scan, reconstruct with very small (< 1/10 expected FWHM) pixels.
- Measure FWHM of resulting profiles in all three directions

Generally, pixels should be 1/3 the expected FWHM or smaller, for any NM application. Combination of recon FOV and image matrix.
Spatial Resolution

Results for 1 cm source

<table>
<thead>
<tr>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trans FWHM</td>
<td>5.11617 (mm)</td>
</tr>
<tr>
<td>Trans FWTM</td>
<td>10.0089 (mm)</td>
</tr>
<tr>
<td>Axial FWHM</td>
<td>5.03312 (mm)</td>
</tr>
<tr>
<td>Axial FWTM</td>
<td>10.7149 (mm)</td>
</tr>
</tbody>
</table>

Results for 10 cm source

<table>
<thead>
<tr>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial FWHM</td>
<td>5.91931 (mm)</td>
</tr>
<tr>
<td>Radial FWTM</td>
<td>12.9862 (mm)</td>
</tr>
<tr>
<td>Tang. FWHM</td>
<td>6.72302 (mm)</td>
</tr>
<tr>
<td>Tang. FWTM</td>
<td>32.1492 (mm)</td>
</tr>
<tr>
<td>Axial FWHM</td>
<td>5.99356 (mm)</td>
</tr>
<tr>
<td>Axial FWTM</td>
<td>11.4418 (mm)</td>
</tr>
</tbody>
</table>
Which count-based PET metric is the best predictor of image quality?

0%  1. Total counts (prompts)
19% 2. True counts
0%  3. Random counts
71% 4. Noise-equivalent counts (NEC)
10% 5. Scatter fraction
Which count-based PET metric is the best predictor of image quality?

4. Noise-equivalent counts (NEC)

Cherry, Sorenson, and Phelps “Physics in Nuclear Medicine”
Why is photon attenuation a large effect in PET (compared to SPECT)?

0%  1. There’s no way to correct it.
81%  2. Both photons must be detected.
  3. The photon energy is high.
0%   4. The photon energy is low.
14%  5. Timing resolution is imperfect.
Why is photon attenuation a large effect in PET (compared to SPECT)??

2. Both photons must be detected

Cherry, Sorenson, and Phelps “Physics in Nuclear Medicine”
What factor is essential to localizing the PET annihilation locations?

1. The 511 keV $\gamma$ energy. (14%)
2. The F-18 half-life. (0%)
3. The anti-parallel $\gamma$ paths. (64%)
4. The scanner sensitivity. (18%)
5. The electron mass. (5%)
What factor is essential to localizing the PET annihilation locations?

3. The anti-parallel $\gamma$ paths

Cherry, Sorenson, and Phelps “Physics in Nuclear Medicine”
What effect does increasing iterations in ML-EM image reconstruction have?

0%  1. Increased patient comfort.
0%  2. Less CPU time.
5%  3. Lower resolution.
76% 4. Increased noise.
19% 5. Shorter scan time.
What effect does increasing iterations in ML-EM image reconstruction have?

4. Increased noise.

Cherry, Sorenson, and Phelps “Physics in Nuclear Medicine”
Which of the following is directly measured in current NEMA NU-2 PET tests?

1. Spatial resolution.  
2. Timing resolution.  
3. Energy resolution.  
4. PET-CT alignment accuracy.  
5. Image reconstruction speed.
Which of the following is directly measured in current NEMA NU-2 PET tests?

1. Spatial Resolution.

“Performance Measurements of Positron Emission Tomographs (PETs)”, NEMA-NU2 2012, National Electrical Manufacturer’s Association
Thank you.